Shell Structure Evolution and Effective In-Medium NN Interaction

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Shell Structure Evolution and Effective In-Medium NN Interaction

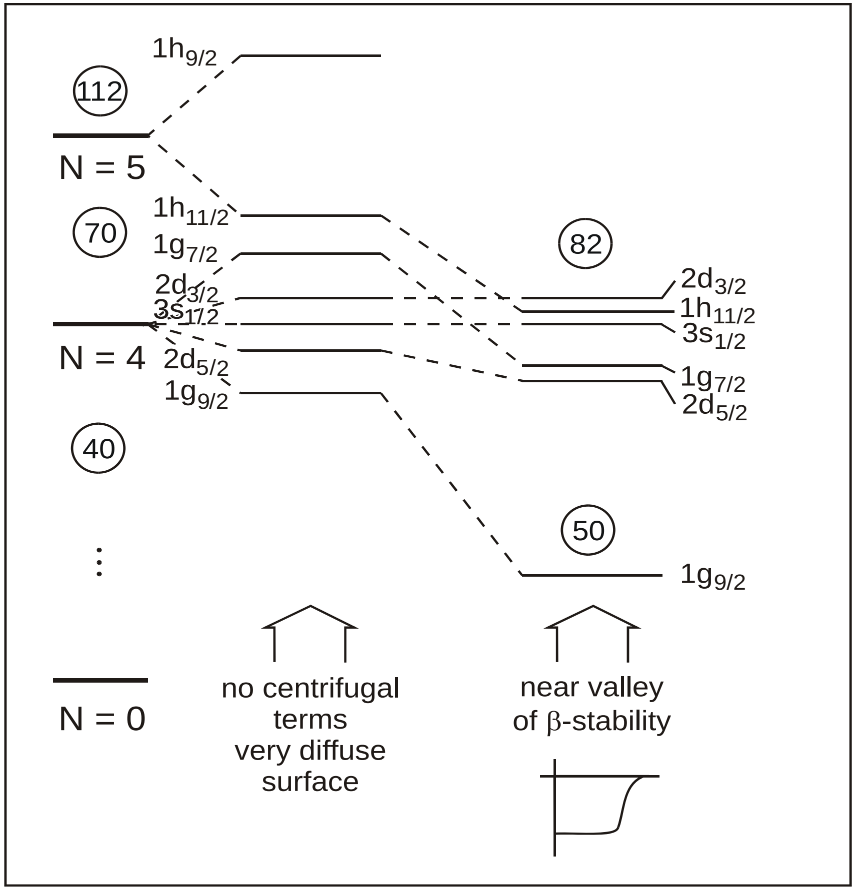
I. Shell model theory and effective interactions (30/09/2009)

II. Monopole term of the effective interaction and evolution of the shell structure (01/10/2009)

Monopole term of the effective interaction and evolution of the shell structure

1. Problem: shell structure changes when N/Z ratio changes
2. Decomposition of the effective interaction into monopole and multipole parts (occupation number formalism)
3. Monopole term: nuclear spherical mean field and effective single-particle energies (ESPE)
4. Multipole term: correlations
5. Mechanism for formation of intruder states
6. Evolution of the shell structure and nuclear effective interaction

Nuclear shell structure as a function of N/Z



Schematically !

Near the beta-stability region:

$$U(k) = \frac{m\omega^2 r_k^2}{2} + \alpha\left(\vec{l}\cdot\vec{l}\right) + \beta\left(\vec{l}\cdot\vec{s}\right)$$

M. Goeppert-Mayer, Phys. Rev. 75(1949)
O. Haxel et al, Phys.Rev. 75 (1949)

Very neutron-rich nuclei:

- Diffuse neutron density
- Uniform distribution of levels
- Quenching of the shell gaps

$$U(k) = \frac{m\omega^2 r_k^2}{2} + \beta\left(\vec{l}\cdot\vec{s}\right)$$

Recent review on shell evolution :
O. Sorlin, M.-G. Porquet, Prog. Nucl. Part. Phys. 61 (2008) 602

HF(B) theory with a Skyrme-type force
J.Dobaczewski et al, PRL72, 981 (1994); PRC50, 2860 (1994); PRC53, 2809 (1996)

Disappearance of magic numbers N=16,20,28 in very neutron-rich nuclei

$$S_{2N}(A,Z) = BE(A,Z) - BE(A-2,Z)$$

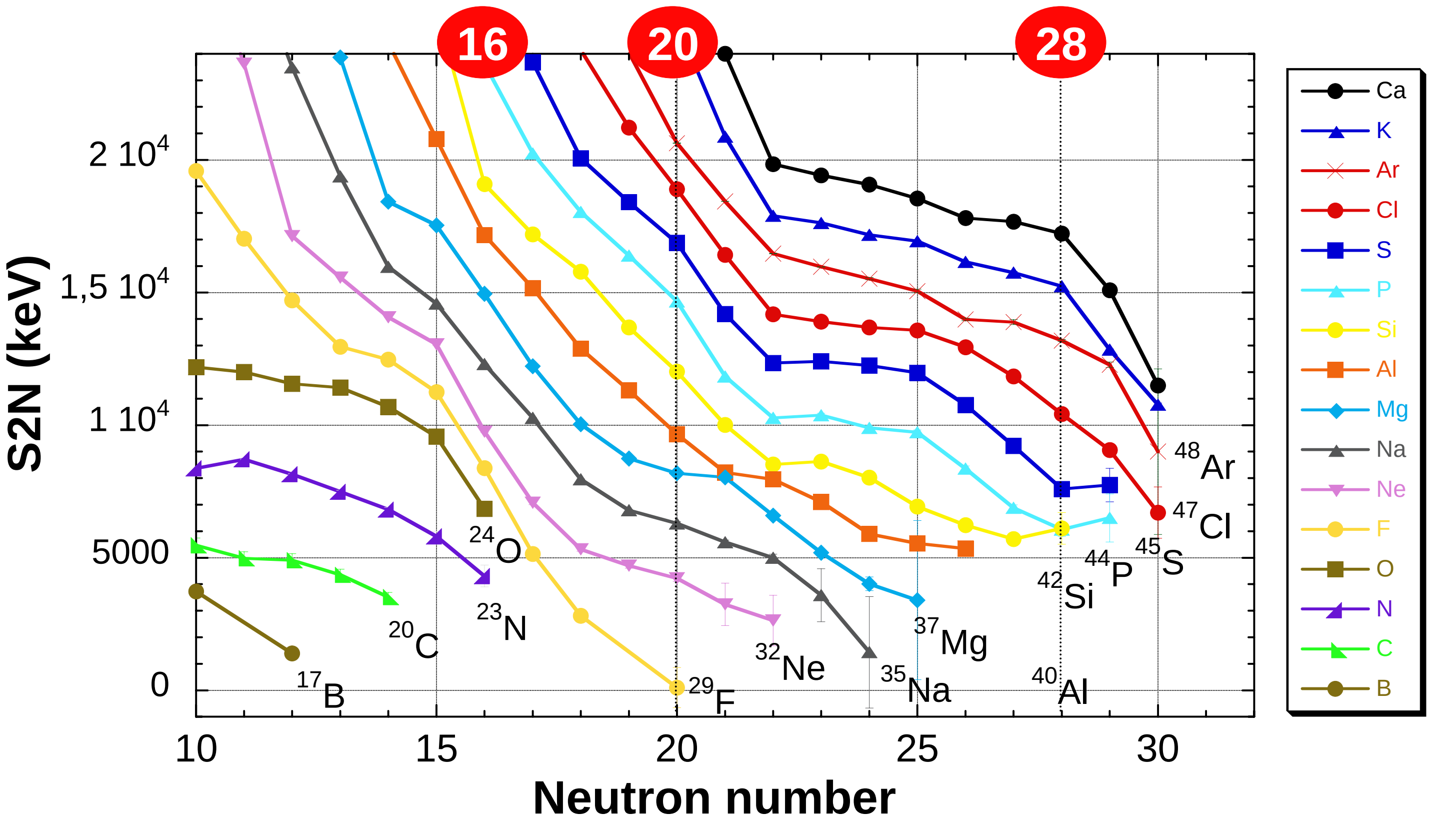


Figure taken from B.Jurado, H.Savajols

Example of a non-magic N=20 nucleus: Mg-isotopes (Z=12)

Energy (MeV)	$^{30}Mg_{18}$	$^{32}Mg_{20}$	$^{34}Mg_{22}$
$E(2^+)$	1.48	**0.89**	0.67
$E(4^+)$		**(2.3)**	2.13
$B(E2; 2^+ \rightarrow 0^+)$ (e2.fm4)	59(5)	**90(16)**	126(25)

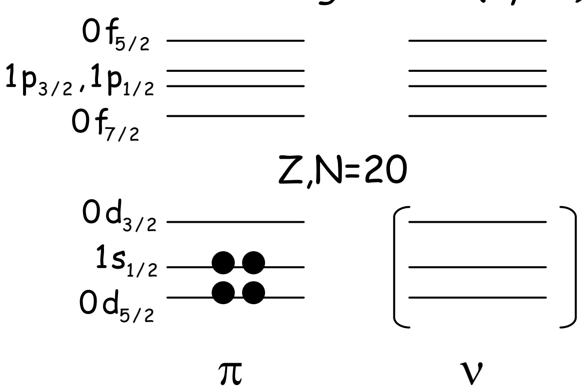
Deformed rotors !
β~0.5 - 0.6

Data on ^{32}Mg: Guillemaud-Muller et al, 1984; Motobayashi et al, 1995; Azaiez (1999)
Theory: E.Caurier et al, Rev. Mod. Phys. (2005)

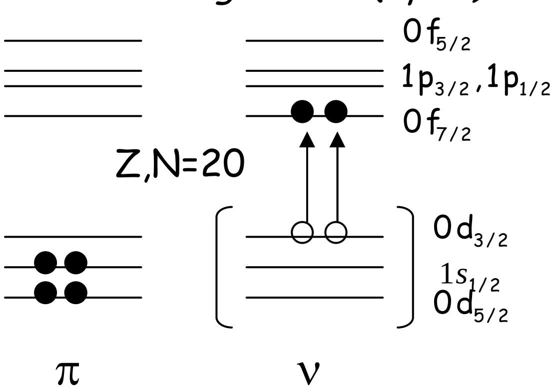
$^{32}Mg_{20}$ (shell model)

Energy (MeV)	Normal	Intruder
$E(0^+)$	0	0 (-1.1)
$E(2^+)$	1.69	**0.93**
$E(4^+)$	2.93	**2.33**
$B(E2; 2^+ \rightarrow 0^+)$ (e2.fm4)	36	**98**

Normal configuration (0p0h)



Intruder configuration (2p2h)



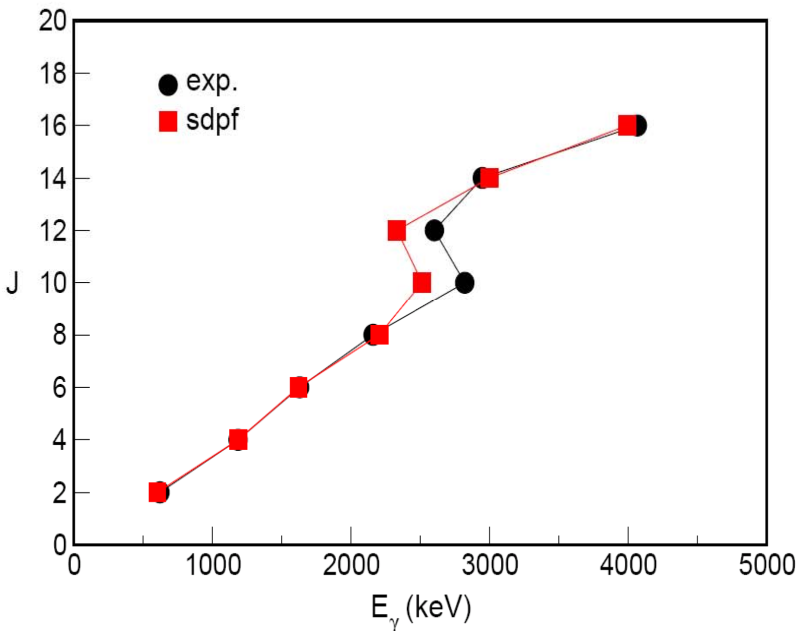
Superdeformation in $^{36}Ar_{18}$

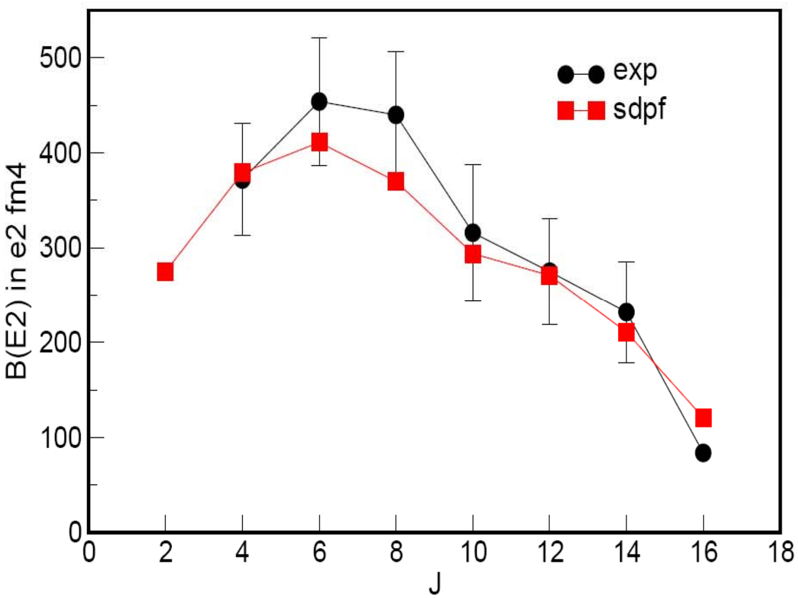
Intruder np-nh configurations can lead even to superdeformation !

$[sd]^{16}[pf]^{0}$ - 0p0h – spherical configuration

$[sd]^{12}[pf]^{4}$ - 4p4h – deformed configuration

E. Caurier et al, Phys.Rev.Lett. 95, 042502 (2005)





Shell-model calculations near shell closures

There is competition between normal and intruder configurations (particle-hole excitations to the next oscillator shell).

Why intruder configurations become favored ?

Monopole and **multipole** parts of the shell model Hamiltonian (second quantization formalism)

Occupation-number formalism (second quantization)

We introduce particle creation and annihilation operators :

$$a^+_\alpha |0\rangle = |\alpha\rangle, \quad \langle 0|(a^+_\alpha)^+ = \langle\alpha| = a_\alpha$$

$$\alpha \equiv (n_\alpha l_\alpha j_\alpha m_\alpha)$$

Satisfying fermionic (anti)commutation relations :

$$\{a^+_\alpha, a_\beta\} = a^+_\alpha a_\beta + a_\beta a^+_\alpha = \delta_{\alpha\beta}$$
$$\{a^+_\alpha, a^+_\beta\} = \{a_\alpha, a_\beta\} = 0$$

A-particle antisymmetric state :

$$a^+_{\alpha_A} a^+_{\alpha_{A-1}} \dots a^+_{\alpha_2} a^+_{\alpha_1} |0\rangle = |\alpha_1\alpha_2 \dots \alpha_{A-1}\alpha_A\rangle$$

(J) and (JT) coupled two-body states :

$$|\alpha\,\beta; J\,M\rangle \equiv |(n_\alpha l_\alpha j_\alpha)(n_\beta l_\beta j_\beta); J\,M\rangle = -\frac{1}{\sqrt{1+\delta_{\alpha\beta}}}\sum_{m_\alpha m_\beta}(j_\alpha m_\alpha j_\beta m_\beta|J\,M)\,a^+_\alpha a^+_\beta|0\rangle =$$
$$\equiv -\frac{1}{\sqrt{1+\delta_{\alpha\beta}}}\left[a^+_\alpha \times a^+_\beta\right]^J_M |0\rangle$$

$$|\alpha\beta; JMTT_z\rangle = -\frac{1}{\sqrt{1+\delta_{\alpha\beta}}}\left[a^+_{\alpha\frac{1}{2}} \times a^+_{\beta\frac{1}{2}}\right]^{JT}_{MT_z}|0\rangle$$

Operators in the occupation-number formalism

Symmetric one-body operator acting on a system of *A* identical fermions :

$$\hat{O} = \sum_{k=1}^{A} \hat{O}(\vec{r}_k)$$

$$\left\langle \alpha \middle| \hat{O} \middle| \beta \right\rangle = \int \phi_\alpha^*(\vec{r})\hat{O}(\vec{r})\phi_\beta(\vec{r})d\vec{r}$$

$\Longrightarrow$

$$\hat{O} = \sum_{\alpha\beta} \left\langle \alpha \middle| \hat{O} \middle| \beta \right\rangle a_\alpha^+ a_\beta$$

For example, the number operator:

$$\hat{N} = \sum_{\alpha\beta} \left\langle \alpha \middle| 1 \middle| \beta \right\rangle a_\alpha^+ a_\beta = \sum_\alpha a_\alpha^+ a_\alpha$$

Symmetric two-body operator acting on a system of *A* identical fermions:

$$\hat{T} = \sum_{j<k=1}^{A} \hat{O}(\vec{r}_j, \vec{r}_k)$$

$$\left\langle \alpha\,\beta \middle| \hat{T} \middle| \gamma\,\delta \right\rangle = \int \phi_\alpha^*(\vec{r}_1)\phi_\beta^*(\vec{r}_2)\hat{T}(\vec{r}_1,\vec{r}_2)\left(1 - \hat{P}_{12}\right)\phi_\gamma(\vec{r}_1)\phi_\delta(\vec{r}_2)d\vec{r}_1 d\vec{r}_2$$

$\Downarrow$

$$\hat{T} = \frac{1}{4}\sum_{\alpha\beta\gamma\delta} \left\langle \alpha\beta \middle| \hat{O} \middle| \gamma\delta \right\rangle a_\alpha^+ a_\beta^+ a_\delta a_\gamma$$

Nuclear many-body Hamiltonian in the occupation-number formalism

$$H = \sum_{k=1}^{A}\left(\frac{p_k^2}{2m_k} + U(k)\right) + \sum_{k<l=1}^{A} W(k,l) - \sum_{k=1}^{A} U(k) = H^{(0)} + V$$

$$\Downarrow$$

$$H = \underbrace{\sum_{\alpha} \varepsilon_{\alpha} a_{\alpha}^{+} a_{\alpha}}_{\text{One-body term}} + \underbrace{\frac{1}{4}\sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | V | \gamma\delta \rangle a_{\alpha}^{+} a_{\beta}^{+} a_{\delta} a_{\gamma}}_{\text{Two-body term}} = H^{(0)} + V$$

One-body term *Two-body term*

The residual interaction can be expressed using coupled TBME:

$$V = -\frac{1}{4}\sum_{\alpha,\beta,\gamma,\delta,J} \langle \alpha\beta \| V \| \gamma\delta \rangle_J \sqrt{(1+\delta_{\alpha\beta})(1+\delta_{\gamma\delta})} \left[\left[a_{\alpha}^{+} \times a_{\beta}^{+} \right]^J \times \left[\tilde{a}_{\gamma} \times \tilde{a}_{\delta} \right]^J \right]_0^{(0)}$$

$$\tilde{a}_{j_\alpha, m_\alpha} = (-1)^{j_\alpha + m_\alpha} a_{j_\alpha, -m_\alpha}$$

Transformation to a multipole representation

$$V = -\frac{1}{4}\sum_{\alpha,\beta,\gamma,\delta,J,T} V^{JT}_{\alpha\beta\gamma\delta}\left[\left[a^+_\alpha \times a^+_\beta\right]^{JT} \times \left[\tilde{a}_\gamma \times \tilde{a}_\delta\right]^{JT}\right]^{(0)}_0$$

Particle-particle representation

$\Downarrow$

$$V = \sum_{\alpha\le\beta,\gamma\le\delta,\lambda} w^{\lambda}_{\alpha\beta\gamma\delta}\left[\left[a^+_\alpha \times \tilde{a}_\beta\right]^{\lambda} \times \left[a^+_\gamma \times \tilde{a}_\delta\right]^{\lambda}\right]^{(0)}_0 \qquad \lambda = (JT)$$

Particle-hole representation

λ=(00),(01) : ***monopole part*** of the two-body interaction

$\lambda\neq$(00),(01) : ***multipole part*** of the two-body interaction

R.K.Bansal, J.B.French, Phys.Lett. 11, 145 (1964)

A.Poves, A.P.Zuker, Phys. Rep. 70, 235 (1981)

M. Dufour, A.P.Zuker, PRC54, 1641 (1996)

Monopole Hamiltonian : spherical mean field (~HF)

$$H_{mon} = \sum_{\alpha} \varepsilon_{\alpha} a_{\alpha}^{+} a_{\alpha} + \sum_{\alpha\le\beta,\gamma\le\delta,T} w_{\alpha\beta\gamma\delta}^{0T} \left[a_{\alpha}^{+} \times \tilde{a}_{\beta} \right]^{(0T)} \left[a_{\gamma}^{+} \times \tilde{a}_{\delta} \right]^{(0T)}$$

$$= \sum_{\alpha} \varepsilon_{\nu_\alpha} \hat{n}_{\nu_\alpha} + \sum_{\alpha} \varepsilon_{\pi_\alpha} \hat{n}_{\pi_\alpha} + \sum_{\alpha\beta} \hat{n}_{\nu_\alpha} \hat{n}_{\pi_\beta} \bar{V}_{\alpha\beta}^{\nu\pi}$$

$$+ \sum_{\alpha\le\beta} \frac{\hat{n}_{\nu_\alpha} \left(\hat{n}_{\nu_\beta} - \delta_{\alpha\beta} \right)}{1 + \delta_{\alpha\beta}} \bar{V}_{\alpha\beta}^{\nu\nu} + \sum_{\alpha\le\beta} \frac{\hat{n}_{\pi_\alpha} \left(\hat{n}_{\pi_\beta} - \delta_{\alpha\beta} \right)}{1 + \delta_{\alpha\beta}} \bar{V}_{\alpha\beta}^{\pi\pi}$$

$$\bar{V}_{\alpha\beta}^{\rho\rho'} = \frac{\sum_J \langle \alpha\beta | V | \alpha\beta \rangle_J (2J+1)}{\sum_J (2J+1)}$$

Centroids of the two-body interaction (angular momentum averaged matrix elements)

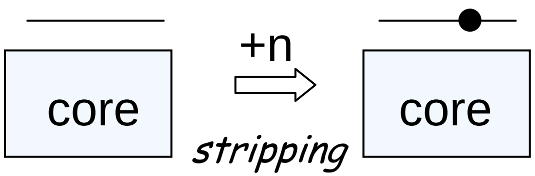
$(\rho, \rho' = \pi \text{ or } \nu)$

R.K.Bansal, J.B.French, Phys.Lett. 11, 145 (1964)
A.Poves, A.P.Zuker, Phys. Rep. 70, 235 (1981)
M. Dufour, A.P.Zuker, PRC54, 1641 (1996)

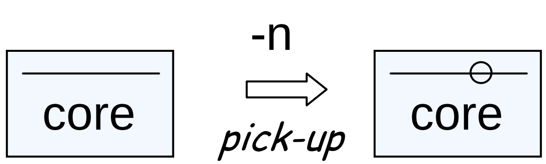
(Effective) Single-Particle Energies

Baranger, J.B.French (1964)

Closed shell nuclei



$$\tilde{\varepsilon}_{nlj} = \sum_f S_{fi}^{+j} E_{fi}^{+j}$$



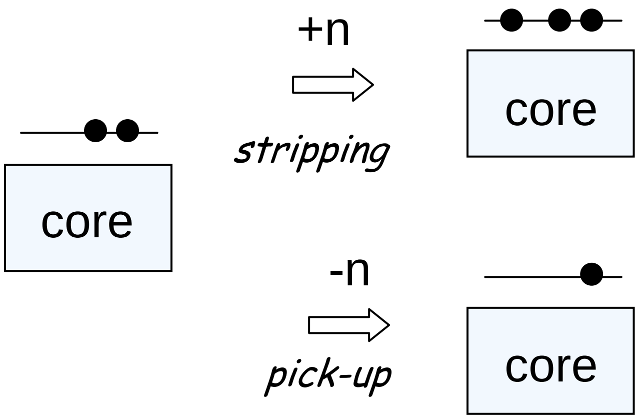
$$\tilde{\varepsilon}_{nlj} = \sum_f S_{fi}^{-j} E_{fi}^{-j}$$

$$E_{fi}^{+j} = E_{n+1,k_f J_f} - E_{n,k_i J_i}$$

$$E_{fi}^{-j} = E_{n,k_i J_i} - E_{n-1,k_f J_f}$$

Excitation energies of final states of 1n-transfer reactions

Nuclei with valence particles



$$\tilde{\varepsilon}_{nlj} = \sum_f S_{fi}^{+j} E_{fi}^{+j} + \sum_f S_{fi}^{-j} E_{fi}^{-j}$$

$$S_{fi}^{+j} = \frac{\left|\left\langle n+1, k_f J_f \middle\| a_{nlj}^{+} \middle\| n k_i J_i \right\rangle\right|^2}{(2j+1)(2J_i+1)}$$

$$S_{fi}^{-j} = \frac{\left|\left\langle n, k_i J_i \middle\| a_{nlj}^{+} \middle\| n-1, k_f J_f \right\rangle\right|^2}{(2j+1)(2J_i+1)}$$

Spectroscopic strengths (~ spectroscopic factors)

$$\sum_f S_{fi}^{+j} + \sum_f S_{fi}^{-j} = 1$$

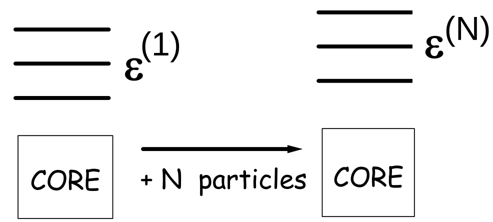
Effective single-particle energies (ESPE) and the monopole part of the Hamiltonian

$$\tilde{\varepsilon}_\alpha = \varepsilon_\alpha + \sum_\beta \bar{V}_{\alpha\beta} \langle n_\beta \rangle$$

ESPE

SPE with respect To the core

Average number of nucleons on j_β-orbital



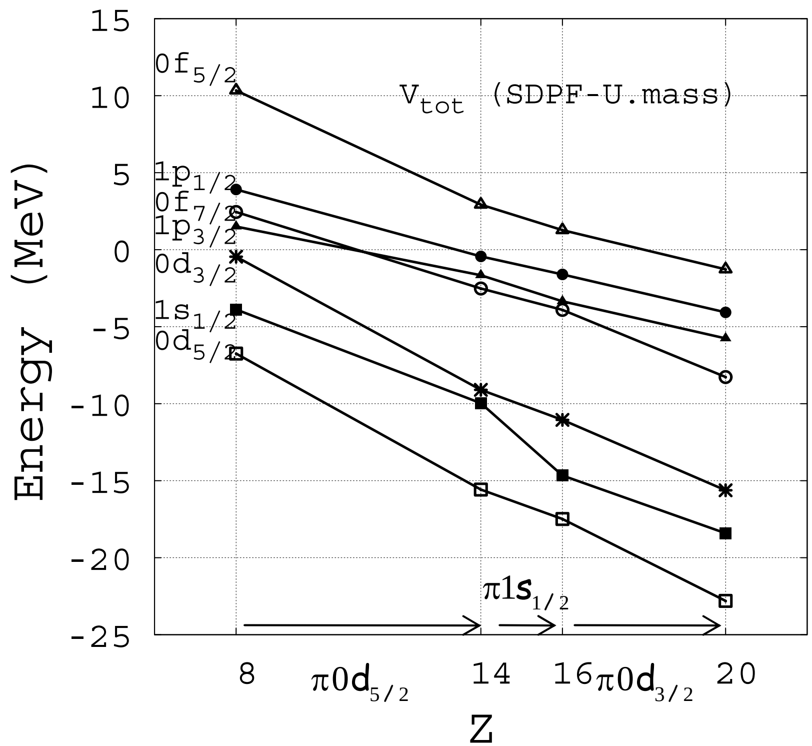
$$\bar{V}_{\alpha\beta} = \frac{\sum_J (2J+1) \langle \alpha\beta | V | \alpha\beta \rangle_J}{\sum_J (2J+1)}$$

Nucleon separation energies obtained from a monopole Hamiltonian

$$\langle H_M \rangle = \sum_\alpha \varepsilon_\alpha n_\alpha + \sum_{\alpha \le \beta} \frac{n_\alpha (n_\beta - \delta_{\alpha\beta})}{1 + \delta_{\alpha\beta}} \bar{V}_{\alpha\beta}$$

If normal filling is assumed, then applicable only for closed-shell nuclei plus or minus one nucleon

Neutron ESPE in N=20 isotones



Multipole Hamiltonian: particle-particle correlations

$$H_{Mult} = H - H_{mon} = \sum_{(ijkl)\lambda} w^{\lambda}_{ijkl} \left[a_i^{+} \times \tilde{a}_j \right]^{\lambda} \left[a_k^{+} \times \tilde{a}_l \right]^{\lambda}$$

$$\lambda \neq (00), (01)$$

Leading terms of the multipole Hamiltonian

particle-hole

λ=(20) : quadrupole-quadrupole
λ=(40) : hexadecapole-hexadecapole
λ=(11) : $(\sigma\tau)(\sigma\tau)$
...

particle-particle

(JT)=(01) : isovector pairing (L=0)
(JT)=(10) : isoscalar pairing (L=0)

M. Dufour, A.P.Zuker, Phys.Rev. C54, 1641 (1996)
E.Caurier et al, Rev.Mod.Phys.77 (2005) 427

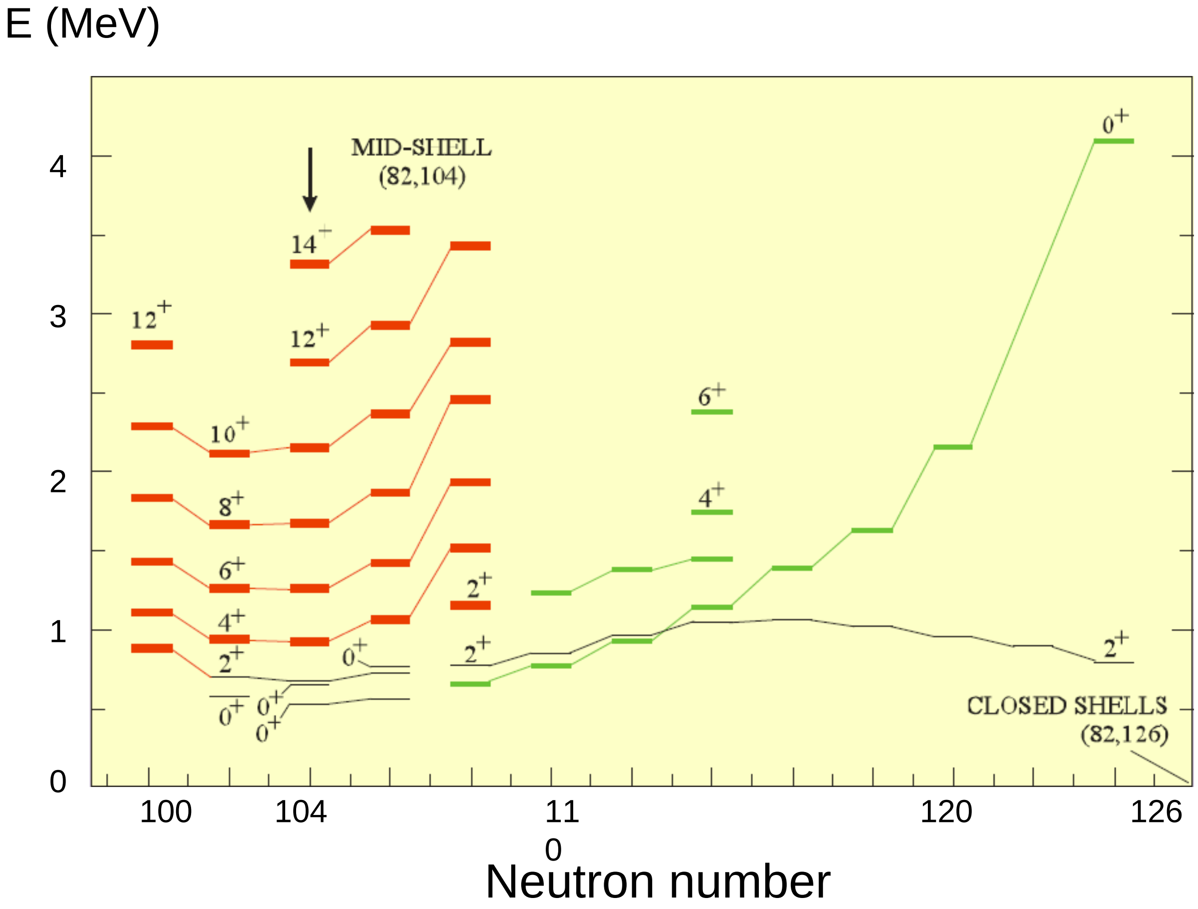
Monopole Hamiltonian ⇔ Multipole Hamltonian

What is the role of monopole and multipole components in the competition between normal and intruder configurations ?

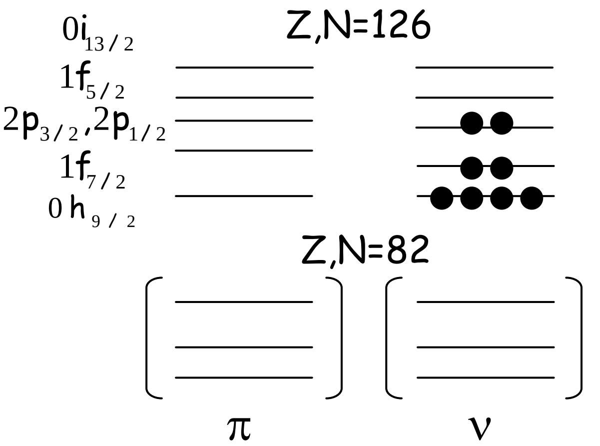
Monopole component of the nuclear **mean field** favoring normal filling and defines magic numbers

Multipole part of the interaction provides **correlation energy** of np-nh excitations (with quadrupole-quadrupole being dominant)

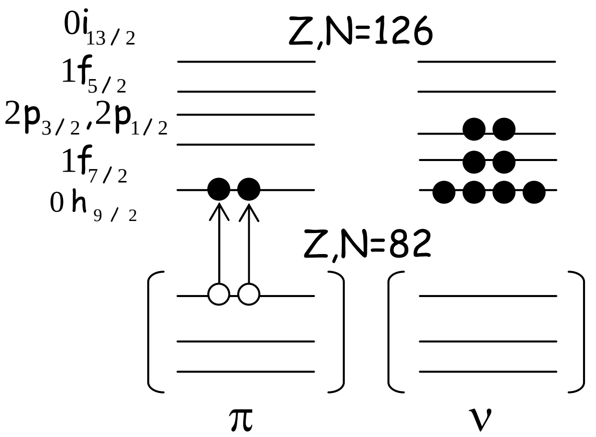
Example 1: Spherical and deformed configurations in Pb-isotopes (Z=82)



Normal configuration (0p0h)

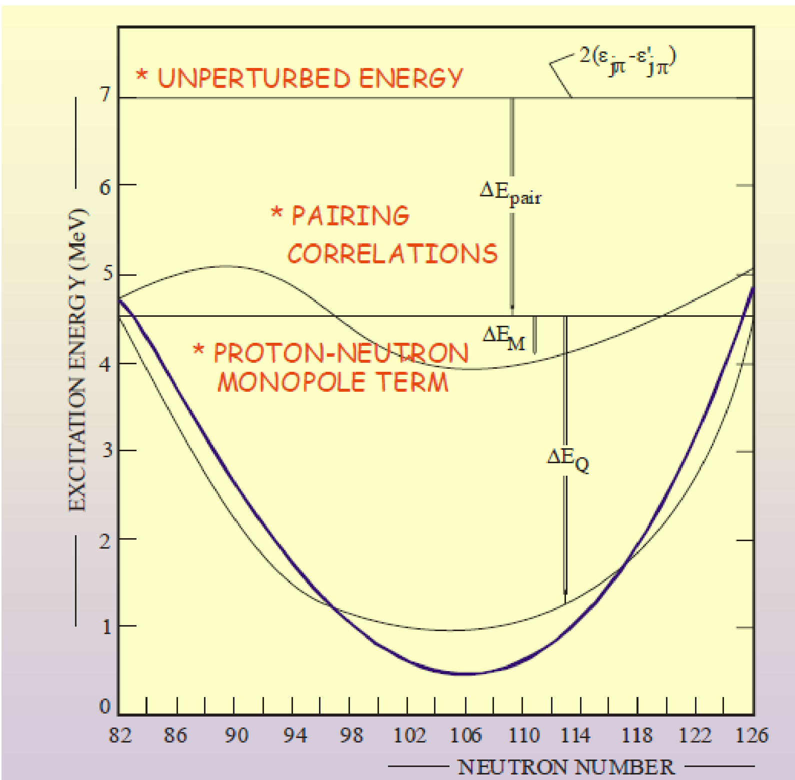


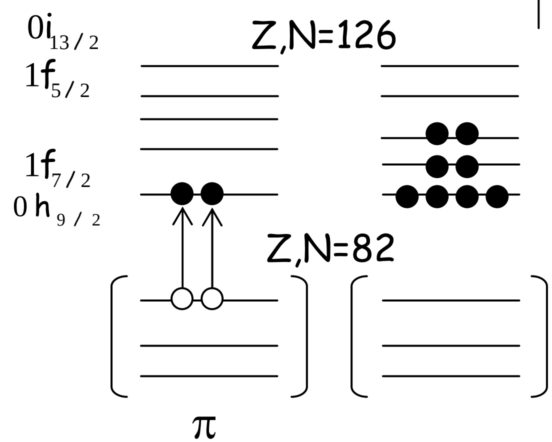
Intruder configuration (2p2h)



Schematic model for energy gain of intruder configurations: Pb-isotopes

K. Heyde et al, Nucl. Phys. A 47 (1985)





$$\Delta E_{mon} = 2\sum_{j_\nu} \langle n_{j_\nu} \rangle \left(\bar{V}_{j_\pi j_\nu} - \bar{V}_{j'_\pi j_\nu} \right)$$

$$\Delta E_{pair} = \Delta E_{pair}(p) + \Delta E_{pair}(h)$$

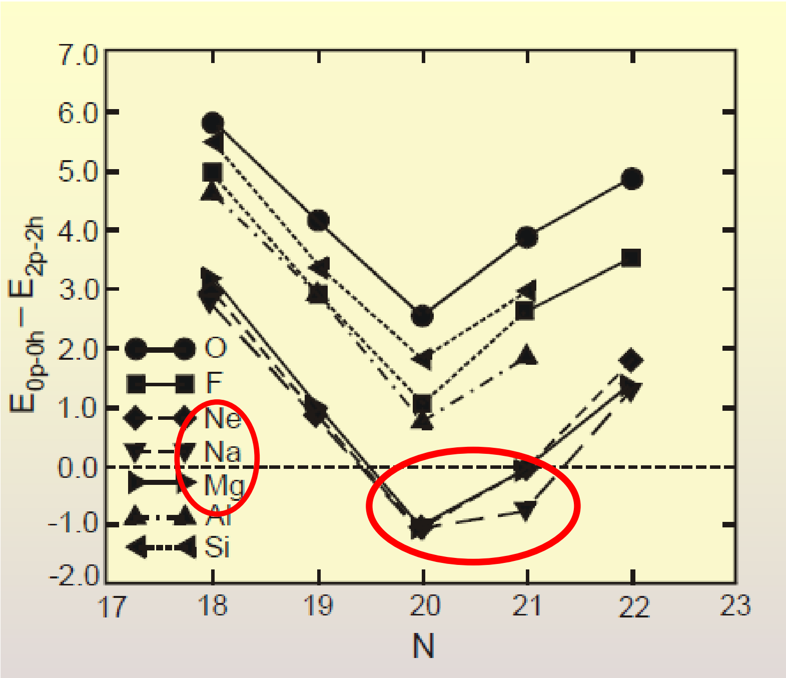
$$\Delta E_Q \approx 4\kappa \sqrt{\Omega_\pi - N_\pi} \cdot \sqrt{\Omega_\nu - N_\nu} \cdot N_\nu$$

Mechanism responsible for shape-coexistence

$$E(0^+_{int\,r}) \approx 2\left(\varepsilon_{j_\pi} - \varepsilon_{j'_\pi}\right) + \Delta E_{pair} + \Delta E_{mon} + \Delta E_Q$$

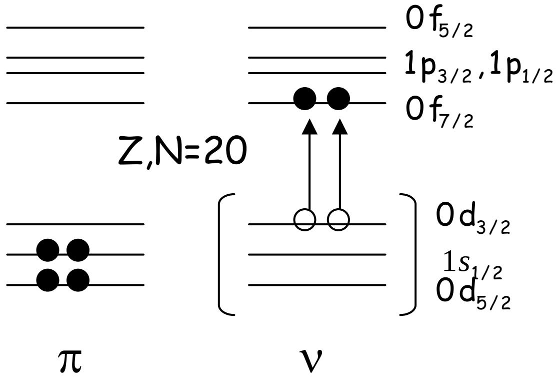
Example 2: N=20 shell gap and the island of inversion

Inversion of normal (0p0h) and intruder (2p2h) configurations

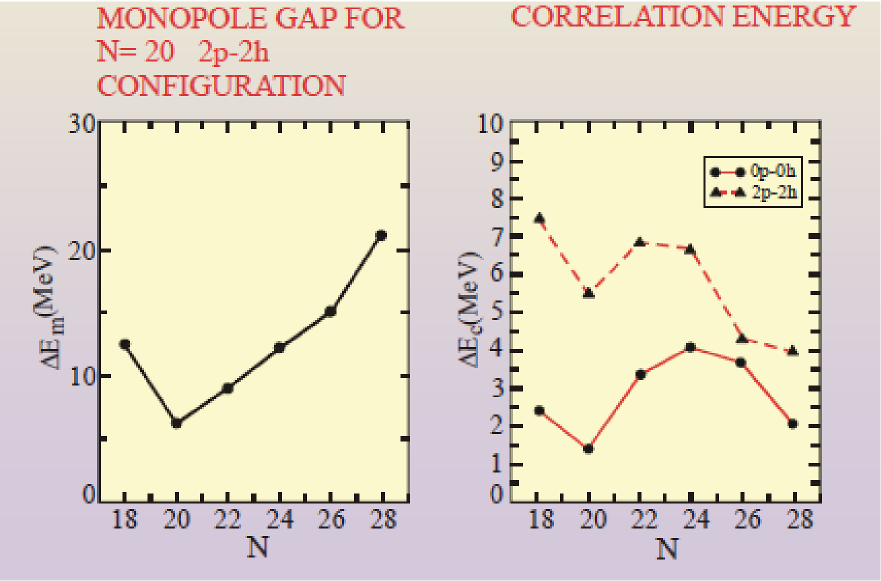


$^{32}Mg_{20}$

Intruder configuration (2p2h)



Mg-isotopes



General mechanism responsible for competition between normal and deformed configurations leading to shape coexistence, deformed and superdeformed structures in (almost) magic nuclei, appearance of 'islands of inversion'

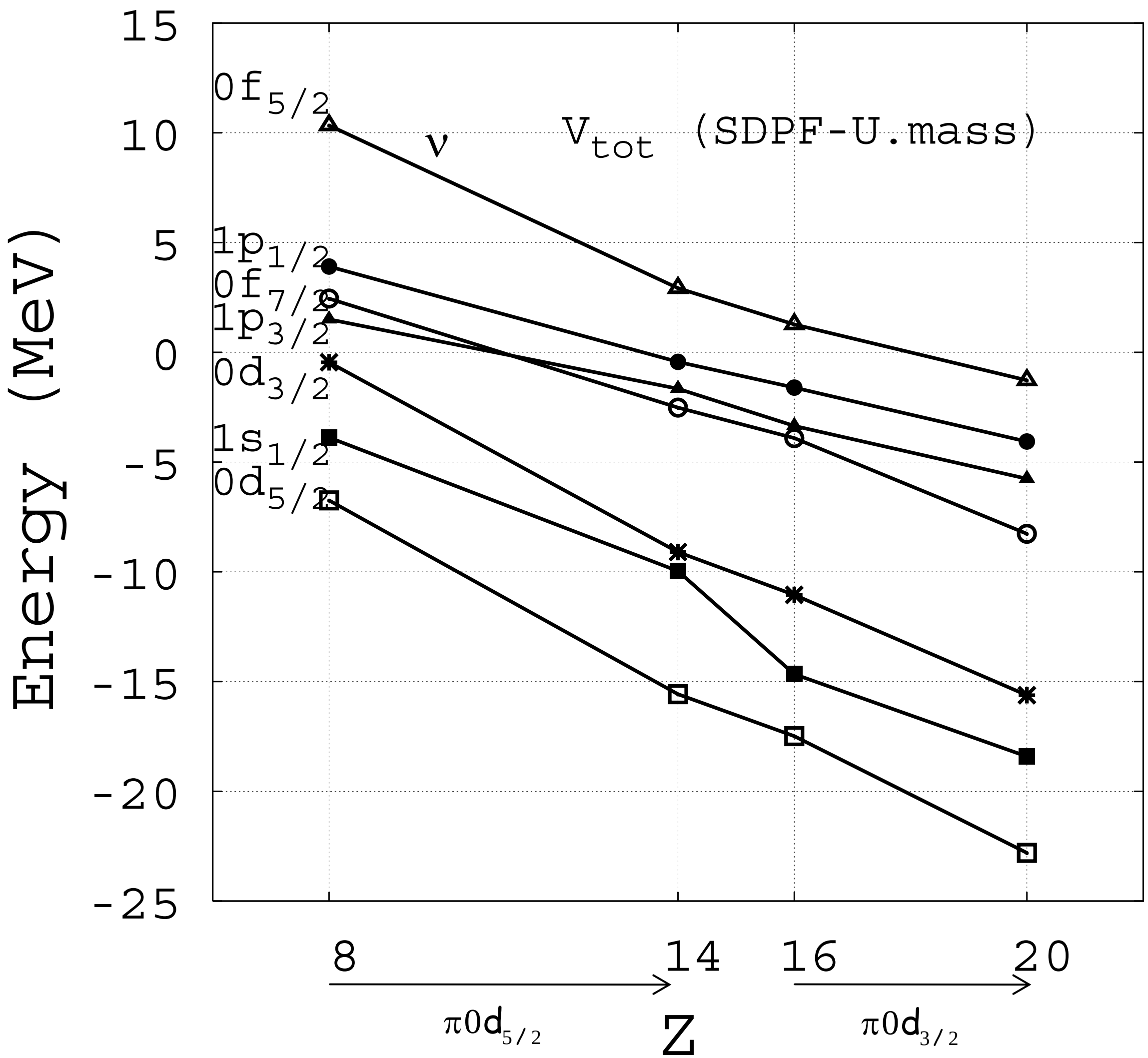
E.Caurier et al, Phys. Rev. C58 (1998) 2033

Monopole Hamiltonian

Can we compare with experiment the monopole part ?

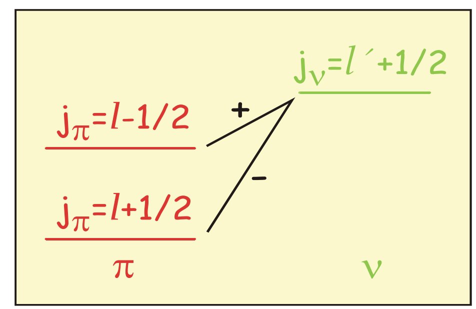
- nuclei with one particle or hole beyond the double-closed shell (~HF)
- nuclei adjacent to semi-magic ones (~HF+BCS)

Neutron ESPE in N=20 isotones from O to Ca (monopole field only !)



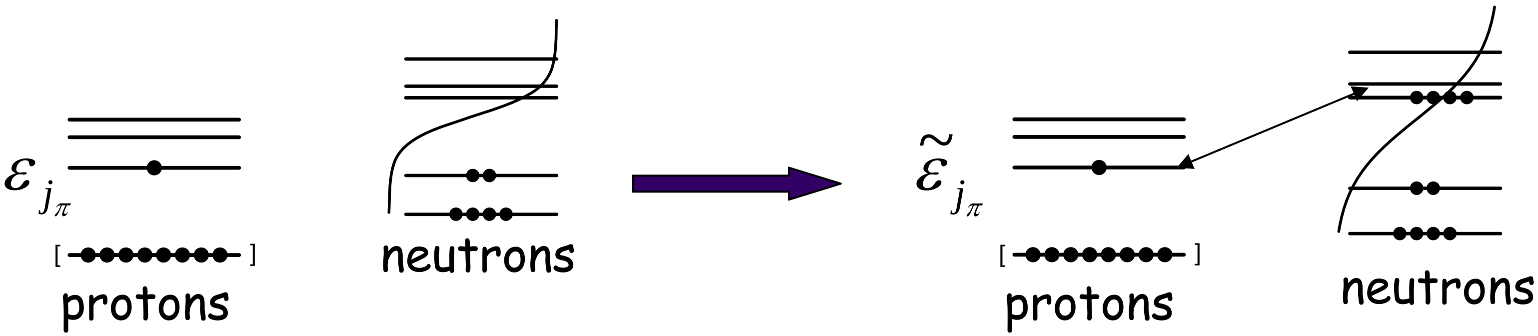
$$\tilde{\varepsilon}_\alpha = \varepsilon_\alpha + \sum_\beta \overline{V}_{\alpha\beta} n_\beta$$

$$\overline{V}_{\alpha\beta} = \frac{\sum_J (2J+1)\langle \alpha\beta | V | \alpha\beta \rangle_J}{\sum_J (2J+1)}$$

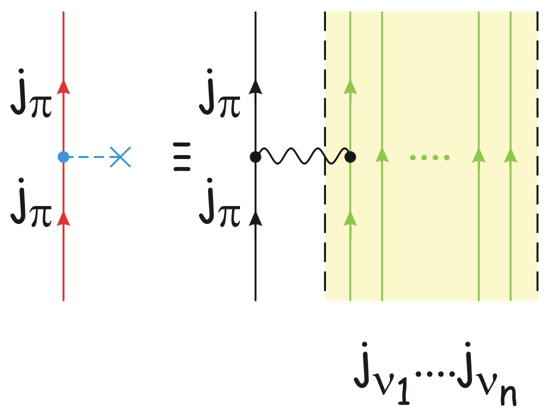


Effective interaction in the (sdpf) shell-model space:
F.Nowacki, A.Poves, Phys. Rev. C89, 1266 (2009)

Nuclei adjacent to semi-magic ones in 'HF+BCS' approximation



$$H = H_{mon} + H_{pair}$$



occupation probabilities (from BCS)

$$\tilde{\varepsilon}_{j_\pi} = \varepsilon_{j_\pi} + \sum_{j_\nu} \overline{V}_{j_\pi j_\nu} (2j_\nu + 1) v^2_{j_\nu}$$

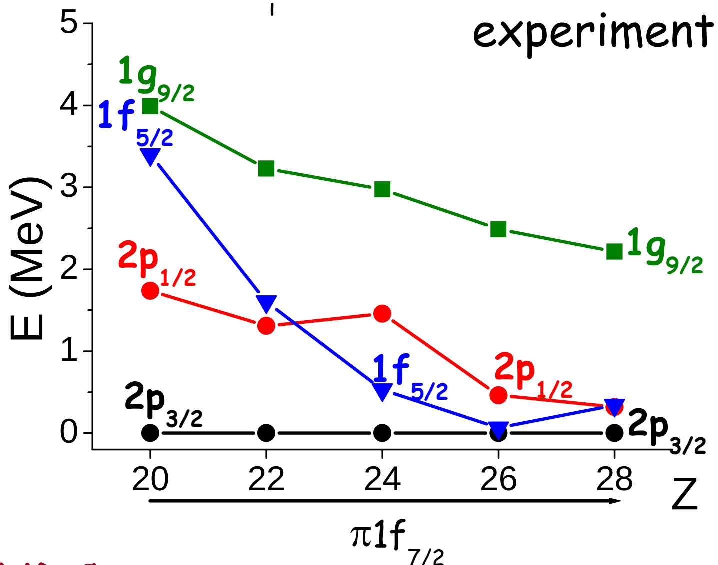
average interaction matrix element

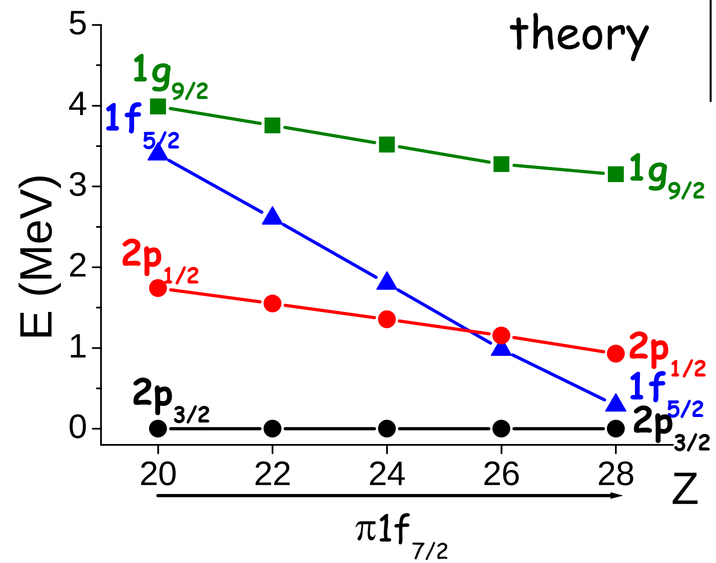
$$\overline{V}_{j_\pi j_\nu} = \frac{\sum_J \langle j_\pi j_\nu | V | j_\pi j_\nu \rangle_J (2J+1)}{\sum_J (2J+1)}$$

A.L.Goodman, NPA267, 1 (1977)
R.A.Sorensen,NPA420, 221 (1984)
K.Heyde et al,NPA466, 189 (1987)
A.P.Zuker, NPA576, 65 (1994)

Monopole shift in N=29 Isotones from Ca to Ni

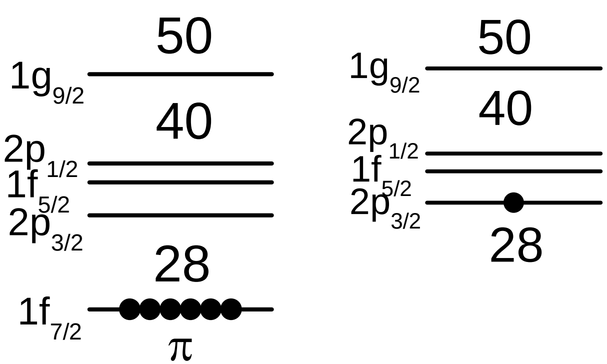
$$\tilde{\varepsilon}(nlj) = \sum_i S^i_{nlj} E^i_{nlj}$$





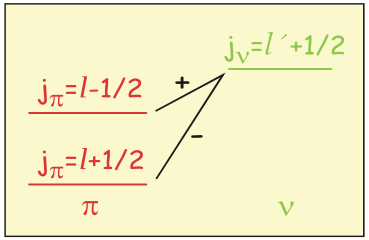
NNDC

Microscopic effective interaction (M. Hjorth-Jensen et al) modified

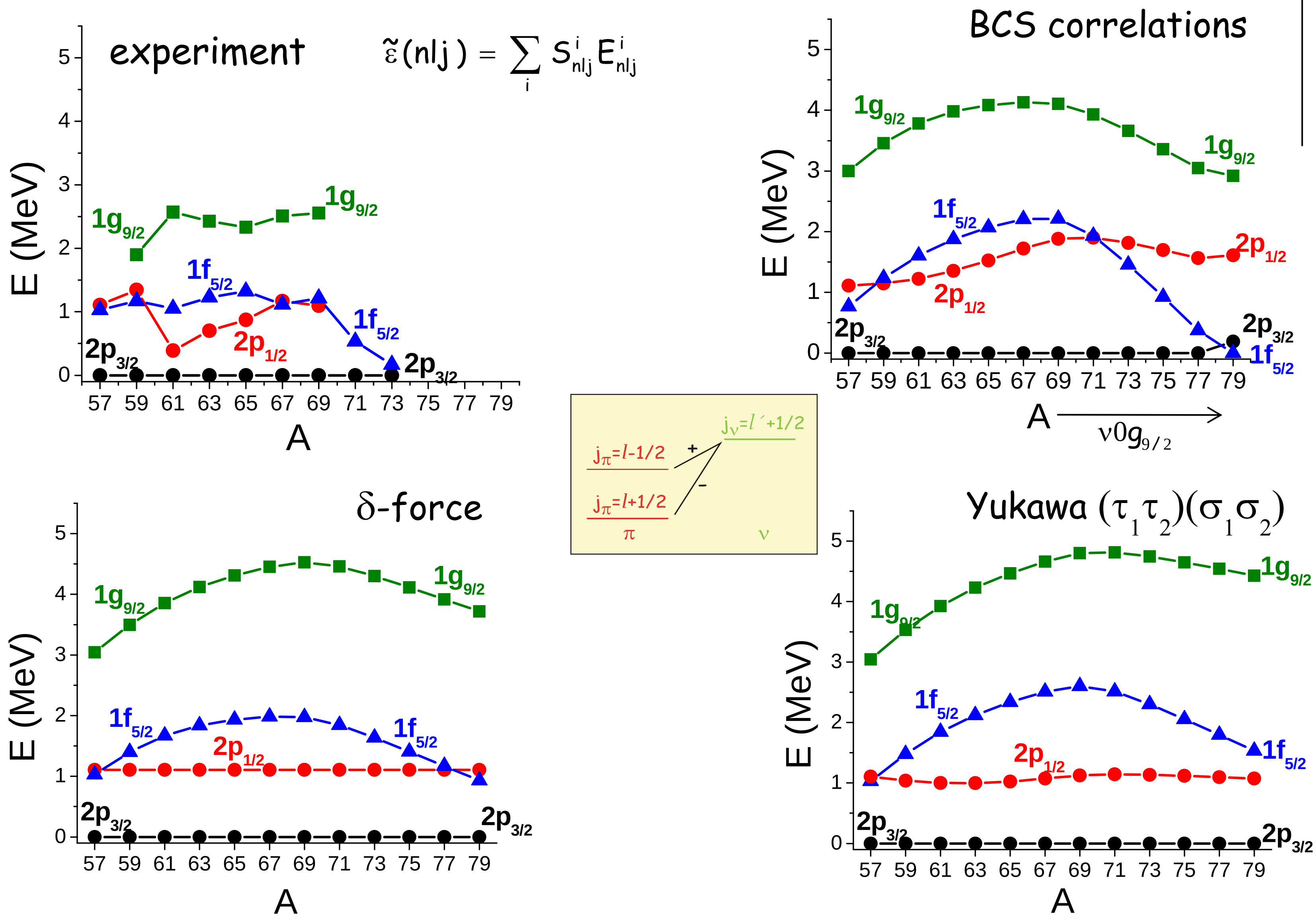


$$\tilde{\varepsilon}_{j_\pi} = \varepsilon_{j_\pi} + \sum_{j_\nu} \overline{V}_{j_\pi j_\nu} (2j_\nu + 1) v^2_{j_\nu}$$

$$\nu 1f_{5/2}, \nu 1g_{9/2} \Leftrightarrow \pi 1f_{7/2}$$

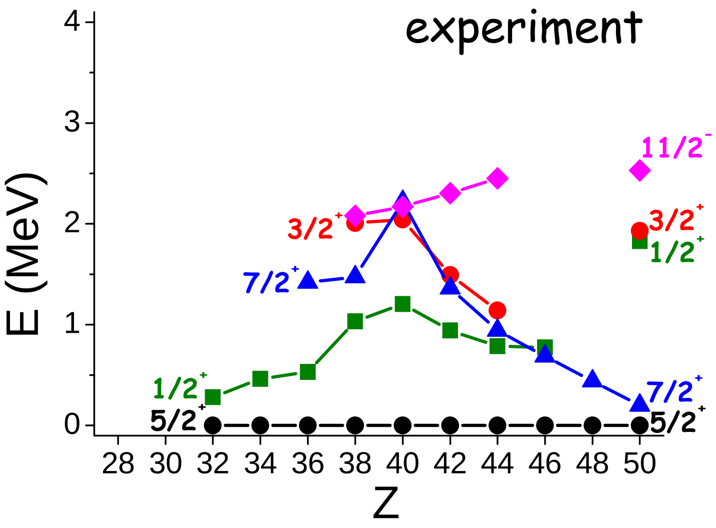


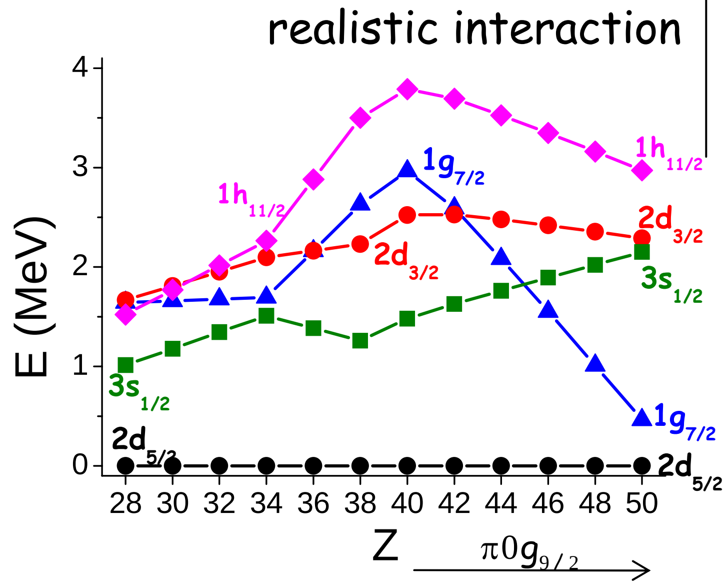
Monopole Shift in Neutron-Rich Cu-Isotopes



N.Smirnova et al, PRC69, 044306 (2004)

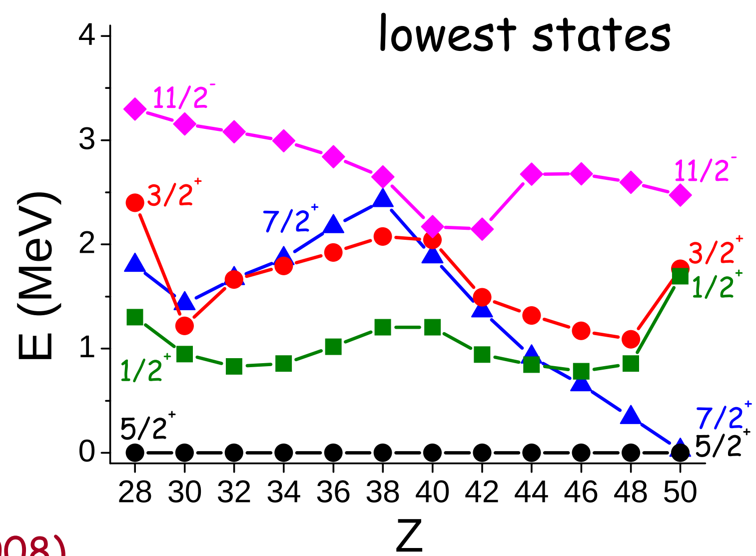
Monopole Shift in N=51 Isotones





NNDC
J.S.Thomas et al, PRC71, 021302 (2005)

Realistic interaction in the $\pi(2p_{1/2}1g_{9/2})$ $\nu(2d3s1g_{7/2}1h_{11/2})$ shell-model space M. Hjorth-Jensen et al (from CD-Bonn, 2001)



Existing analysis: H. Grawe et al, EPJA25 (2005)
O.Sorlin, M.-G.Porquet, Prog.Part.Nucl.Phys. 61 (2008)

Can we understand and describe evolution of the shell structure in terms of the interaction ?

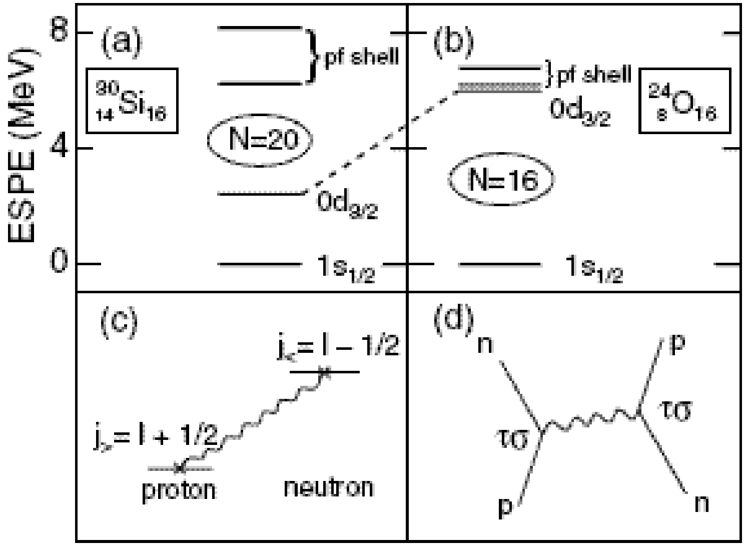


Figure taken from T.Otsuka et al, Phys.Rev.Lett.87 (2001) 082502

$$V_{\sigma\tau}(1,2) = f_{\sigma\tau}(r)\left(\vec{\sigma}_1\cdot\vec{\sigma}_2\right)\left(\vec{\tau}_1\cdot\vec{\tau}_2\right)$$

$$\bar{V}^{T}_{jj'} = \frac{\sum_J \langle jj'|V|jj'\rangle_{JT}(2J+1)}{\sum_J (2J+1)}$$

T.Otsuka et al, Phys.Rev.Lett.95 (2005)

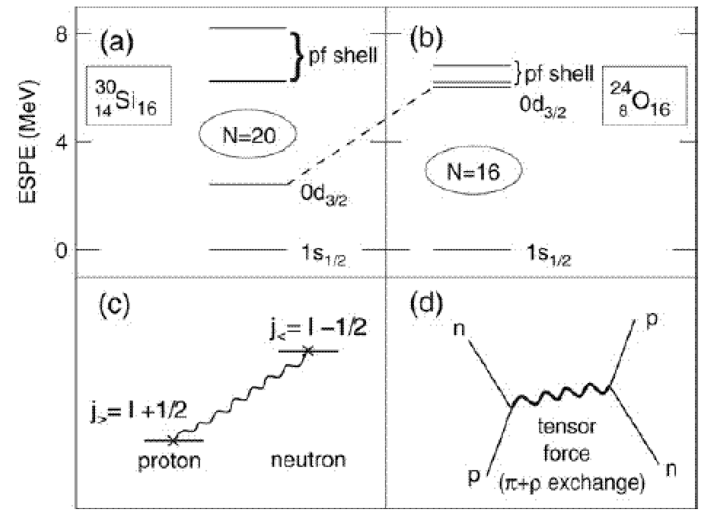


Figure taken from T.Otsuka, D. Abe, Prog. Part. Nucl. Phys. 59 (2007) 425

$$V_t(1,2) = f_t(r)\left(\left[\vec{\sigma}_1\otimes\vec{\sigma}_2\right]^{(2)}\cdot Y^{(2)}\right)\left(\vec{\tau}_1\cdot\vec{\tau}_2\right)$$

The most general form of the two nucleon potential compatible with symmetry principles

$$V(1,2) = V_c(1,2) + V_T(1,2) + V_{LS}(1,2)$$

$$V_c(1,2) = A(r) + C(r)(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + D(r)(\vec{\tau}_1 \cdot \vec{\tau}_2) + F(r)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2)$$

$$V_T(1,2) = V_T(r)\frac{3(\vec{r} \cdot \vec{\sigma}_1)(\vec{r} \cdot \vec{\sigma}_2) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)}{r^2}(1 + a\vec{\tau}_1 \cdot \vec{\tau}_2)$$

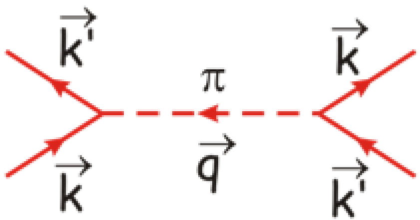
$$V_{LS}(1,2) = V_{LS}(r)(\vec{L} \cdot \vec{S})(1 + b\vec{\tau}_1 \cdot \vec{\tau}_2)$$

These terms naturally appear in meson exchange models

ONE-PION EXCHANGE - IMPORTANT PART OF NN INTERACTION

- ELASTIC SCATTERING IN MOMENTUM SPACE

$$V^{\pi NN}(\vec{q} = \vec{k}' - \vec{k}) = \frac{g_\pi^2}{4M^2} \frac{(\vec{\sigma}_i . \vec{q})(\vec{\sigma}_j . \vec{q})}{\vec{q}^2 + m_\pi^2}$$

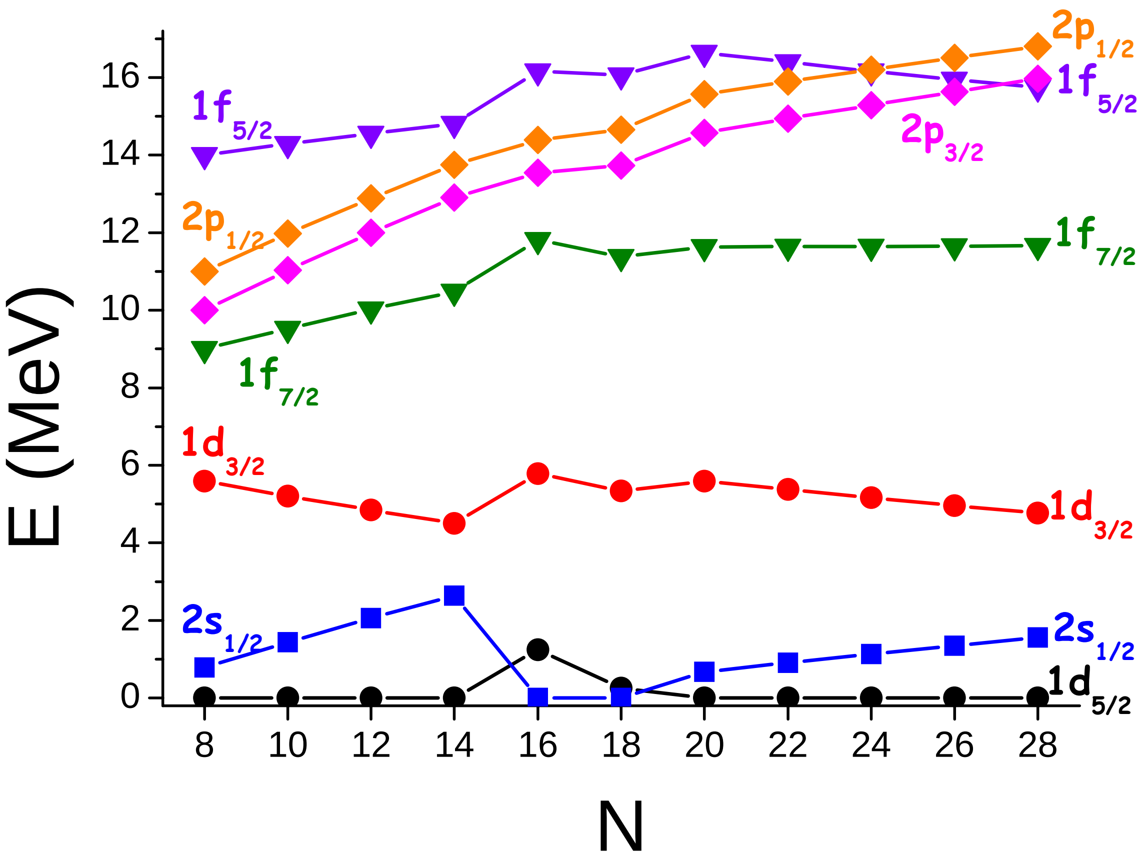


- POTENTIAL (FOURIER TRANSFORM) IN COORDINATE SPACE

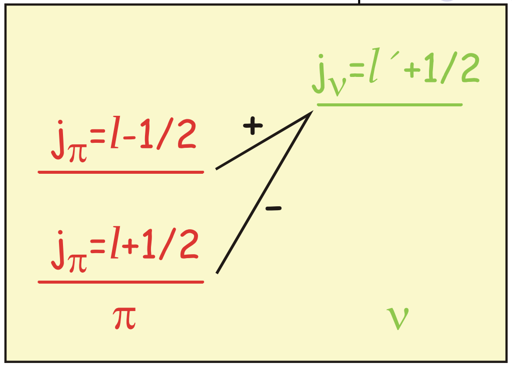
$$V_\pi^{OPEP} = \frac{g_\pi^2}{4M^2} \frac{1}{3} m_\pi \, \vec{\tau}_i . \vec{\tau}_j \left\{ \vec{\sigma}_i . \vec{\sigma}_j + \left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2}\right) \underbrace{(3\vec{\sigma}_i . \hat{r} \;\; \vec{\sigma}_j . \hat{r} - \vec{\sigma}_i . \vec{\sigma}_j)}_{\text{Tensor part}} \right\} \frac{e^{-\mu r}}{\mu r}$$

Effects of the Spin-Isospin Central Force

$$V(1,2) = V_{Yuk}(r)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2)$$



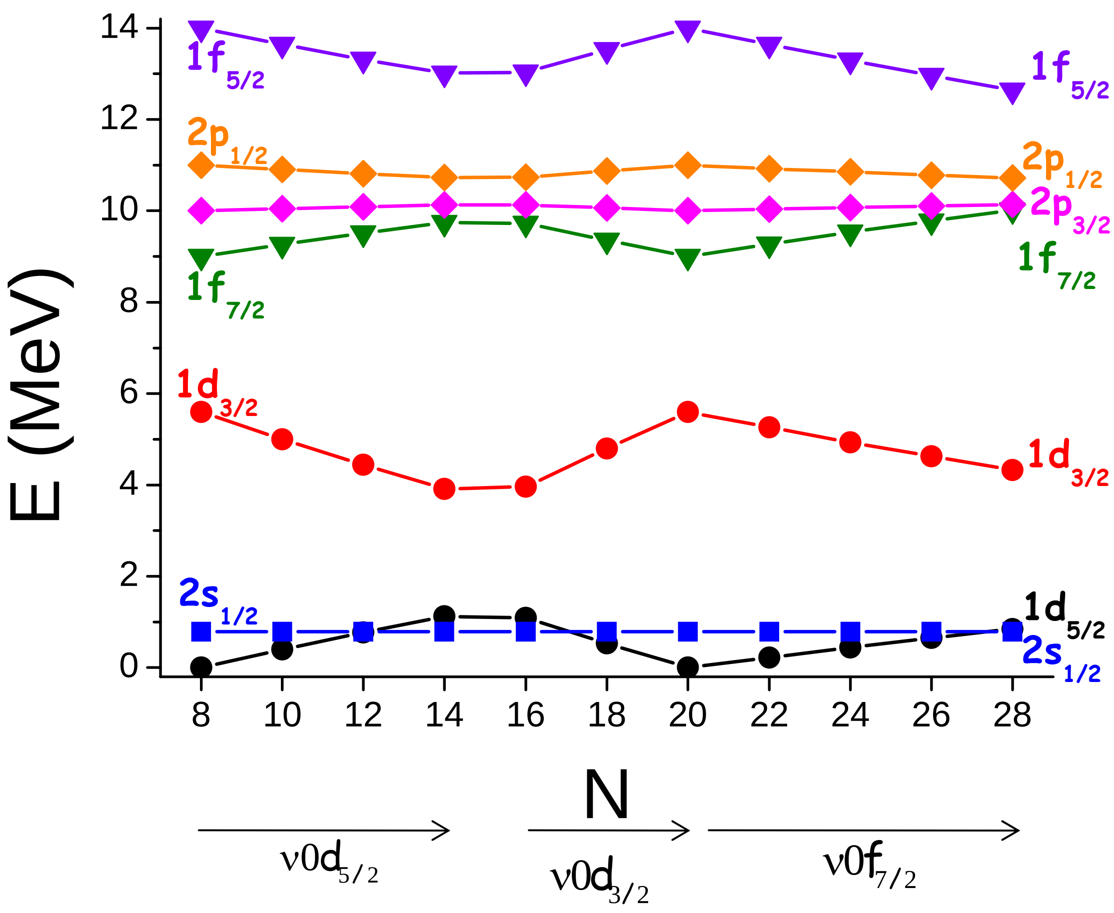
(sdpf)-shell model space

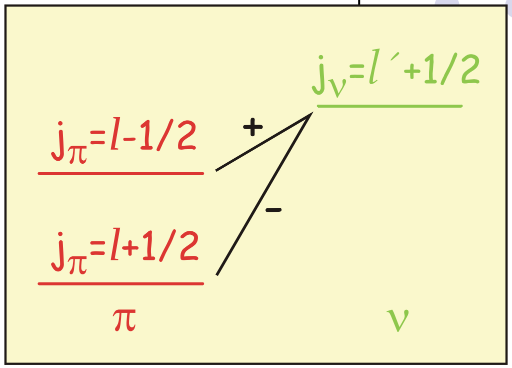


Generic property ?

Effects of the Tensor Force

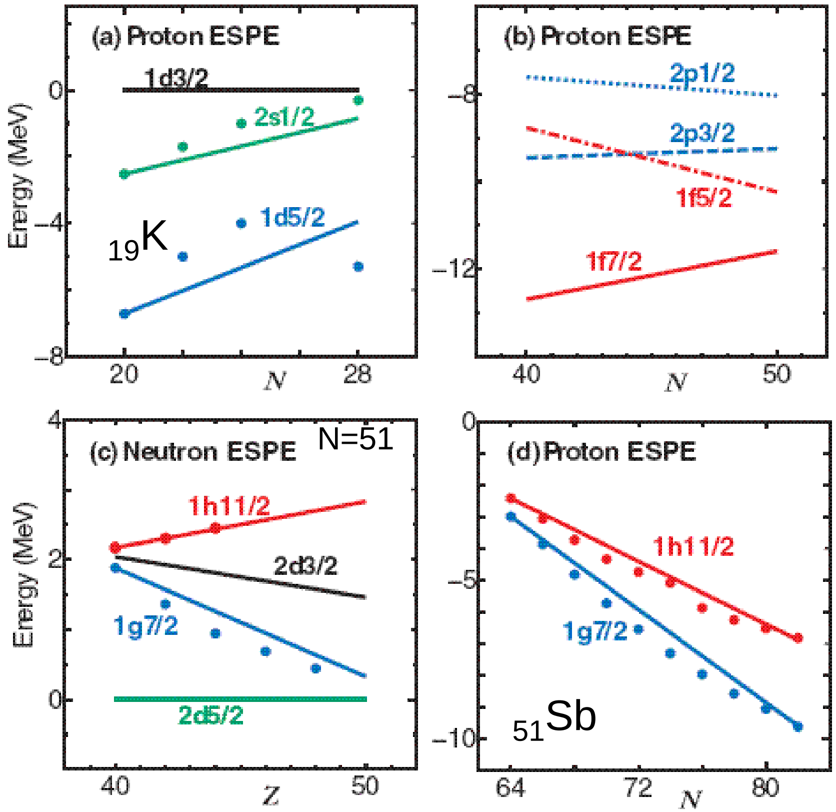
$$V_T(1,2) = V_{Yuk}(r)\left(\frac{3(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})}{|\vec{r}_1-\vec{r}_2|^2} - (\vec{\sigma}_1\cdot\vec{\sigma}_2)\right)(\vec{\tau}_1\cdot\vec{\tau}_2)$$





Generic property

Tensor force : how important it is ?



Tensor force from the (π+ρ) exchange model
with cut-off at 0.8 fm

T.Otsuka et al, PRL95 (2005)

$$(2j_> + 1)\bar{V}^T_{j_>,j'} + (2j_< + 1)\bar{V}^T_{j_<,j'} = 0$$

N.Smirnova et al, AIP Proc. (2006)

$$V_t(1,2) = f_t(r)\left(\left[\vec{\sigma}_1 \otimes \vec{\sigma}_2\right]^{(2)} \cdot Y^{(2)}\right)\left(\vec{\tau}_1 \cdot \vec{\tau}_2\right)$$

Tensor term in other models :

T. Otsuka et al, PRL 97 (2006) 162501
J. Dobaczewski, nucl-th/0604043
B.A.Brown et al, PRC74 (2006) 061303
M.Grasso et al, PRC76 (2007) 044319
T. Lesinski et al, PRC76 (2007) 014312
S.Sugimoto et al, PRC76 (2007) 054310
M. Zalewski et al, PRC77 (2008) 024316
....

Generic properties are well described, but how different terms of the bare NN-interaction are renormalized in nuclear medium ?
How can we disentangle quantitatively different terms of the **effective** interaction ?

Spin-tensor decomposition of the two-body interaction

$$S_1^{(0)}=1, S_2^{(0)}=\left[\vec{\sigma}_1\times\vec{\sigma}_2\right]^{(0)}$$

Scalar

$$S_3^{(1)}=\left(\vec{\sigma}_1+\vec{\sigma}_2\right), S_4^{(1)}=\left[\vec{\sigma}_1\times\vec{\sigma}_2\right]^{(1)}, S_5^{(1)}=\left(\vec{\sigma}_1-\vec{\sigma}_2\right)$$

Vector

$$S_6^{(2)}=\left[\vec{\sigma}_1\times\vec{\sigma}_2\right]^{(2)}$$

Tensor

$$V_t(1,2)=f_t(r)\left(\left[\vec{\sigma}_1\times\vec{\sigma}_2\right]^{(2)}\cdot Y^{(2)}\right)$$

$$V(1,2)=\sum_{k=0,1,2}\left(S^{(k)}\cdot Q^{(k)}\right)=\sum_{k=0,1,2}V^{(k)}$$

$$\left\langle(\alpha\beta):LS,JMTM_T\left|V^{(k)}\right|(\gamma\delta):L'S',JMTM_T\right\rangle=(2k+1)(-1)^J\begin{Bmatrix}L & S & J\\ S' & L' & k\end{Bmatrix}\sum_{J'}(-1)^{J'}(2J'+1)\begin{Bmatrix}L & S & J'\\ S' & L' & k\end{Bmatrix}$$
$$\times\left\langle(\alpha\beta):LS,J'MTM_T\left|V(1,2)\right|(\gamma\delta):L'S',J'MTM_T\right\rangle$$

Central (Triplet-even, Triplet -odd, Singlet-even, Singlet-odd)
Vector (LS and ALS)
Tensor (even and odd)

} From L,S-selection rules

J.P.Elliott et al, NPA121 (1968) 241;
M.W.Kirson, PLB47 (1973) 110; K.Klingenbeck et al, PRC15 (1977) 1483
E.Osnes, D. Strottman, Phys. Rev. C 45 (1992) 662

Application to the monopole Hamiltonian

$$V = \sum_{k=0,1,2} (Q_k \cdot S_k) = \sum_{k=0,1,2} V^{(k)}$$

$\longrightarrow$

$$\bar{V}_{jj'} = \sum_{k=0,1,2} \bar{V}^{(k)}_{jj'}$$

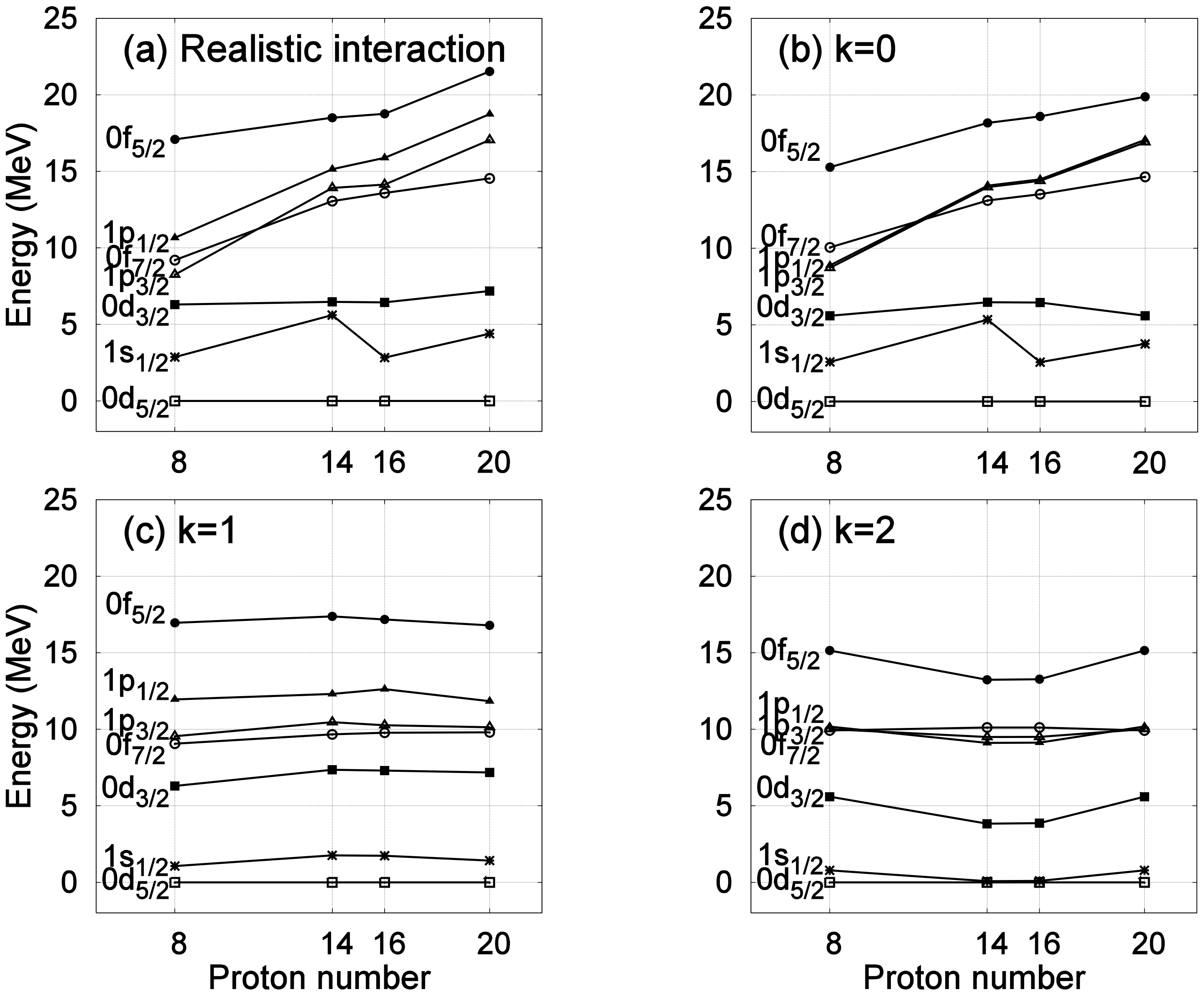
$$\bar{V}^{T}_{jj'} = \frac{\sum_J \langle jj' | V | jj' \rangle_{JT} (2J+1)}{\sum_J (2J+1)}$$

The effect of different rank terms is totally additive, i.e. it provides a very convenient way for comparison

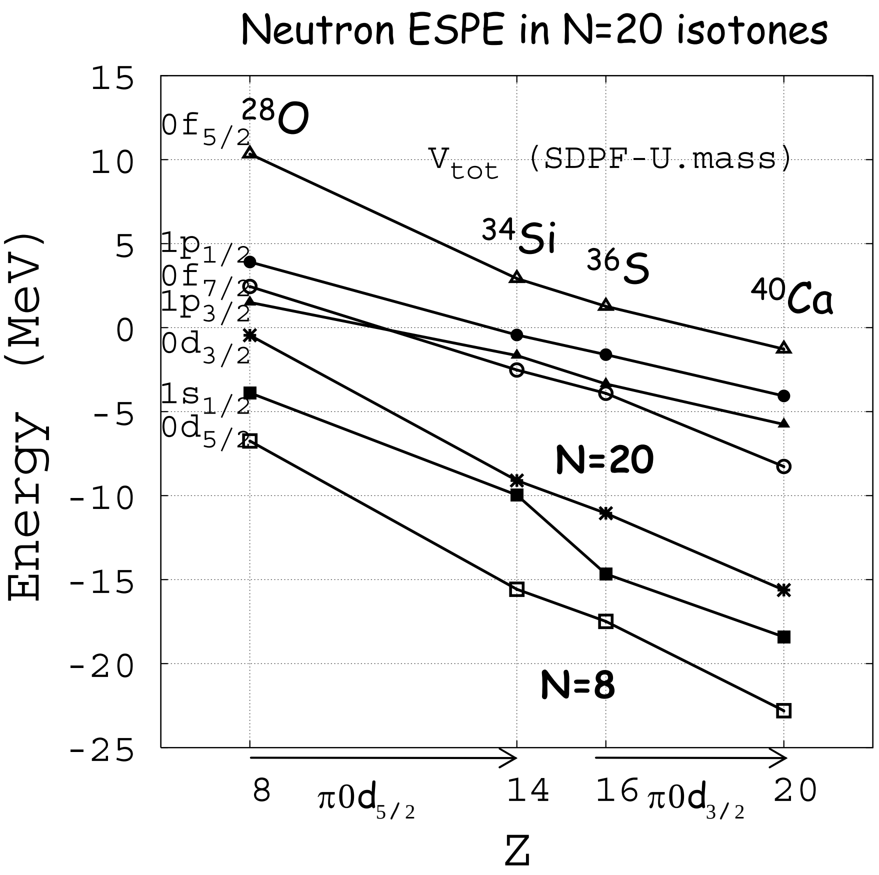
Analysis is possible for a model space containing all spin-orbit partners from a give oscillator shell :
p, sd, psd, sdpf, pf,...
$\Rightarrow$ Limited to lighter nuclei

Analysis of sdpf-space using interaction from F.Nowacki, A.Poves, Phys. Rev. C79 (2009)

Neutron ESPE in N=20 isotones from O to Ca



Contribution of different two-body terms to the change of N=20 shell gap



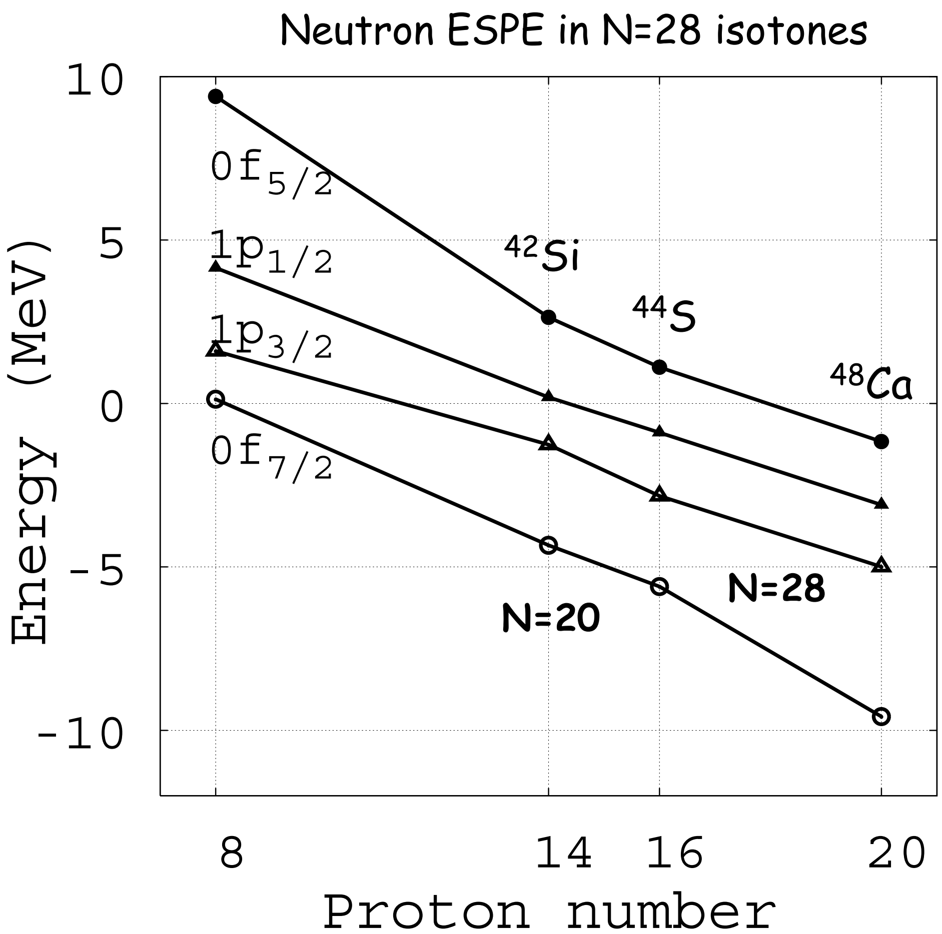
	$\nu 0d_{3/2}$-$1s_{1/2}$ (MeV)	$\nu 0f_{7/2}$-$0d_{3/2}$ (MeV)	$\nu 0f_{7/2}$-$0d_{3/2}$ (MeV)
Filling orbital	$\pi 0d_{5/2}$ $^{28}O \rightarrow ^{34}Si$	$\pi 0d_{5/2}$ $^{28}O \rightarrow ^{34}Si$	$\pi 0d_{3/2}$ $^{36}S \rightarrow ^{40}Ca$
Total	**-2.57**	**3.68**	**0.21**
Central	-1.87	2.17	1.99
Vector	0.36	-0.45	-0.45
Tensor	-1.06	1.96	-1.93

Both, central and tensor components are important in formation/disappearance of N=16 and N=20 shell gaps !

- Central part
- Tensor part

N. Smirnova, B. Bally, K.Heyde, submitted (2009)

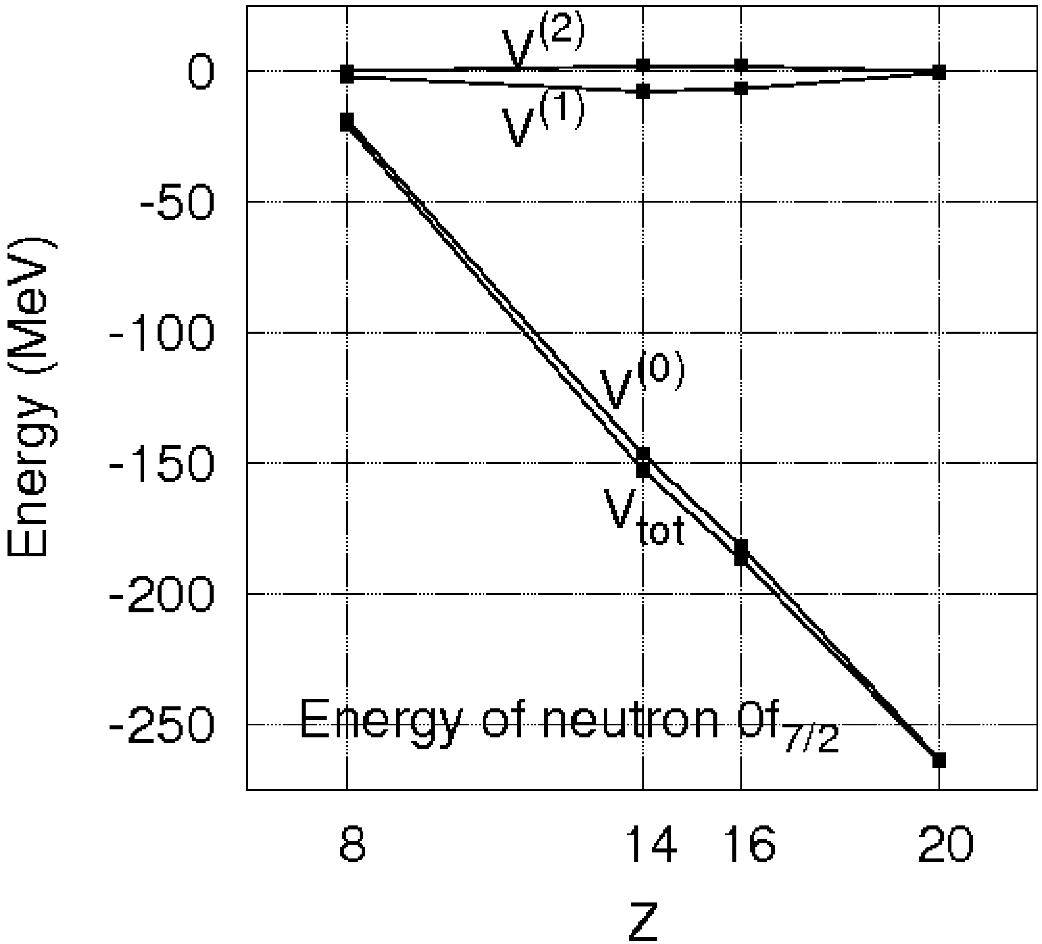
Contribution of different two-body terms to the change of N=28 shell gap



Energy gap	ν1p$_{3/2}$-0f$_{7/2}$ (MeV)	ν1p$_{3/2}$-0f$_{7/2}$ (MeV)
Empting orbital	π0d$_{5/2}$ ^{42}Si $\rightarrow$ (^{36}O)	π0d$_{3/2}$ ^{48}Ca $\rightarrow$ ^{44}S
Total	**-1.60**	**-1.81**
Central	-2.03	-1.31
Vector	0.23	0.18
Tensor	-0.67	0.68

- Central part
- Tensor part
- Radial overlap

Two-body contribution to the monopole binding energy of neutron $0f_{7/2}$ orbital in N=20 isotones from O to Ca



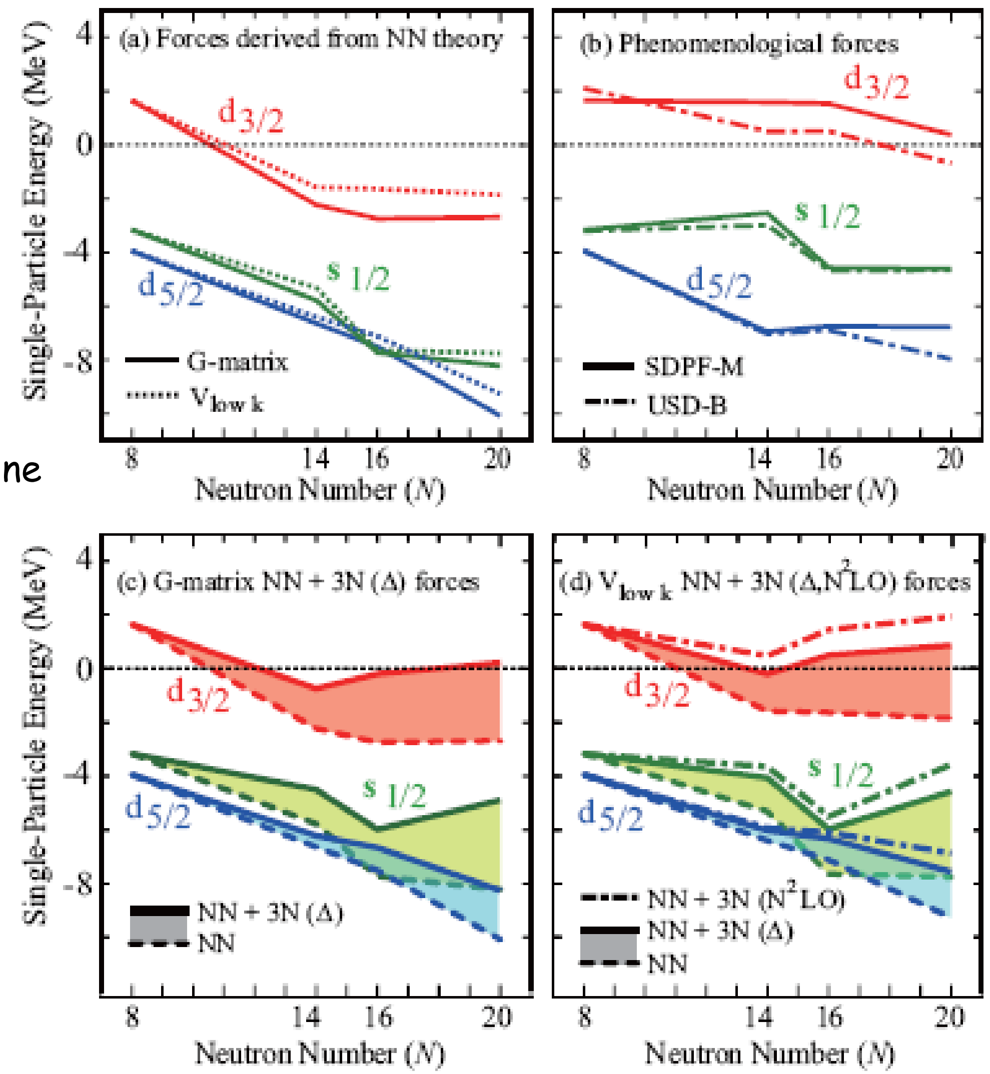
The main contribution comes from the central component !

Monopole effect of a three-body force

In pf-shell:

Asymmetric LS term amplified in the fitted effective NN interaction compared to the microscopic one

Neutron ESPE in O-isotopes



From T.Otsuka et al; nucl-th (08/2009)

Summary

- Shell-model Hamiltonian can be separated in the monopole and multipole parts. The monopole part of the shell-model Hamiltonian provides us with a schematic spherical mean field, while the multipole part is responsible for different types of particle-particle correlations

- Interplay between monopole field and multipole correlations determine the energy balance between the so-called normal and intruder (often deformed) configurations and will lead to the appearance of coexisting structures, 'islands of inversion' and so on.

- The monopole field is typically studied with respect to the shell structure evolution in a wide N/Z range (towards very neutron-rich nuclei). Main contributions come from both central, as well as tensor force (the latter being the most important terms when looking for the evolution of spin-orbit partners)

- Perspective: to understand the role of the three-body component

- Useful for further understanding of the microscopic effective interaction, as well for adjusting energy density functionals for the models based on them