Shell Structure Evolution and Effective In-Medium NN Interaction

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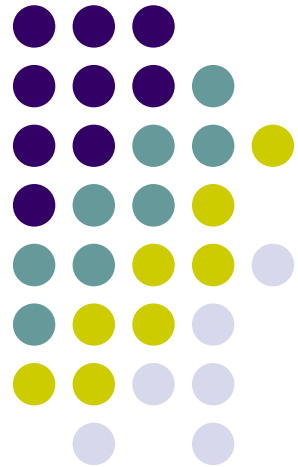

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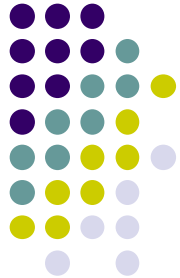
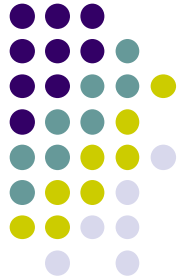


I. Shell model theory and effective interactions (30/09/2009)

II. Monopole term of the effective interaction and evolution of the shell structure (01/10/2009)

Complexity and regularity in nuclear spectra



Single-particle motion near Fermi surface

Pairing phenomena

Shape multipole vibrations

Rotation of deformed nuclei

General non-relativistic many-body problem

$$H = \sum_{k=1}^{A} \frac{p_k^2}{2m_k} + \sum_{k<l=1}^{A} W(k,l) + \sum_{k<l<m=1}^{A} W(k,l,m) + \ldots$$

$$k \equiv \{\vec{r}_k, \vec{\sigma}_k, \vec{\tau}_k\}, \quad \vec{p}_k = -i\hbar\vec{\nabla}_k$$

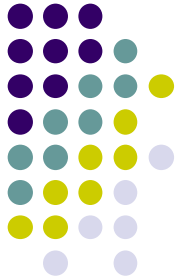
May be formally reduced as follows :

$$H = \underbrace{\sum_{k=1}^{A} \frac{p_k^2}{2m_k} + \sum_{k=1}^{A} U(k)}_{H^{(0)}} + \underbrace{\sum_{k<l=1}^{A} W(k,l) - \sum_{k=1}^{A} U(k)}_{V_{res}}$$

Mean-field theories: search for the 'best' mean-field potential starting from a given two-body interaction + correlations
(lectures by M. Grasso, Th.Duguet)

Shell-model type theories: schematic average potential + residual interaction

Self-consistent mean-field potential (Hartree-Fock method)



We solve self-consistently HF equations, starting from V(r,r') and an initial guess for the wave function :

$$\left\{-\frac{h^2}{2m}\Delta + U_H(\vec{r})\right\}\phi_\alpha(\vec{r}) + \int U_{Ex}(\vec{r},\vec{r}')\phi_a(\vec{r}')d\vec{r}' = \varepsilon_a\phi_a(\vec{r})$$

$$U_H(\vec{r}) = \sum_{j=1}^{N}\int \phi_j^*(\vec{r}')V(\vec{r},\vec{r}')\phi_j(\vec{r}')d\vec{r}'$$

Direct (Hartree) term

Exchange (Fock) term

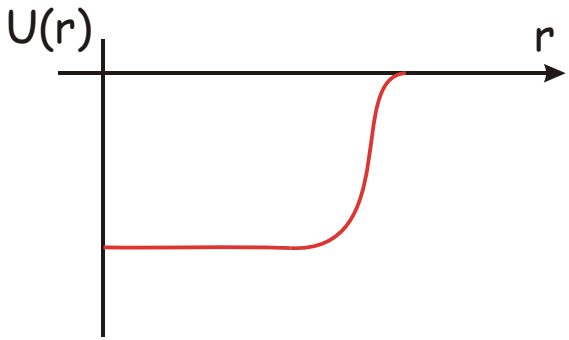
$$U_{Ex}(\vec{r},\vec{r}') = \sum_{j=1}^{N}\phi_j^*(\vec{r})V(\vec{r},\vec{r}')\phi_j(\vec{r}')d\vec{r}'$$

$$U_H(\vec{r}) = \int \rho(\vec{r}')V(\vec{r},\vec{r}')d\vec{r}'$$

$$\rho(\vec{r}) = \sum_j \left|\phi_j(\vec{r})\right|^2$$

$$V(\vec{r},\vec{r}') \propto \delta(\vec{r}-\vec{r}')$$

$$U_H(\vec{r}) \propto \rho(\vec{r})$$



Shell-model approach

We start with a many-body Hamiltonian

$$H = \sum_{k=1}^{A} \frac{\vec{p}_k^{\,2}}{2m_k} + \sum_{k<l=1}^{A} W(k,l) \quad \text{where} \quad k \equiv \{\vec{r}_k, \vec{\sigma}_k, \vec{\tau}_k\}, \quad \vec{p}_k = -i\hbar\vec{\nabla}_k$$

and introduce a mean-field U(k) :

$$H = \underbrace{\sum_{k=1}^{A}\left(\frac{\vec{p}_k^{\,2}}{2m_k} + U(k)\right)}_{\hat{H}^{(0)} = \sum_{k=1}^{A} h(k)} + \underbrace{\sum_{k<l=1}^{A} W(k,l) - \sum_{k=1}^{A} U(k)}_{\text{Residual interaction}}$$

$$h(k)\phi_\alpha(\vec{r}_k) = \varepsilon_\alpha \phi_\alpha(\vec{r}_k) \Longrightarrow \{\varepsilon_\alpha, \phi_\alpha(\vec{r}_k)\}$$

Single-particle energy
Single-particle wave function

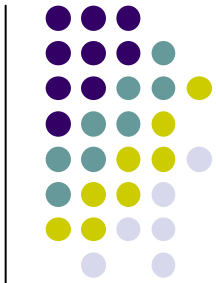
Spherical mean-field :

$$\alpha = \{n_\alpha, l_\alpha, j_\alpha, m_\alpha\}$$

Construction of a basis :

$$\Phi(1,\ldots,A) = \frac{1}{\sqrt{A!}} \det\{\phi_{\alpha_1}(1),\ldots,\phi_{\alpha_A}(A)\}$$

Particle in a spherically symmetric potential: wave function and quantum numbers



$$-\frac{\hbar^2}{2m}\Delta\phi(\vec{r})+U(r)\phi(\vec{r})=\varepsilon\phi(\vec{r})$$

$$\phi(\vec{r})=\frac{R(r)}{r}Y(\vartheta,\varphi)$$

$$-\frac{\hbar^2}{2m}R''(r)+\frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}R(r)+U(r)R(r)=\varepsilon R(r)$$

$$\hat{l}^2Y_{lm_l}(\vartheta,\varphi)=l(l+1)Y_{lm_l}(\vartheta,\varphi)$$

$$\hat{l}_zY_{lm_l}(\vartheta,\varphi)=mY_{lm_l}(\vartheta,\varphi)$$

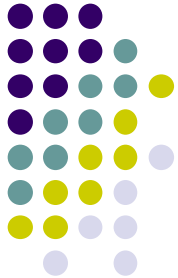
$$\phi_{nlm_l}(\vec{r})=\frac{R_{nl}(r)}{r}Y_{lm_l}(\vartheta,\varphi)$$

n is the radial quantum number (number of zeros)

Transformation with respect to the Parity operation $\hat{P}(\vec{r}\rightarrow-\vec{r})$

$$\hat{P}\phi_{nlm_l}(\vec{r})=\hat{P}\left(R_{nl}(r)Y_{lm_l}(\vartheta,\varphi)\right)=R_{nl}(r)\hat{P}Y_{lm_l}(\vartheta,\varphi)=R_{nl}(r)(-1)^lY_{lm_l}(\vartheta,\varphi)=(-1)^l\phi_{nlm_l}(\vec{r})$$

Particle with spin in a spherically-symmetric potential



$$-\frac{\hbar^2}{2m}\Delta\phi(\vec{r})+U(r)\phi(\vec{r})+f_{so}(r)\vec{l}\cdot\vec{s}=\varepsilon\phi(\vec{r})$$

The total wave functions takes then a form :

$$\phi_{nljm}(\vec{r})=\frac{R_{nlj}(r)}{r}\underbrace{\left[Y_l(\vartheta,\varphi)\times\chi_{1/2}\right]_m^{(j)}}_{\sum_{m_l m_s}\left(lm_l\,\frac{1}{2}m_s\,|jm\right)Y_{lm_l}(\vartheta,\varphi)\chi_{1/2m_s}}$$

$$-\frac{\hbar^2}{2m}R''(r)+\frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}R(r)+\left(U(r)+a_{so}f_{so}(r)\right)R(r)=\varepsilon R(r)$$

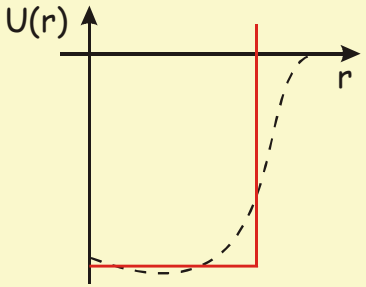
Normalization condition:

$$\int\left|\phi_{nljm}(\vec{r})\right|^2 d\vec{r}=\int_0^\infty\left|R_{nlj}(r)\right|^2 dr=1$$

Some examples of schematic potentials

Above results can be obtained approximately using

- Square-well potential + strong spin-orbit term

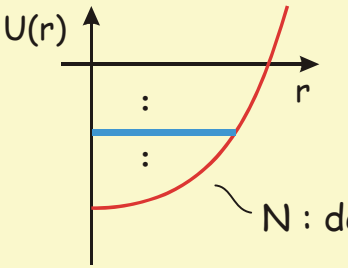


ex. N = 4

→ $J_{l+1/2}(kr)$

s ——— l=0 (100)
d ——— l=2 (93)
g ——— l=4 (75.5)

- Harmonic oscillator potential + orbital + spin-orbit term



→ $(\nu r)^l e^{\frac{-\nu^2 r^2}{2}} L_{n-1}^{l+1/2}(\nu^2 r^2)$

$(\nu = \sqrt{\frac{m\omega}{\hbar}})$

$$U(r) = \frac{m\omega^2 r^2}{2}$$

$$\varepsilon_N = \hbar\omega(2n + l + 3/2)$$
$$= \hbar\omega(N + 3/2)$$

$N = 0,1,2,\ldots,$
$l = 0,1,2,\ldots,N$
$n = 0,1,2,\ldots,(N-l)/2$

Harmonic oscillator with centrifugal and spin-orbit terms

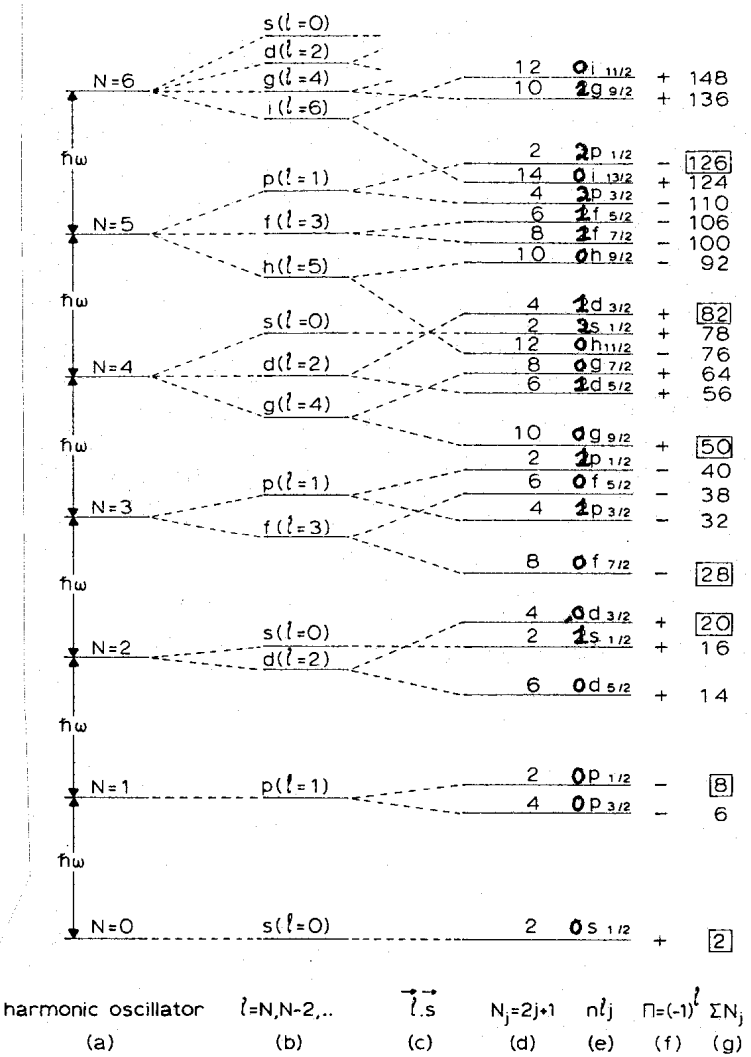
$$U(r) = \frac{m\omega^2 r^2}{2} + \alpha\left(\vec{l}\cdot\vec{l}\right) + \beta\left(\vec{l}\cdot\vec{s}\right)$$

M.Göppert-Mayer (1949)
H.Jensen, O.Haxel, H.E.Suess (1949)

Nuclei near stability line (most known at that time nuclei)

- Magic numbers
- Spin and parities of the ground states of most odd-A nuclei
- Most of the magnetic moments of g.s. of odd-A nuclei

Harmonic oscillator potential possesses many symmetries which make it a preferable choice for the **basis** for solution of the A-body problem



Isospin

$$V_{\pi\pi} \approx V_{\nu\nu} \approx V_{\pi\nu}$$

$$\pi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \nu = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

We introduce isospin operators :

$$\vec{t} = \frac{\vec{\tau}}{2}, \quad \tau_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tau_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tau_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Single-particle wave functions for a proton and a neutron can be expressed as

$$\phi_\pi(\vec{r}) = \phi(\vec{r}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \phi(\vec{r}) \theta_{t=1/2, m_t=-1/2}$$

$$\phi_\nu(\vec{r}) = \phi(\vec{r}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \phi(\vec{r}) \theta_{t=1/2, m_t=+1/2}$$

Or, explicitly for a nucleon wave function we have:

$$\phi_{nljm,tm_t}(\vec{r}) = \frac{R_{nlj}(r)}{r} \left[Y_l(\theta, \varphi) \times \chi_{1/2} \right]^{(j)}_m \theta_{tm_t}$$

Isospin and classification of nuclear states

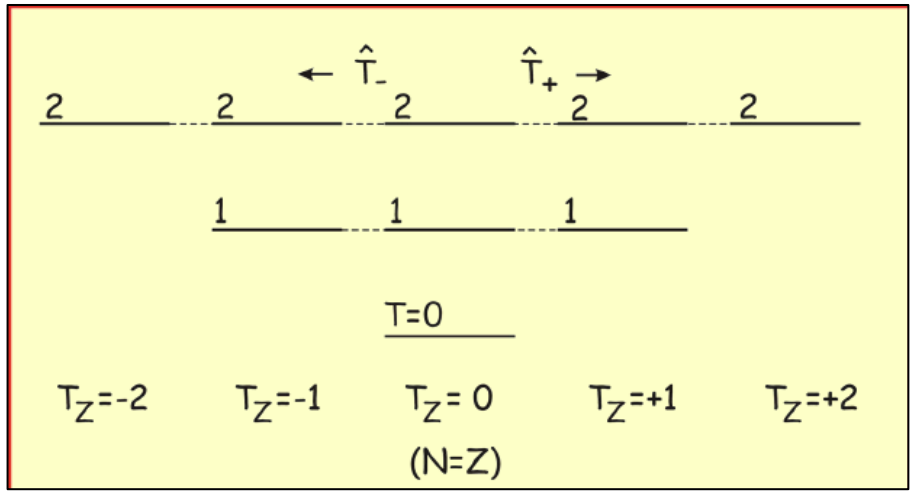
Isospin operators can be used as angular momentum to construct isospin states for many nucleons

$$\hat{\vec{T}} = \sum_{i=1}^{A} \hat{\vec{t}}_i, \quad \hat{T}_z = \sum_{i=1}^{A} \hat{t}_{z,i}$$

If Hamiltonian is charge-independent: $\left[\hat{H}, \hat{T}\right] = 0$

then the nuclear states of a nucleus with N neutrons and Z protons (A=N+Z) can be characterized by certain values of T and T_z

$$T_z = \frac{1}{2}(N-Z), \quad \frac{1}{2}(N-Z) \le T \le \frac{A}{2}$$



Realistic situation : $m_\pi \approx m_\nu$; $V_{Coulomb}$

$$\hat{H} = \hat{H}^{(0)} + \hat{H}^{(1)} + \hat{H}^{(2)}$$

$$E(T, T_z) = a(T) + b(T)T_z + c(T)T_z^2$$

Isobaric multiplet mass equation (IMME)

E.Wigner

Two-particle wave function

Normalized, antisymmetrized and coupled to a certain J two-particle wave function have a form :

$$\Phi^{\alpha\beta}_{JMTM_T}(1,2) = \left\{\left[\phi_\alpha(\vec{r}_1)\times\phi_\beta(\vec{r}_2)\right]^{JM} + (-1)^{j_\alpha+j_\beta+J+T}\left[\phi_\beta(\vec{r}_1)\times\phi_\alpha(\vec{r}_2)\right]^{JM}\right\} \cdot \Theta_{TM_T} \Big/ \sqrt{2(1+\delta_{\alpha\beta})}$$

where the two-nucleon isospin part can be expressed as

$$\alpha = \{n_\alpha, l_\alpha, j_\alpha, m_\alpha\}$$

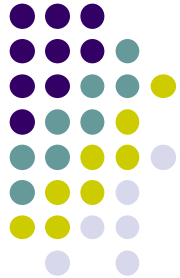
$$\Theta_{1,1} = \theta_{1/2,1/2}(1)\theta_{1/2,1/2}(2)$$
$$\Theta_{1,-1} = \theta_{1/2,-1/2}(1)\theta_{1/2,-1/2}(2)$$
$$\Theta_{1,0} = 1/\sqrt{2}\left[\theta_{1/2,1/2}(1)\theta_{1/2,-1/2}(2) + \theta_{1/2,-1/2}(1)\theta_{1/2,1/2}(2)\right]$$
$$\Theta_{0,0} = 1/\sqrt{2}\left[\theta_{1/2,1/2}(1)\theta_{1/2,-1/2}(2) - \theta_{1/2,-1/2}(1)\theta_{1/2,1/2}(2)\right]$$

$$J = |j_\alpha - j_\beta|, \ldots, j_\alpha + j_\beta; \quad M = -J, \ldots, J$$

Important remark : in case $j_\alpha = j_\beta = j$ J+T is always odd !

$$(\nu 0d_{5/2})^2 : J = 0,2,4$$
$$(\pi 0d_{5/2}\nu 0d_{5/2}) : J = 0,2,4\ (T=1);\quad J = 1,3,5\ (T=0)$$

Many-particle wave function



1. Coefficients of fractional parentage (cfp's) allow to construct many-particle wave function totally antisymmetric and coupled to a certain J (T) value

$$\Phi\left(j^n\chi;JM\right)=\sum_{\chi'J'}\left[j^{n-1}\left(\chi'J'\right)j\,|\}j^n\chi J\right]\Phi\left(j^{n-1}\chi';J'M'\right)\phi_{jm}$$

n-particle wave function — *one-particle CFP* — *(n-1)-particle wave function*

Macfarlane, French (1960)

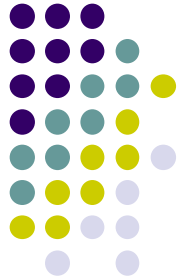
2. M-scheme (alternative method to construct normalized, antisymmetric states of A fermions)

$$\Phi\left(1,2,\ldots,A\right)=\frac{1}{\sqrt{A!}}\begin{vmatrix}\phi_{\alpha_1}\left(\vec{r}_1\right) & \phi_{\alpha_1}\left(\vec{r}_2\right) & \cdots & \phi_{\alpha_1}\left(\vec{r}_A\right)\\ \phi_{\alpha_2}\left(\vec{r}_1\right) & \phi_{\alpha_2}\left(\vec{r}_2\right) & \cdots & \phi_{\alpha_2}\left(\vec{r}_A\right)\\ \vdots & & & \\ \phi_{\alpha_A}\left(\vec{r}_1\right) & \phi_{\alpha_A}\left(\vec{r}_2\right) & \cdots & \phi_{\alpha_A}\left(\vec{r}_A\right)\end{vmatrix}$$

$$M=\sum_{i=1}^{A}m_i$$

and then project on good J (T)

Solution of a many-body Schrödinger equation: diagonalization of the residual interaction



- Construct a basis in the valence space (coupled J-states or in M-scheme)

$$\Phi_{J,k} = \left\{ \left(j_\alpha\right)^{n_\alpha}_{J_\alpha} \left(j_\beta\right)^{n_\beta}_{J_\beta} \cdots \right\}_{J,k}$$

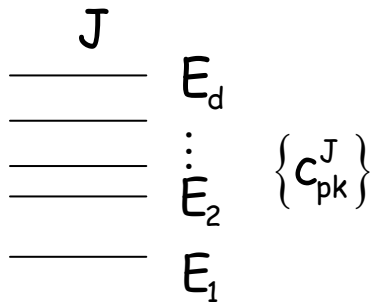
- The eigenfunction is then expanded in terms of basis functions :

$$\Psi_{J,p} = \sum_{k=1}^{d} c^{J}_{pk} \Phi_{J,k}$$

*(**d** is the number of basis functions, or basis dimension)*

- Solve eigenvalue equation

$$\hat{H}\Psi_{J,p} = E_{J,p}\Psi_{J,p} \quad \Rightarrow \quad \begin{pmatrix} H_{11} & H_{12} & \cdots & H_{1d} \\ H_{21} & H_{22} & \cdots & H_{2d} \\ \vdots & & \ddots & \\ H_{d1} & H_{d2} & \cdots & H_{dd} \end{pmatrix} \quad \Rightarrow$$



$$\hat{H} = \hat{H}^{(0)} + \hat{V}$$

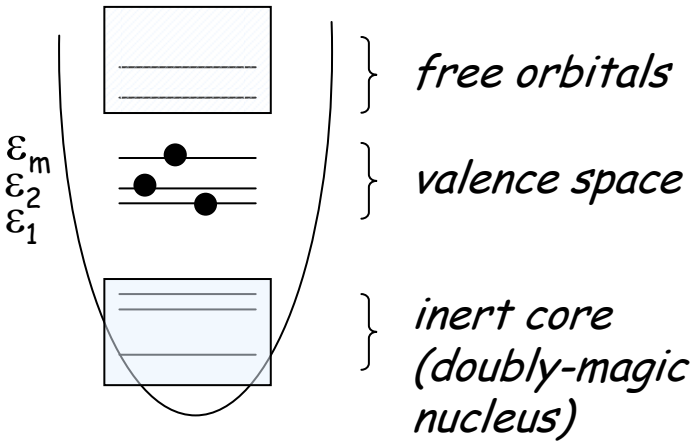
$$H_{lk} \equiv \left\langle \Phi_l \middle| \hat{H} \middle| \Phi_k \right\rangle = E^{(0)}_k \delta_{lk} + V_{lk}, \quad E^{(0)}_k = n_\alpha \varepsilon_\alpha + n_\beta \varepsilon_\beta + \ldots$$

Basis dimension and choice of the model space

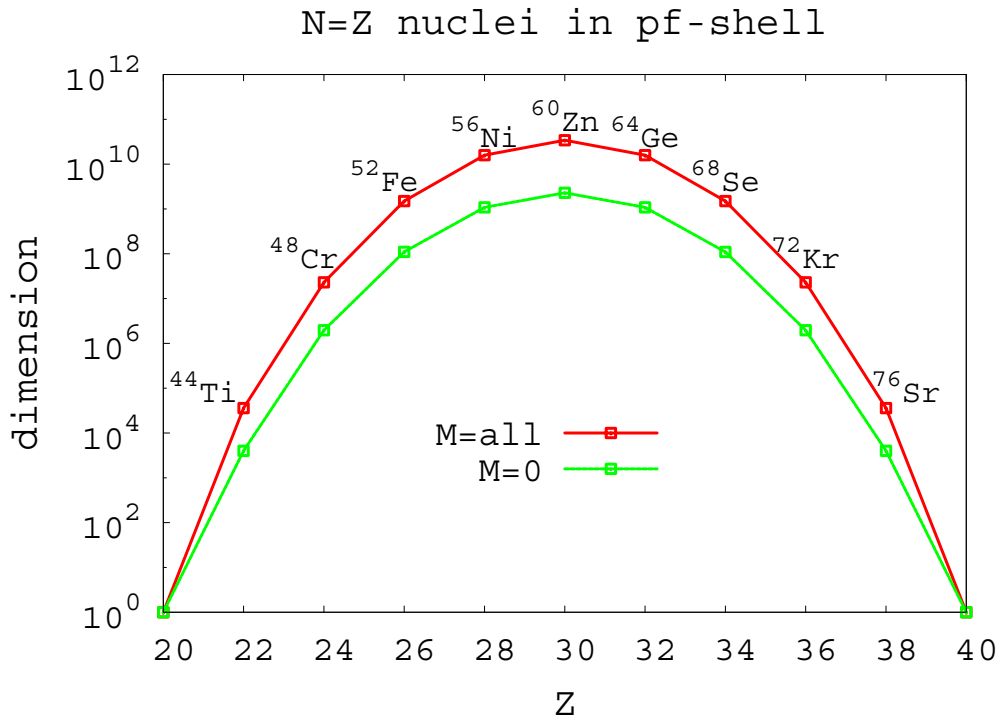
Basis dimension grows quickly with the number of single-particle states involved and a number of nucleons

$$\dim \approx \binom{N_\nu}{n_\nu} \cdot \binom{N_\pi}{n_\pi} = \frac{N_\nu!}{n_\nu!(N_\nu - n_\nu)!} \cdot \frac{N_\pi!}{n_\pi!(N_\pi - n_\pi)!}$$

This is why in conventional shell model calculations are done for valence nucleons beyond a closed shell core typically in one (two) oscillator shells



$$\dim\left({}^{60}\mathrm{Zn}\right) \approx \binom{20}{10} \cdot \binom{20}{10} = 3.4 \times 10^{10}$$



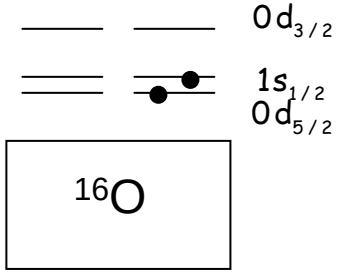
Practical shell model ^{18}O in *sd*-space (0^+ states)

Input: ε_α, $\langle\alpha\beta|V|\gamma\delta\rangle_{JT}$ USD interaction

$$\varepsilon(0d_{5/2}) = BE\left[{}^{17}_{8}O_9\right] - BE\left[{}^{16}_{8}O_8\right] = -3.948\,MeV$$

$$\varepsilon(1s_{1/2}) = \varepsilon(0d_{5/2}) + E_{ex}({}^{17}O;1/2^+_1) = -3.165\ MeV$$

$$\varepsilon(0d_{3/2}) = \varepsilon(0d_{5/2}) + E_{ex}({}^{17}O;3/2^+_1) = 1.647\ MeV$$



Basis $|\Phi\rangle_{JT}$: $\left|(0d_{5/2})^2\right\rangle_{01}$; $\left|(1s_{1/2})^2\right\rangle_{01}$; $\left|(0d_{3/2})^2\right\rangle_{01}$

$$H_{11} = 2\varepsilon(0d_{5/2}) + \underbrace{\left\langle (0d_{5/2})^2 |V| (0d_{5/2})^2 \right\rangle_{01}}_{\text{-2.28 MeV}}$$

$$H_{22} = 2\varepsilon(1s_{1/2}) + \underbrace{\left\langle (1s_{2/2})^2 |V| (1s_{1/2})^2 \right\rangle_{01}}_{\text{-2.125 MeV}}$$

$$H_{33} = 2\varepsilon(0d_{3/2}) + \underbrace{\left\langle (0d_{3/2})^2 |V| (0d_{3/2})^2 \right\rangle_{01}}_{\text{-2.185 MeV}}$$

$$H_{12} = \left\langle (0d_{5/2})^2 |V| (1s_{1/2})^2 \right\rangle_{01} = \text{-0.1325 MeV}$$

$$H_{23} = \left\langle (1s_{1/2})^2 |V| (0d_{3/2})^2 \right\rangle_{01} = \text{-1.0835 MeV}$$

$$H_{13} = \left\langle (0d_{5/2})^2 |V| (0d_{3/2})^2 \right\rangle_{01} = \text{-3.186 MeV}$$

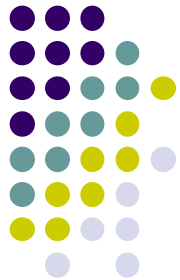
$$\begin{vmatrix} H_{11}-E & H_{12} & H_{13} \\ H_{12} & H_{22}-E & H_{23} \\ H_{13} & H_{23} & H_{33}-E \end{vmatrix} = 0$$

0^+_3 ——— 14.1

0^+_2 ——— 4.3

0^+_1 ——— 0.0

Shell-model codes



M-scheme codes

- Antoine (Caurier)
- Mshell (Mizusaki)
- Redstick (Ormand, Johnson)
- Vecsse (Sebe)
- Oxbash (Brown et al) ->(JT)
- Oslo code (Engeland)
- …

Coupled codes (J-scheme)

- Nathan (Caurier, Nowacki)
- DUPSM (Novoselsky, Vallières)
- Ritsschil (Zwarts)
- …

Max basis dimensions : ~10^{10}

Exact diagonalization by Lanczos algorithm

$$\hat{H}|1\rangle = E_{11}|1\rangle + E_{12}|2\rangle$$
$$\hat{H}|2\rangle = E_{21}|1\rangle + E_{22}|2\rangle + E_{23}|3\rangle$$
$$\ldots$$

$$E_{11} = \langle 1|\hat{H}|1\rangle,\ E_{12}|2\rangle = (\hat{H} - E)|1\rangle$$
$$E_{21} = E_{12},\ E_{22} = \langle 2|\hat{H}|2\rangle,$$
$$E_{23}|3\rangle = (\hat{H} - E_{22})|2\rangle - E_{21}|1\rangle$$

$$\begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix} \Rightarrow \begin{pmatrix} E_{11} & E_{12} & 0 \\ E_{21} & E_{22} & E_{23} \\ 0 & E_{32} & E_{33} \end{pmatrix} \Rightarrow \begin{pmatrix} E_{11} & E_{12} & 0 & 0 \\ E_{21} & E_{22} & E_{23} & 0 \\ 0 & E_{32} & E_{33} & E_{34} \\ 0 & 0 & \ldots & \ldots \end{pmatrix} \Rightarrow$$

Convergence of the lowest eigenstates

Effective operators

Hilbert space → Model space

$$\hat{V}_{NN} \Rightarrow \hat{V}_{eff}$$

$$\hat{H}\Psi = E\Psi \Rightarrow \hat{H}_{eff}\Psi^{M} = \left(\hat{H}^{(0)} + \hat{V}_{eff}\right)\Psi^{M} = E\Psi^{M}$$

$$\left\langle \Psi \middle| \hat{O} \middle| \Psi \right\rangle \Rightarrow \left\langle \Psi^{M} \middle| \hat{O}_{eff} \middle| \Psi^{M} \right\rangle$$

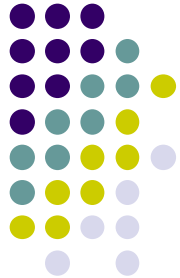
Practical approaches to get TBME: $\left\langle \alpha\beta \middle| V_{eff} \middle| \gamma\delta \right\rangle_{JT}$

i. Schematic interaction (parameterized interaction potential between two nucleons in a nuclear medium)

ii. Empirical effective interaction (fit of the TBME to energy levels of nuclei to be described within the chosen model space)

iii. Microscopic interaction (derived from a bare NN-force, see scheme above)

(i) Schematic (parameterized) interaction



Some examples :

$$V(1,2) = -V_0\, e^{-\mu r}/\mu r$$ *Yukawa potential*

$$V(1,2) = -V_0 \delta(\vec{r}_1 - \vec{r}_2)$$
$$V(1,2) = -V_0 \delta(\vec{r}_1 - \vec{r}_2)(1 + \alpha \vec{\sigma}_1 \cdot \vec{\sigma}_2)$$ *δ-force*

$$V(1,2) = -V_0 \delta(r_1 - r_2)\delta(r_1 - R)$$ *Surface δ-interaction (SDI)*

...

$$V(1,2) = \chi Q \cdot Q$$
$$Q_\mu = r^2 Y_{2\mu}(\Omega_r)$$ *Quadrupole-quadrupole interaction*

...

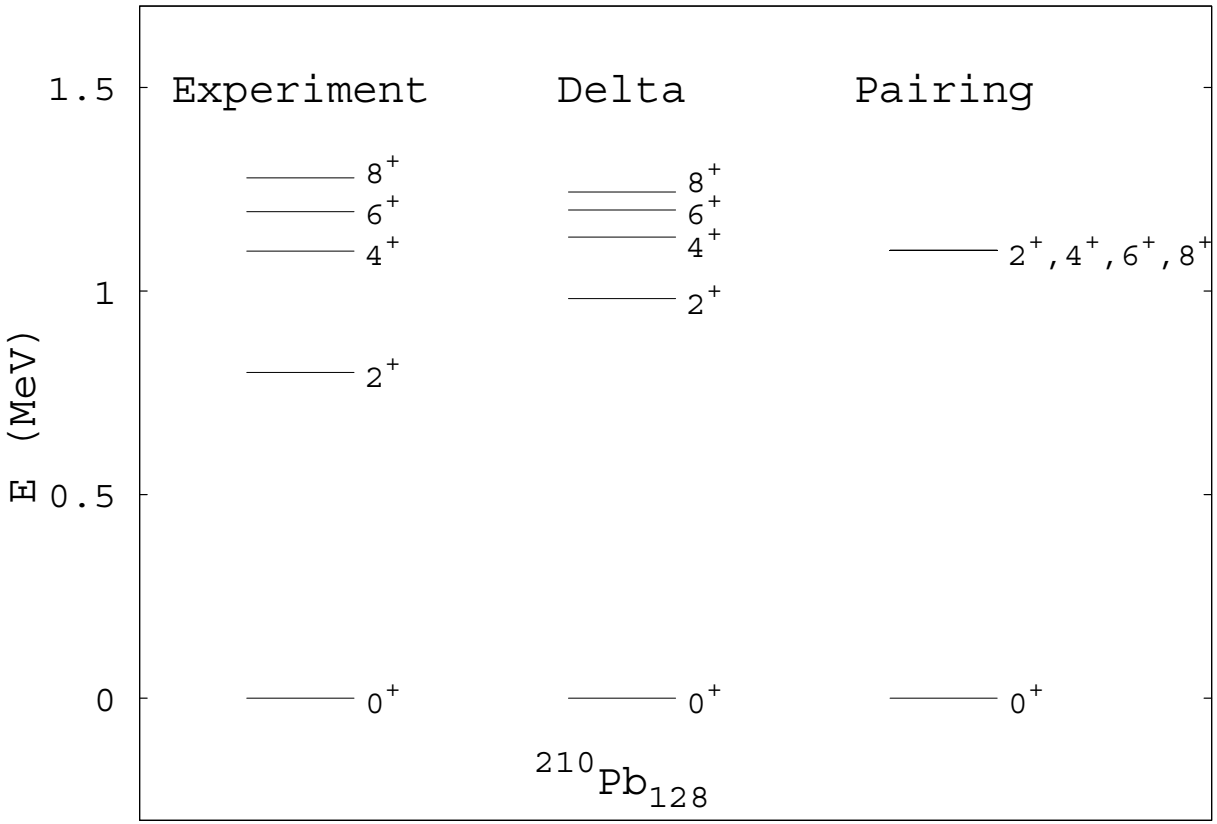
A few parameters (interaction strengths) are fitted to reproduce energy levels in a certain region of (a few)neighboring nuclei
$\Rightarrow$ local description only !

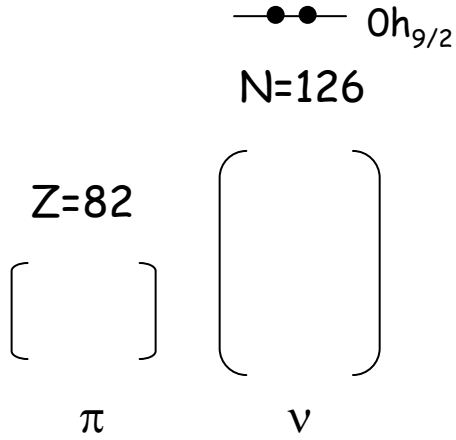
Example 1: $^{210}Pb\ (\nu 0h_{9/2})^2$

$$V_{delta}(1,2) = -V_0\,\delta(\vec{r}_1 - \vec{r}_2)$$

$$V_{pair}(1,2) = -GS_+ \cdot S_-$$

(creates and annihilates a J=0 pair of fermions in the j-shell





$$\left\langle \alpha^2 \left| V_{pair}(1,2) \right| \alpha^2 \right\rangle_{J=0,T=1} = -1/2\,(2j_\alpha + 1)\,G$$

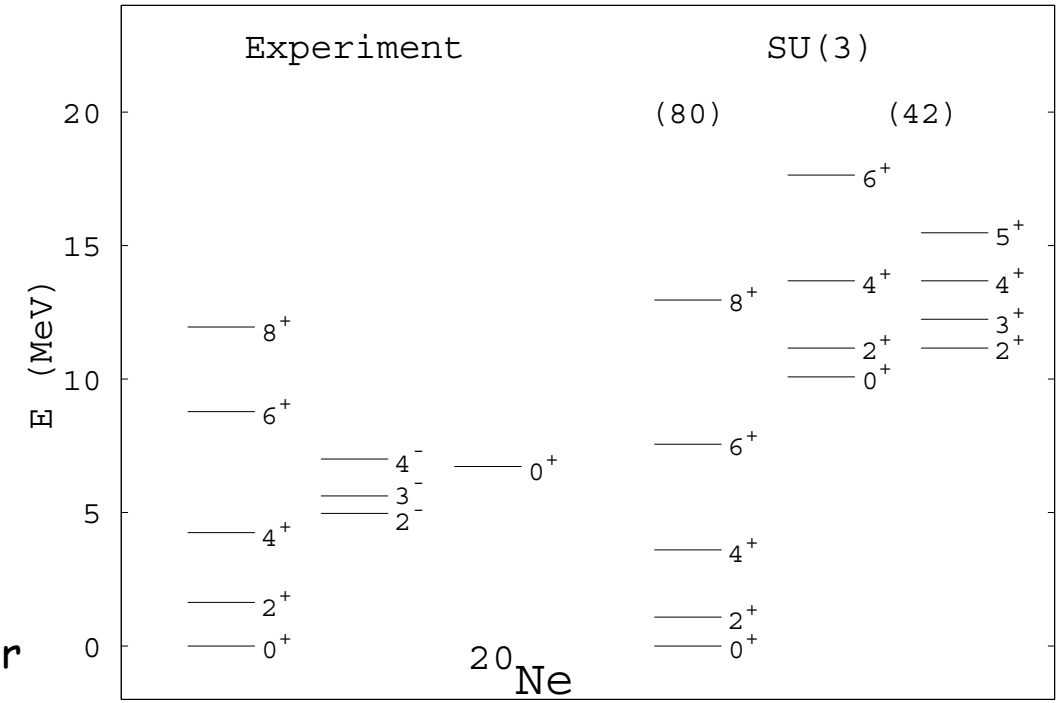
Example 2: ^{20}Ne (Z=10,N=10) and SU(3) model of Elliott

$$H = \sum_{k=1}^{A}\left[\frac{p_k^2}{2m} + \frac{1}{2}m\omega^2 r_k^2\right] - \chi Q\cdot Q$$

J.P.Elliott (1958)

Rotational classification of nuclear states (mixing of many spherical configurations)

Q is an algebraic quadrupole operator



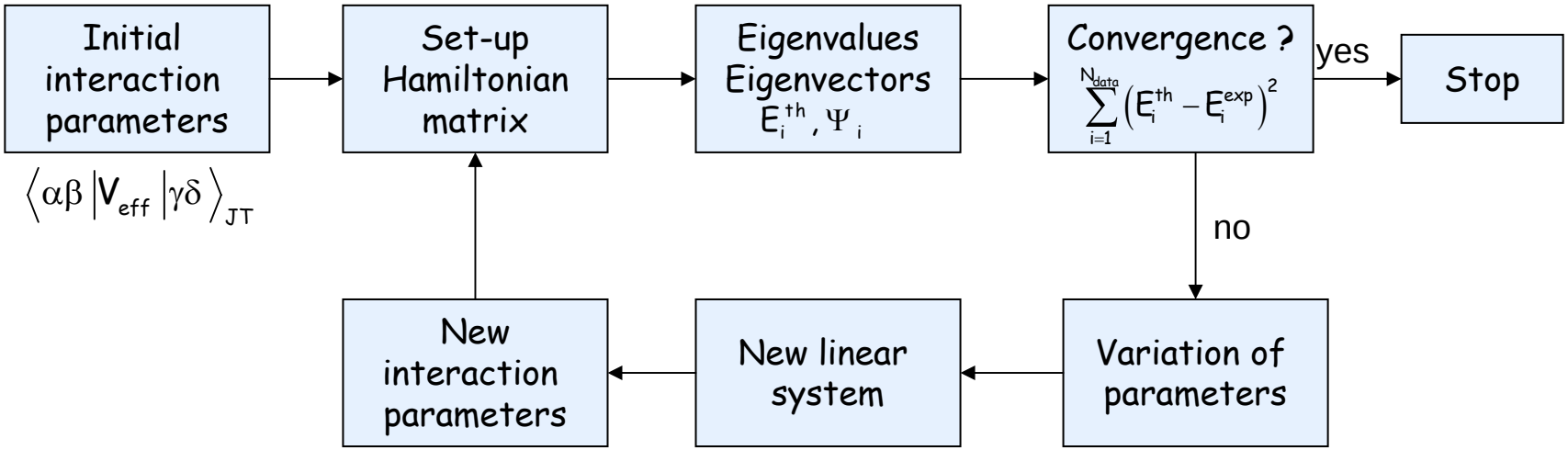
$$Q_\mu = \sqrt{\frac{4\pi}{5}}\left(\sum_k r_k^2 Y_{2\mu}(\Omega_r)/b^2 + b^2\sum_k p_k^2 Y_{2\mu}(\Omega_p)/\hbar^2\right)$$

$$L_\mu = \sum_k [r_k \times p_k]_\mu / \hbar$$

SU(3) generators

(ii) Empirical V_{eff} (least-square-fit method)

All two-body matrix elements (TBME) between valence nucleons in a model space are considered as free parameters.



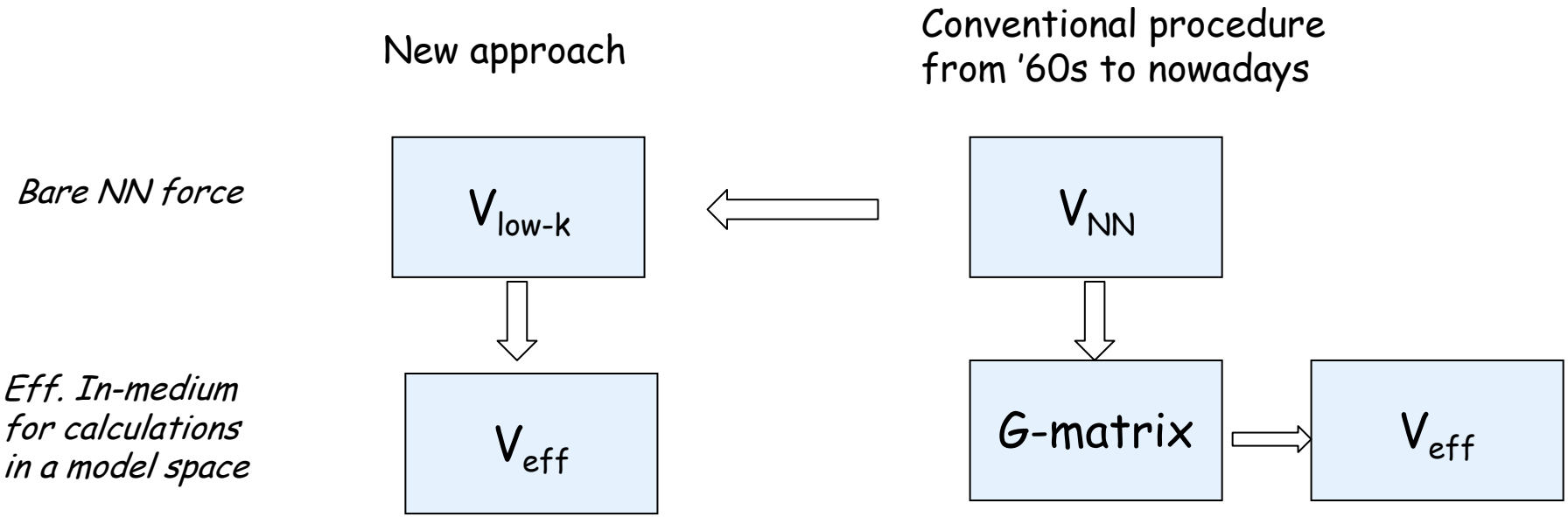
0p-shell: ^{4}He – ^{16}O 15 TBME
1s0d-shell: ^{16}O – ^{40}Ca 63 TBME
1p0f-shell: ^{40}Ca – ^{80}Zr 195 TBME

Cohen, Kurath (1965)
Brown, Wildenthal, USD (1988)
Tokyo-MSU, GXPF1 (2002,2004)

Linear combination method

(iii) Microscopic effective interaction

A bare NN-potential - CD-Bonn, Nijmegen II, AV18, chiral N3LO potential - requires regularization and modification to be applied for many-nucleon systems in a restricted model space.



renormalization group technique to V_{NN} in a momentum space

S. Bogner et al, Phys. Rep. 386 (2003)
lecture by Th. Duguet

expansion of effective interaction in terms of the nuclear reaction matrix G

M. Hjorth-Jensen et al, Phys.Rep.261 (1995)

Towards microscopic effective shell-model interaction: general principles (I)

$\hat{H} = \hat{H}^{(0)} + \hat{V}$ with the unperturbed (HO) Hamiltonian $\hat{H}^{(0)}\Phi_k = E_k^{(0)}\Phi_k$

unperturbed wave functions (basis)

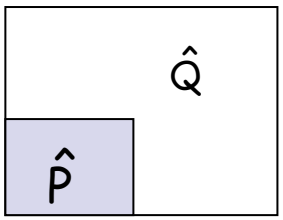
We would like to solve the Schrödinger equation for H to find a set of true wave functions

$$\hat{H}\Psi = \left(\hat{H}^{(0)} + \hat{V}\right)\Psi = E\Psi$$

true wave functions → $\Psi = \sum_{k=1}^{\infty} a_k \Phi_k$

$$\hat{H}_{eff}\Psi^M = \left(\hat{H}^{(0)} + \hat{V}_{eff}\right)\Psi^M = E\Psi^M$$

model wave functions → $\Psi^M = \sum_{k\in M} a_k \Phi_k$



$$\hat{P} = \sum_{k\in M} |\Phi_k\rangle\langle\Phi_k|$$

$$\hat{Q} = \sum_{k\notin M} |\Phi_k\rangle\langle\Phi_k|$$

$$\hat{P} + \hat{Q} = 1$$

$$\hat{P}^2 = \hat{P}, \hat{Q}^2 = \hat{Q}$$

$$\hat{P}\hat{Q} = \hat{Q}\hat{P} = 0$$

$$\left[\hat{P}, \hat{H}^{(0)}\right] = \left[\hat{Q}, H^{(0)}\right] = 0$$

$$\Psi^M = \hat{P}\Psi$$

$$\hat{P}\left(\hat{H} - E\right)\Psi = 0$$

$$\hat{Q}\left(\hat{H} - E\right)\Psi = 0$$

$$\hat{V}_{eff} = \hat{V} + \hat{V}\frac{\hat{Q}}{E - \hat{H}^{(0)}}\hat{V}_{eff}$$

$$\Psi = \Psi^M + \frac{\hat{Q}}{E - \hat{H}^{(0)}}\hat{V}\Psi$$

$$\hat{V}_{eff}\Psi^M = \hat{V}\Psi$$

Towards microscopic effective shell-model interaction from: general principles (II)

Separation between the core and valence nucleons :

$$E = \underbrace{E_c^{(0)} + \Delta E_c}_{\text{core energy}} + \underbrace{E_v^{(0)}}_{\text{unperturbed energy of valence nucleons}} + \Delta E_{cv}$$

Brandow (1967)
Linked, folded diagrams only

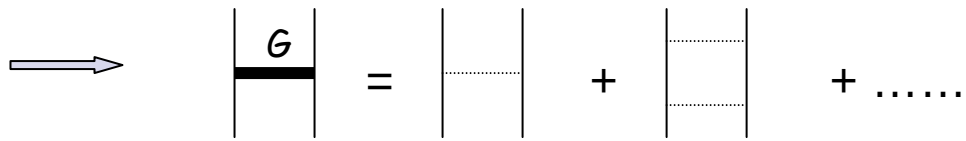
$$\hat{V}_{eff} = \hat{V} + \hat{V}\frac{\hat{Q}}{E_v^{(0)} - \hat{H}_v^{(0)}}\hat{V}_{eff}$$
$$= \hat{V} + \hat{V}\frac{\hat{Q}}{E_v^{(0)} - \hat{H}_v^{(0)}}\hat{V} + \hat{V}\frac{\hat{Q}}{E_v^{(0)} - \hat{H}_v^{(0)}}\hat{V}\frac{\hat{Q}}{E_v^{(0)} - \hat{H}_v^{(0)}}\hat{V} + \dots$$

$$\hat{H}_v^{(0)} = \hat{H}^{(0)} - E_c^{(0)}$$

Old two-step approach to solve this equation :

1. Computation of the reaction matrix, or **G-matrix** (Brueckner theory)

$$\hat{G}(\omega) = \hat{V} + V\frac{\hat{Q}_{2p}}{\omega - \hat{H}_{2p}^{(0)}}\hat{G}(\omega)$$

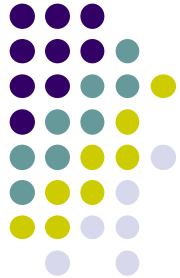


2. Expansion of the $\mathbf{V_{eff}}$ in terms of the G-matrix

$$\hat{V}_{eff} = \hat{G} + \hat{G}\frac{\hat{Q}'}{E_v^{(0)} - H_v^{(0)}}\hat{V}_{eff}$$
$$= \hat{G} + \hat{G}\frac{\hat{Q}'}{E_v^{(0)} - \hat{H}_v^{(0)}}\hat{G} + \hat{G}\frac{\hat{Q}'}{E_v^{(0)} - \hat{H}_v^{(0)}}\hat{G}\frac{\hat{Q}'}{E_v^{(0)} - \hat{H}_v^{(0)}}\hat{G} + \dots$$

Lots of complications and problems with evaluating of this expansion…
$V_{low\text{-}k}$ may provide a new approach to follow…

… still phenomenological adjustment required



Microscopic effective 2-body interactions (either G-matrix or V_{low-k}) fail to reproduce nuclear properties when the number of valence particles increases: the monopole part of the interaction if deficient (lack of 3-body forces)

$\Rightarrow$ phenomenological adjustment to data

E.Caurier et al, Rev. Mod. Phys. 77 (2005) 427

- Monopole part of the interaction adjusted (KB3, KB3G for pf-shell)

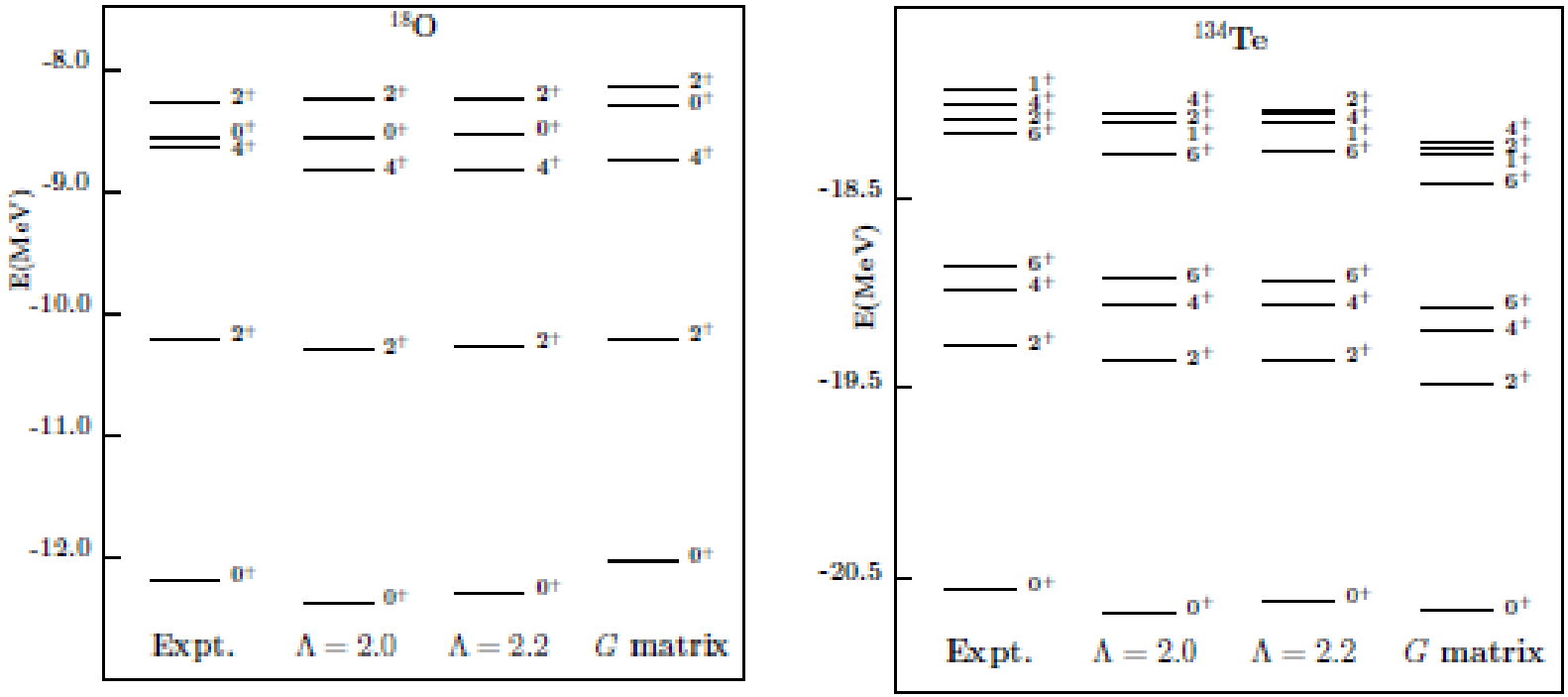
A.Poves, A.P.Zuker, Phys. Rep. 70 (1981)
G. Martinez-Pinedo et al, Phys. Rev. C55 (1997)

- Least-square fit of all the m.e. – by a linear-combination method (GXPF1 for pf-shell)

B.A.Brown, W.A.Richter, Phys. Rev. C74 (2006)
M. Honma et al, Phys. Rev. C65 (2002); idem 69 (2004)

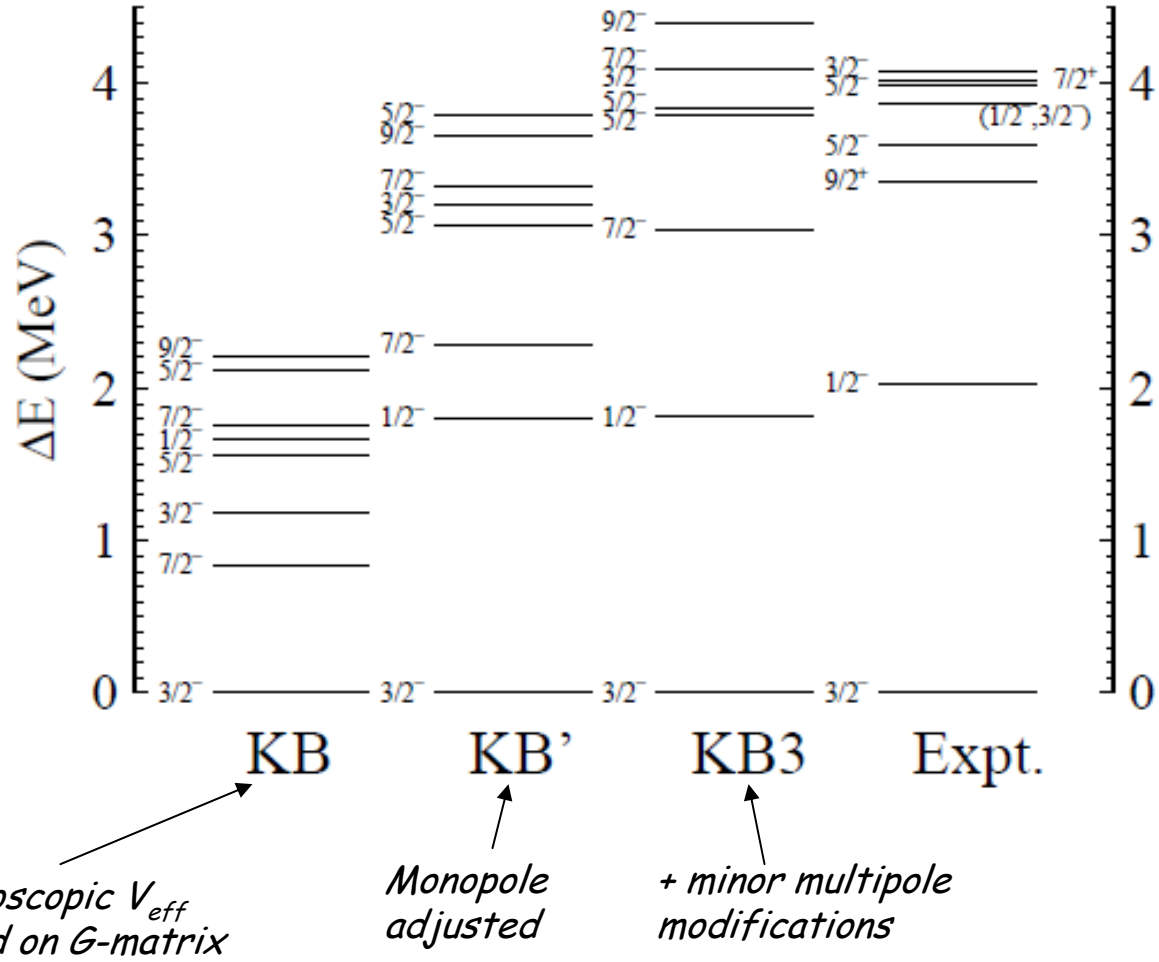
If the model space contains all important degrees of freedom, the shell model is extremely powerful !

V_{low-k} versus G-matrix



From L.Coraggio, A.Covello et al, Prog. Part. Nucl. Phys. 62 (2009) 135

Microscopic effective interaction (G-matrix based) before and after adjustment: ^{49}Ca

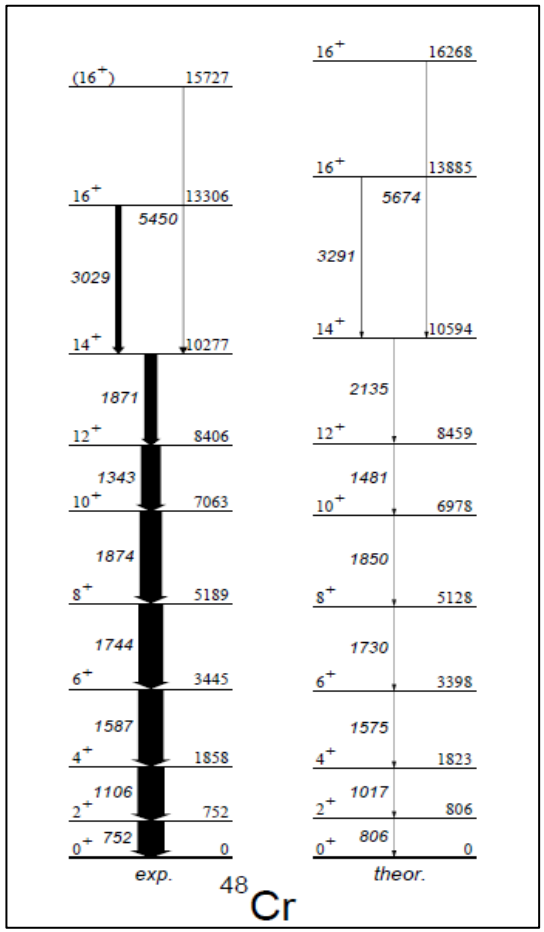


Kuo, Brown (1965)

Poves, Zuker (1981)

Figure from E. Caurier et al, Rev. Mod. Phys. 77 (2005) 427

^{48}Cr in *pf*-shell model space



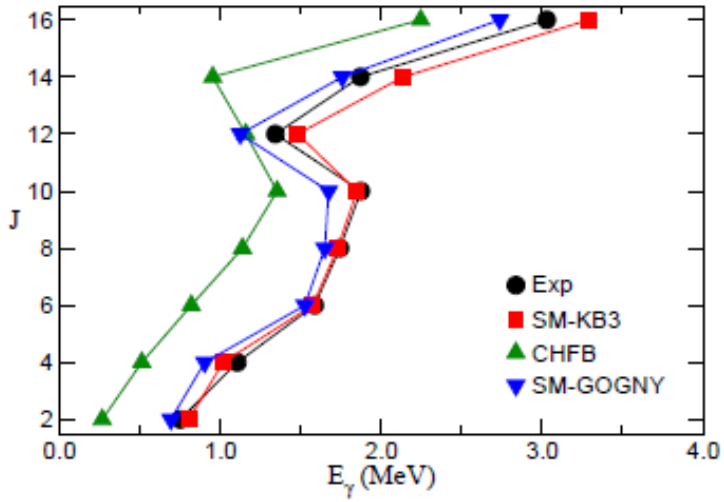
KB3 (semi-empirical interaction in *pf*-shell model space) Strasbourg-Madrid

For J<10 :

$$E_J \sim J(J+1)$$

$$Q_0 = \frac{(J+1)(2J+3)}{3K^2 - J(J+1)} Q_{spec}(J), \quad K \neq 1$$

$$B(E2; J \rightarrow J-2) = \frac{5}{16\pi} e^2 \left| (JK\,20 \mid J-2, K) \right|^2 Q_0^2$$

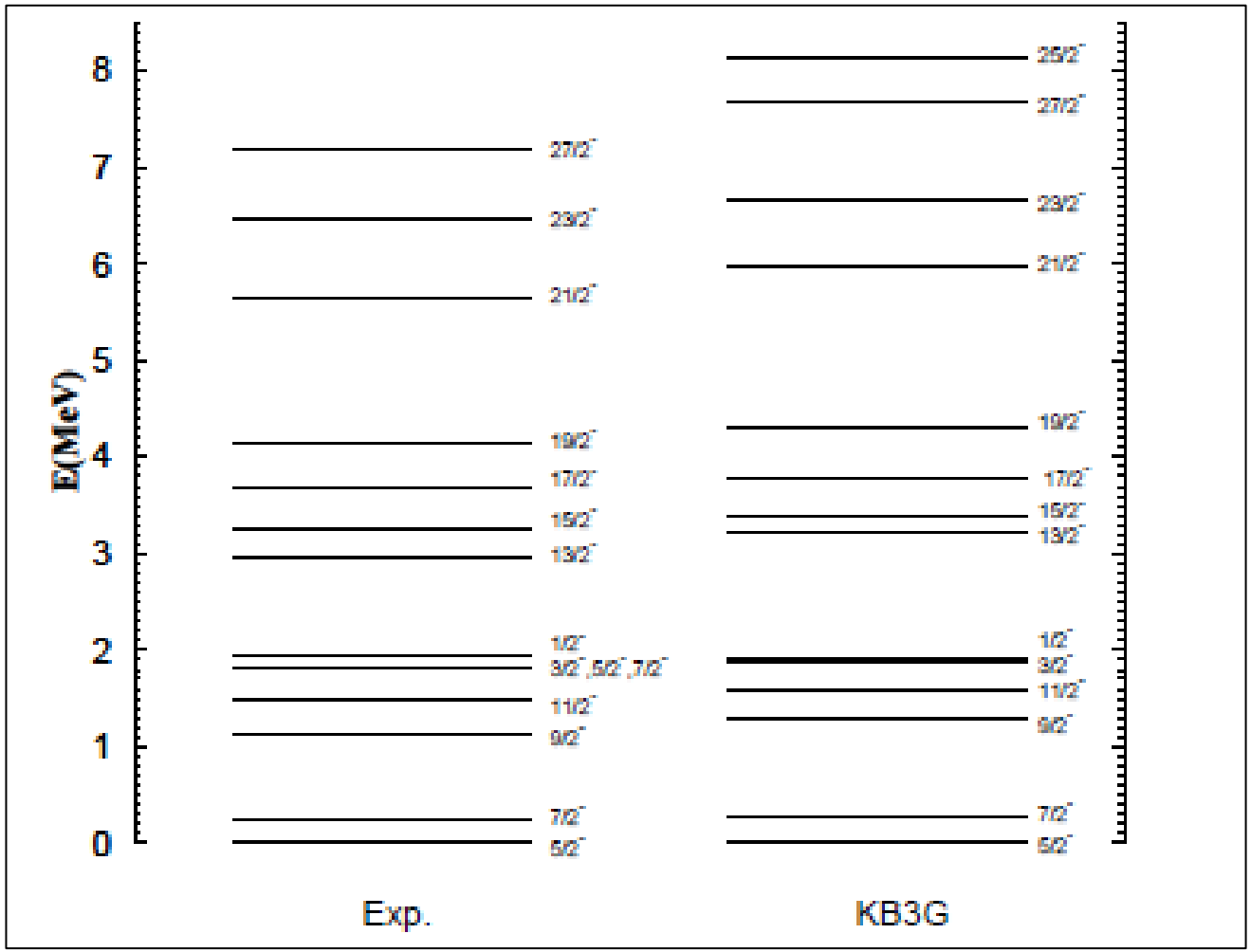


J<10: collective rotation
J=10-12 : backbending phenomenon (competition between rotation and alignment of $0f_{7/2}$ particles)
J>12 : spherical states

Figures from E.Caurier et al, Rev. Mod. Phys. 77 (2005) 427

Semi-empirical (microscopic, adjusted) effective interactions

$^{51}Mn_{26}$



Poves et al (2001)

E.Caurier et al, Rev. Mod. Phys. 77 (2005) 427

Ab-initio No-Core Shell Model

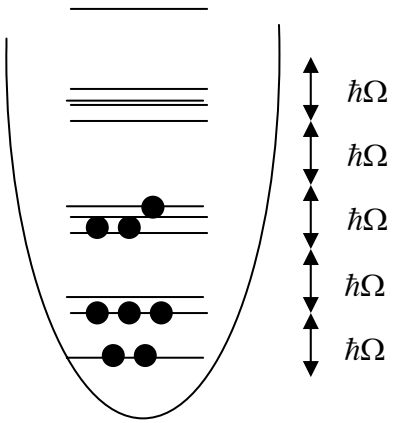
Shell model calculations for all A nucleons in N$\hbar\Omega$ space.

$$\hat{H}_A \Psi(1,\ldots,A) = E \Psi(1,\ldots,A)$$

Navratil, Barrett and collaborators (1996,1998,2000,…)

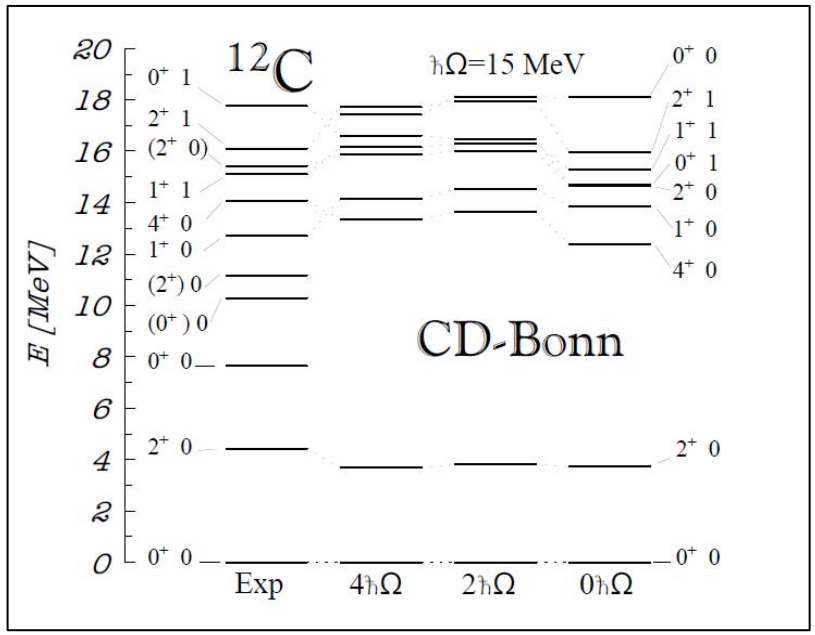
V_{eff} from unitary transformation method (exact decoupling of m.e.)

Okubo (1954), Da Providencia, Shakin (1964)
Suzuki, Lee (1980), Suzuki, Okamoto (1983)



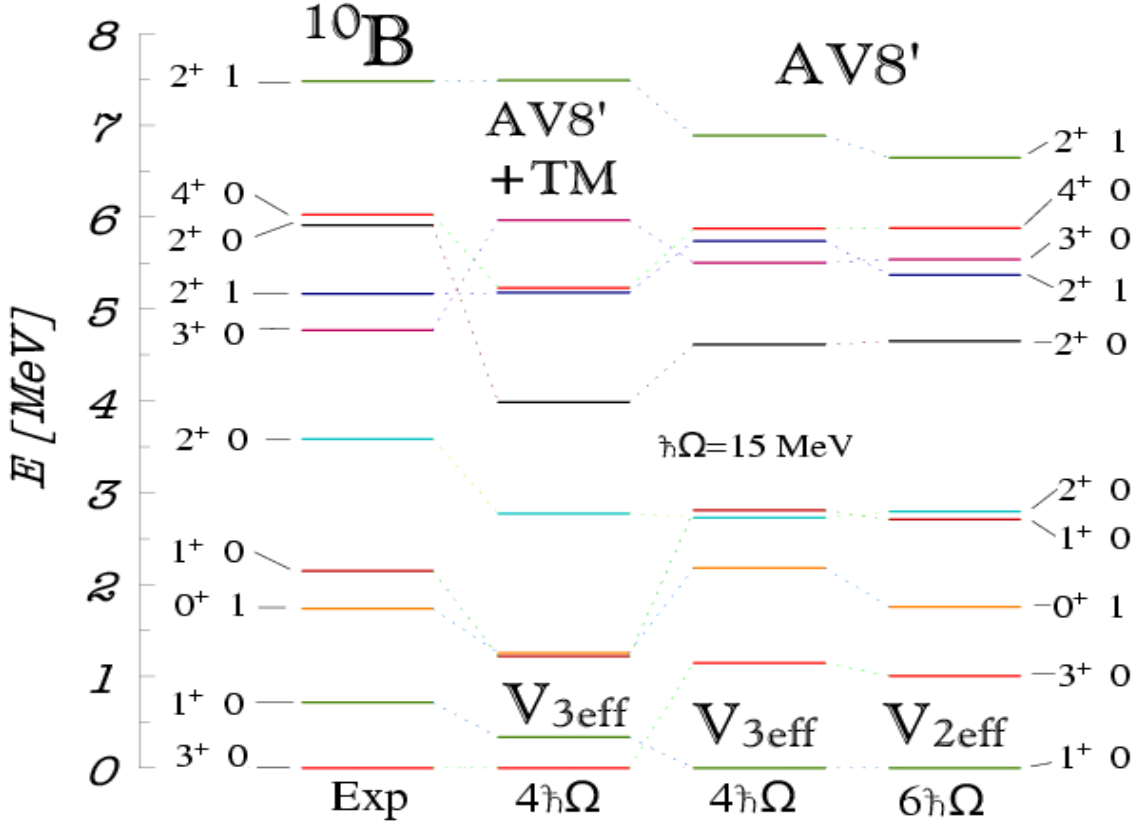
$$\tilde{H} = X^{-1} H_A X$$

$$Q\tilde{H}P = 0, \; H_{eff} = P\tilde{H}P$$



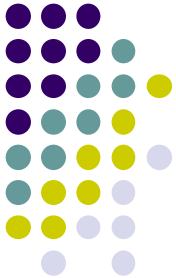
From P. Navratil, J.Vary, B.Barrett, Phys. Rev. Lett.84 (2000)

Three-body force in no-core approach



Figures taken from: E.Ormand, P. Navrátil

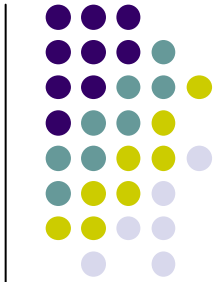
Summary and Conclusions



- Shell model represent a powerful theoretical model to describe low-energy nuclear spectroscopy
- Having got $E_{J,k}$, $\Psi_{J,k}$ one can calculate matrix elements of operators to compare with experiment (spectroscopic factors, static and transition electromagnetic moments - Q, μ, B(E2), …, weak decays - β, $\beta\beta$, lifetimes, etc)
- There is a clear link to the NN interaction, although more developments in the effective interaction theory is required

The shell model as unified view of nuclear structure
E.Caurier et al, Rev. Mod. Phys. 77 (2005) 427

References



Shell-model theory

Shell-Model Applications in Nuclear Spectroscopy
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North-Holland (1977)

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Effective interactions

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