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Modern Theory of Nuclear Forces

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Derivation of nuclear forces

Nuclear forces are defined as irreducible (i.e. non-iterative) contributions to the amplitude and can be derived using various methods.

S-matrix-based method

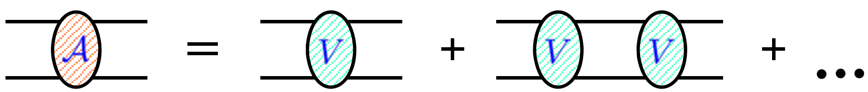
Robilotta, da Rocha '97; Kaiser et al. '97,'01,...; Higa et al. '03,'04; ...

Idea: the potential is derived through (perturbative) matching to the scattering amplitude.

$$\mathcal{A} \;=\; V + V\,G_0\,V + V\,G_0\,V\,G_0\,V + \ldots$$

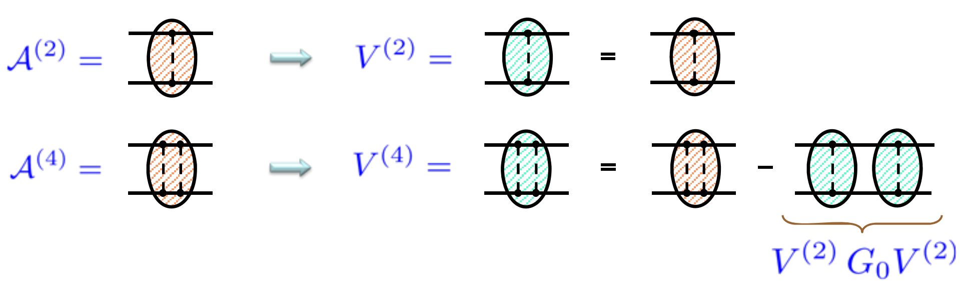
calculated using standard methods (for $\mathcal{A}$) — *matching to* $\mathcal{A}$ *allows to define* V

calculate in ChPT ⟹



⟸ *define* V *by matching to* $\mathcal{A}$

For example:

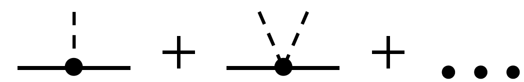


Derivation of nuclear forces

Old-fashioned time-ordered perturbation theory

Weinberg '90,'91; Ordonez et al. '92,'94; van Kolck '94

Consider mesons interacting with non-relativistic nucleons: $H = H_0 + H_I\,, \quad H_I =$



Schrödinger equation:

$$\begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix} = E \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix}$$

nucleonic states $|N\rangle,\ |NN\rangle,\ \ldots$

states with mesons $|N\pi\rangle,\ |N\pi\pi\rangle,\ \ldots$

projectors

← *can not solve (infinite-dimensional eq.)*

Effective Schrödinger equation for $|\phi\rangle$:

$$|\psi\rangle = \frac{1}{E - \lambda H \lambda} H |\phi\rangle \quad \Longrightarrow \quad \left(H_0 + V_{\rm eff}^{\rm t-o}(E)\right) |\phi\rangle = E |\phi\rangle$$

where $$V_{\rm eff}^{\rm t-o}(E) = \eta H_I \eta + \eta H_I \lambda \frac{1}{E - \lambda H \lambda} \lambda H_I \eta$$

$$= \eta H_I \eta + \eta H_I \frac{\lambda}{E - H_0} H_I \eta + \eta H_I \frac{\lambda}{E - H_0} H_I \frac{\lambda}{E - H_0} H_I \eta + \ldots$$

- $V_{\rm eff}^{\rm t-o}$ depends on E
- $|\phi\rangle$ not orthonormal: $\langle \phi_i | \phi_j \rangle = \langle \Psi_i | \Psi_j \rangle - \langle \psi_i | \psi_j \rangle = \delta_{ij} - \langle \phi_i | H_I \left(\frac{1}{E - \lambda H \lambda} \right)^2 H_I | \phi_j \rangle$

Derivation of nuclear forces

Method of unitary transformation

Taketani, Mashida, Ohnuma'52, Okubo '54, E.E., Glöckle, Meißner '98,'00,'05

Find a unitary operator U such that: $\tilde{H} \equiv U^\dagger \begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda \tilde{H} \lambda \end{pmatrix}$

- no dependence on energy (per construction),
- unitary transformation preserves the norm of $|\phi\rangle$

How to compute U?

It is convenient to parameterize U in terms of the operator $A = \lambda A \eta$ *(Okubo '54)*:

$$U = \begin{pmatrix} \eta(1 + A^\dagger A)^{-1/2} & -A^\dagger(1 + AA^\dagger)^{-1/2} \\ A(1 + A^\dagger A)^{-1/2} & \lambda(1 + AA^\dagger)^{-1/2} \end{pmatrix}$$

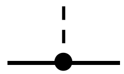
Require that $\eta \tilde{H} \lambda = \lambda \tilde{H} \eta = 0$ $\Longrightarrow$ $\lambda \left(H - [A,\ H] - AHA \right) \eta = 0$

The major problem is to solve the nonlinear decoupling equation.

Notice: similar methods widely used in particle & nuclear physics (Lee-Suzuki) and to deal with few- and many-body problems.

Derivation of nuclear forces

Example: expansion in powers of the coupling constant

$H_I =$  $\propto g \quad \Longrightarrow \quad$ ansatz: $A = A^{(1)} + A^{(2)} + A^{(3)} + \ldots$

Recursive solution of the decoupling equation $\lambda\,(H - [A,\ H] - AHA)\,\eta = 0$

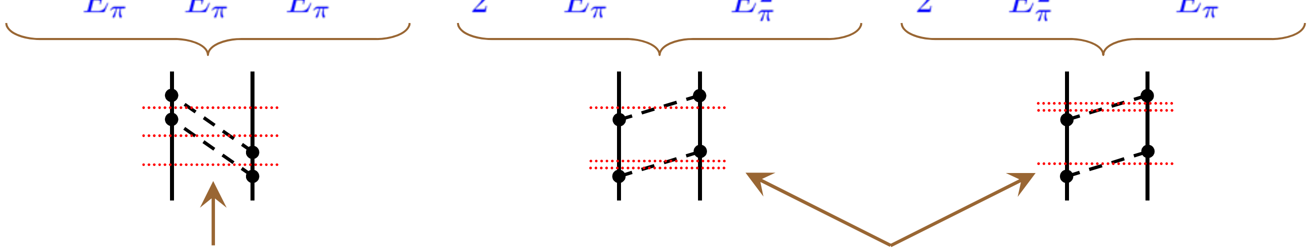
$$g^1: \quad \lambda(H_I - [A^{(1)},\ H_0])\eta = 0 \quad \Longrightarrow \quad A^{(1)} = -\lambda \frac{H_I}{E_\eta - E_\lambda}\eta$$

$$g^2: \quad \lambda(H_I\, A^{(1)} - [A^{(2)},\ H_0])\eta = 0 \quad \Longrightarrow \quad A^{(2)} = -\lambda \frac{H_I\, A^{(1)}}{E_\eta - E_\lambda}\eta$$

$$\ldots$$

In the static approximation, i.e. in the limit $m \to \infty$, one has: $E_\eta - E_\lambda \sim E_\pi$. One obtains:

$$\begin{aligned} V_{\rm eff} \;=\; & -\eta H_I \frac{\lambda}{E_\pi} H_I \eta \\ & -\eta H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \eta + \frac{1}{2}\eta H_I \frac{\lambda}{E_\pi} H_I \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta + \frac{1}{2}\eta H_I \frac{\lambda}{E_\pi^2} H_I \eta H_I \frac{\lambda}{E_\pi} H_I \eta + \ldots \end{aligned}$$



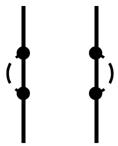
same as in old-fashioned perturbation theory

wave-function renormalization (missing in old-fashioned perturbation theory)

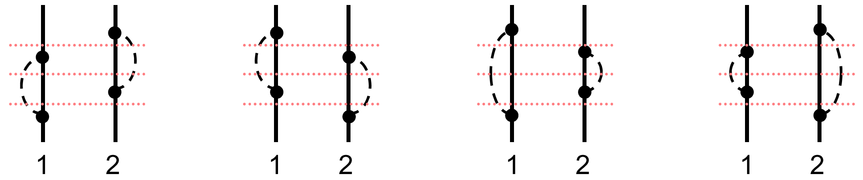
Derivation of nuclear forces

Consider self-energy insertions at 2 non-interacting nucleons:

Expect no contributions to the 2N Hamilton operator!



- **old-fashioned perturbation theory**



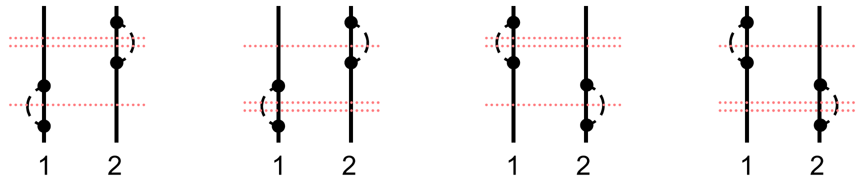
$$
\begin{aligned}
V_{\rm eff}^{\rm t-o} &= -\eta H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \eta \\
&= \mathcal{M}\left(-\frac{2}{\omega_1\omega_2(\omega_1+\omega_2)} - \frac{1}{\omega_1^2(\omega_1+\omega_2)} - \frac{1}{\omega_2^2(\omega_1+\omega_2)}\right) \\
&= \mathcal{M}\left(-\frac{1}{\omega_1^2\omega_2} - \frac{1}{\omega_1\omega_2^2}\right)
\end{aligned}
$$

common isospin, spin & momentum structure (depends on the form of H_I)

What is wrong ??

- **method of unitary transformation**

Additional contributions
(wave-function renormalization)



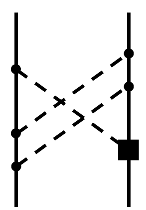
$$
V_{\rm eff} = V_{\rm eff}^{\rm t-o} + \frac{1}{2}\eta H_I \frac{\lambda}{E_\pi} H_I \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta + \frac{1}{2}\eta H_I \frac{\lambda}{E_\pi^2} H_I \eta H_I \frac{\lambda}{E_\pi} H_I \eta = V_{\rm eff}^{\rm t-o} + \mathcal{M}\left(\frac{1}{\omega_1^2\omega_2} + \frac{1}{\omega_1\omega_2^2}\right) = 0
$$

Derivation of nuclear forces

Application to chiral Lagrangians *(E.E. et al., '98)*

expansion in g ⟹ chiral expansion

Power counting



$\sim \left(\frac{Q}{\Lambda}\right)^{\nu}$

Count powers of Q using dimensional analysis
Alternatively: count powers of Λ!

The only source of Λ are the coupling constants ⟹ $\nu = -2 + \sum_i V_i \kappa_i$

$$\mathcal{L}_i = c_i \, (N^\dagger (\ldots) N)^{\frac{n_i}{2}} \; \pi^{p_i} \; (\partial_\mu, M_\pi)^{d_i} \quad \Longrightarrow \quad [c_i] = (mass)^{-\kappa_i} \quad \textit{with} \quad \kappa_i = d_i + \tfrac{3}{2} n_i + p_i - 4$$

Remember:

$\kappa_i < 0$ – *relevant (superrenorm.)*
$\kappa_i = 0$ – *marginal (renorm.)*
$\kappa_i > 0$ – *irrelevant (nonrenorm.)*

Examples:

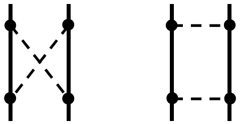
$N^\dagger \boldsymbol{\tau} \vec{\sigma} N \cdot \vec{\nabla} \boldsymbol{\pi} \longrightarrow \kappa_i = 1$

$(N^\dagger N)\,(N^\dagger N) \longrightarrow \kappa_i = 2$

- expansion in coupling constant ($H_i \sim g^{n_i}$) ⟷ chiral expansion ($H_i \sim (Q/\Lambda)^{\kappa_i}$)
- perturbation theory works since all $\kappa_i > 0$ (as a consequence of χ–symmetry)

Derivation of nuclear forces

Example: chiral 2π-exchange potential proportional to g_A^4 :



$$V_{2\pi}^{(2)}(q) = -\eta H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \eta + \frac{1}{2}\eta H_I \frac{\lambda}{E_\pi} H_I \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta + \frac{1}{2}\eta H_I \frac{\lambda}{E_\pi^2} H_I \eta H_I \frac{\lambda}{E_\pi} H_I \eta$$

$$= -\frac{g_A^4}{2(2F_\pi)^4} \int \frac{d^3 l}{(2\pi)^3} \frac{\omega_+^2 + \omega_+\omega_- + \omega_-^2}{\omega_+^3 \omega_-^3 (\omega_+ + \omega_-)} \left\{ \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left(\vec{l}^{\,2} - \vec{q}^{\,2} \right)^2 + 6 (\vec{\sigma}_2 \cdot [\vec{q} \times \vec{l}\,])(\vec{\sigma}_1 \cdot [\vec{q} \times \vec{l}\,]) \right\}$$

where $\omega_\pm = \sqrt{(\vec{q} \pm \vec{l}\,) + 4M_\pi^2}$

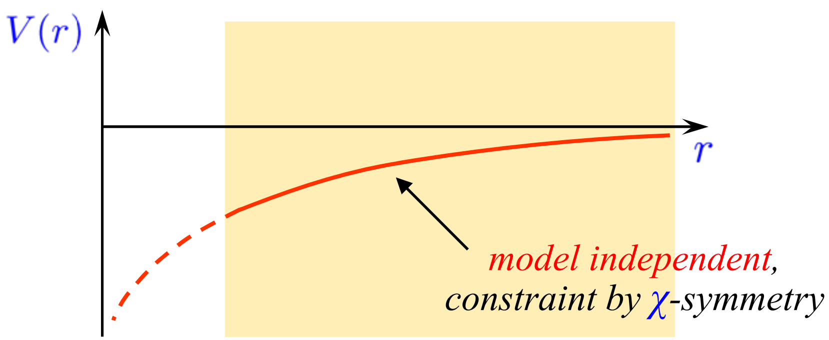
The integral has logarithmic and quadratic divergences, which can be absorbed into the short-range counter terms:

$$V_{\rm cont} = (\alpha_1 + \alpha_2\, q^2)\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \alpha_3 (\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) + \alpha_4 (\vec{\sigma}_1 \cdot \vec{\sigma}_2)\, q^2$$

Coordinate space representation:

$$V_{2\pi}^{(2)}(q) \longrightarrow V_{2\pi}^{(2)}(r)$$

The large-r behavior (i.e. the long-range part) of the potential is uniquely determined and does not depend on regularization.



Further reading

Nuclear potentials from field theory

- *Tamm, J. Phys. (USSR) 9 (45) 449; Dancoff, Phys. Rev. 78 (50) 382*
- *Okubo, Prog. Theor. Phys. (Japan) 12 (54) 603*
- *Fukuda, Sawada, Taketani, Prog. Theor. Phys. (Japan) 12 (54) 156*
- *Friar, Ann. Phys. 104 (77) 380*
- *Phillips, Reports on Progress in Physics XXII (59) 562 [review article]*

Applications to chiral EFT (selected papers)

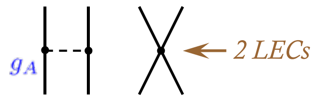
- *Ordonez, Ray, van Kolck, Phys. Rev. C53 (96) 2086*
- *Kaiser, Brockmann, Weise, Nucl. Phys. A625 (97) 758*
- *E.E., Glöckle, Meißner, NPA637 (98) 107; A714 (03) 535*
- *E.E., Eur. Phys. J. A34 (07) 197*

Two-nucleon force

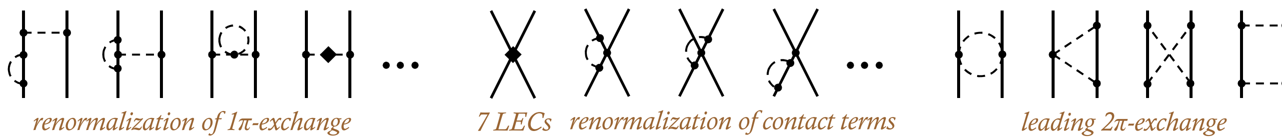
Ordonez et al. '94; Friar & Coon '94; Kaiser et al. '97; E.E. et al. '98,'03; Kaiser '99-'01; Higa et al. '03; …

Chiral expansion of the 2N force: $V_{2N} = V_{2N}^{(0)} + V_{2N}^{(2)} + V_{2N}^{(3)} + V_{2N}^{(4)} + \ldots$

- **LO:**

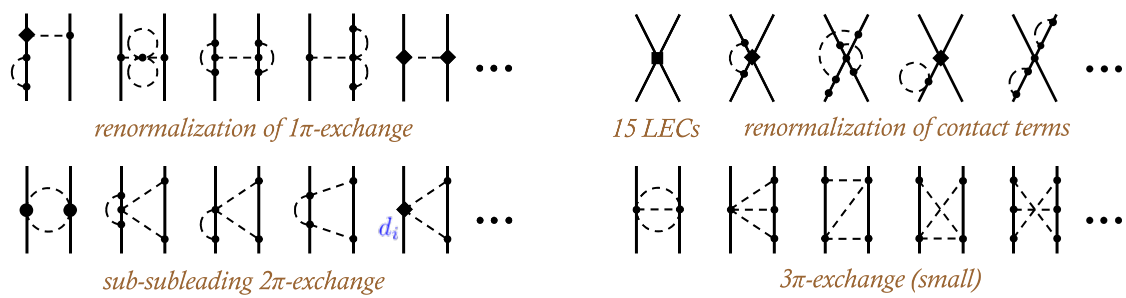


- **NLO:**



- **N²LO:**

- **N³LO:**



\+ isospin-breaking corrections…

van Kolck et al. '93,'96; Friar et al. '99,'03,'04; Niskanen '02; Kaiser '06; E.E. et al. '04,'05,'07; …

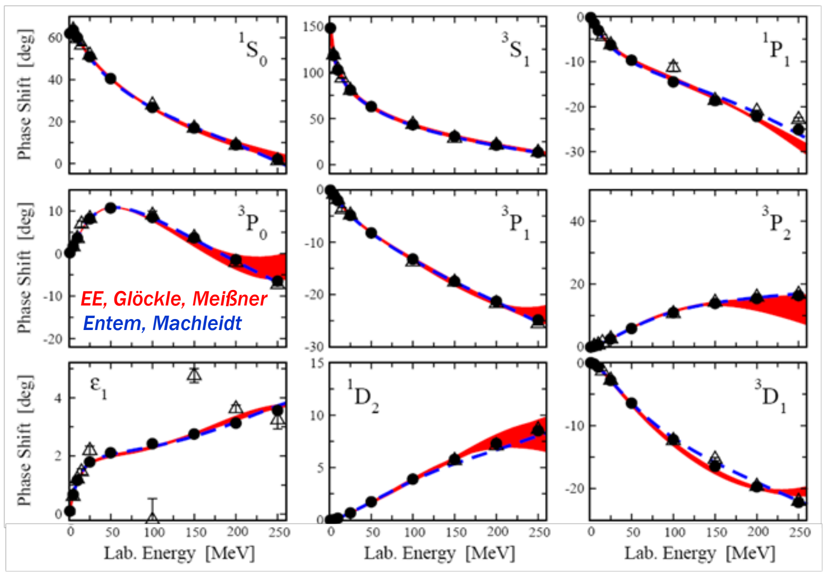
Results based on EFT with explicit Δ(1232) degrees of freedom available up to N²LO

Ordonez, Ray, van Kolck '96; Kaiser, Gerstendorfer, Weise '98; Krebs, E.E., Meißner '07,'08

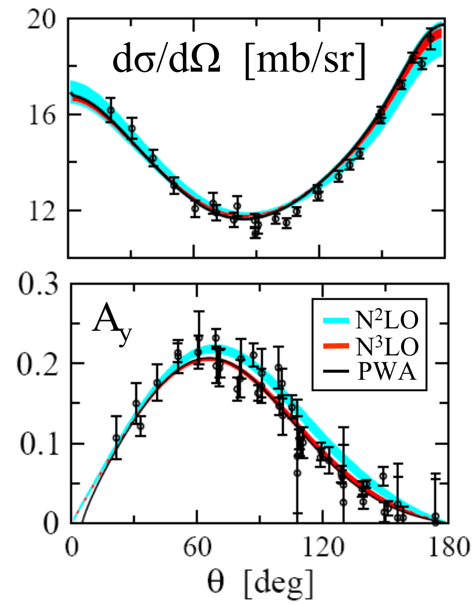
Two nucleons up to N^{3}LO

Entem, Machleidt '04; E.E., Glöckle, Meißner '05

Neutron-proton phase shifts at N^{3}LO



np scattering at 50 MeV

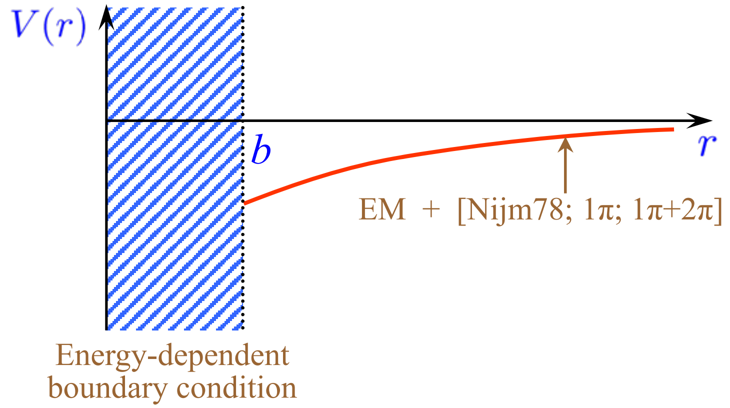


Deuteron binding energy & asymptotic normalizations A_s and η_d

	NLO	N^{2}LO	N^{3}LO	Exp
E_d [MeV]	$-2.171\ldots-2.186$	$-2.189\ldots-2.202$	$-2.216\ldots-2.223$	$-2.224575(9)$
A_S [fm$^{-1/2}$]	$0.868\ldots0.873$	$0.874\ldots0.879$	$0.882\ldots0.883$	$0.8846(9)$
η_d	$0.0256\ldots0.0257$	$0.0255\ldots0.0256$	$0.0254\ldots0.0255$	$0.0256(4)$

2π-exchange & NN phase shifts

Chiral 2π-exchange potential upto N^{2}LO has been tested in an energy-dependent proton-proton partial-wave analysis,
Rentmeester et al.'99,'03

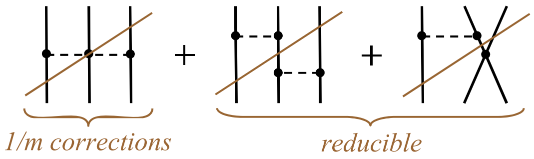


	$b = 1.4$ fm		$b = 1.8$ fm	
	#BC	$\chi^2_{\rm min}$	#BC	$\chi^2_{\rm min}$
Nijm78	19	1968.7	–	–
OPE	31	2026.2	29	1956.6
OPE + TPE(l.o.)	28	1984.7	26	1965.9
OPE + χTPE	23	1934.5	22	1937.8

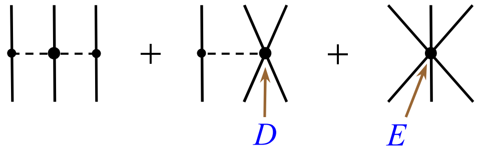
Similar results obtained based on the distorted-wave methods
Birse & McGovern'04, Birse'07

Three-nucleon force

- **NLO:** does not contribute
  Weinberg '91; Coon & Friar '94; van Kolck '94; E.E. et al.,'98; …



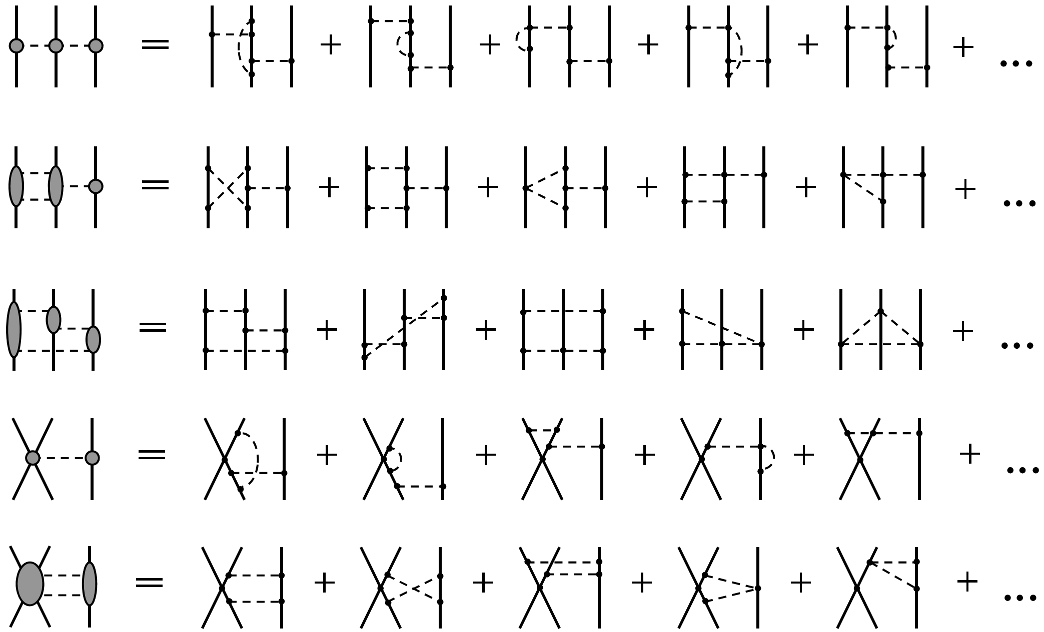
- **N²LO:** first nonvanishing contributions
  van Kolck '94; E.E. et al. '02



- **N³LO:** work in progress
  Bernard, E.E., Krebs, Meißner '07
  Ishikawa, Robilotta '07

  — no free parameters

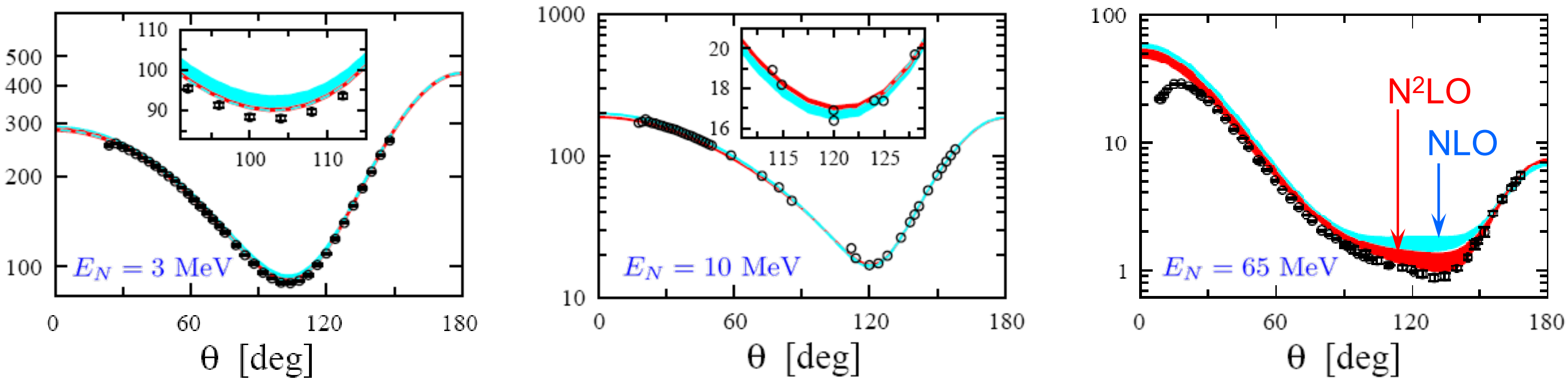
  — chiral symmetry plays essential role



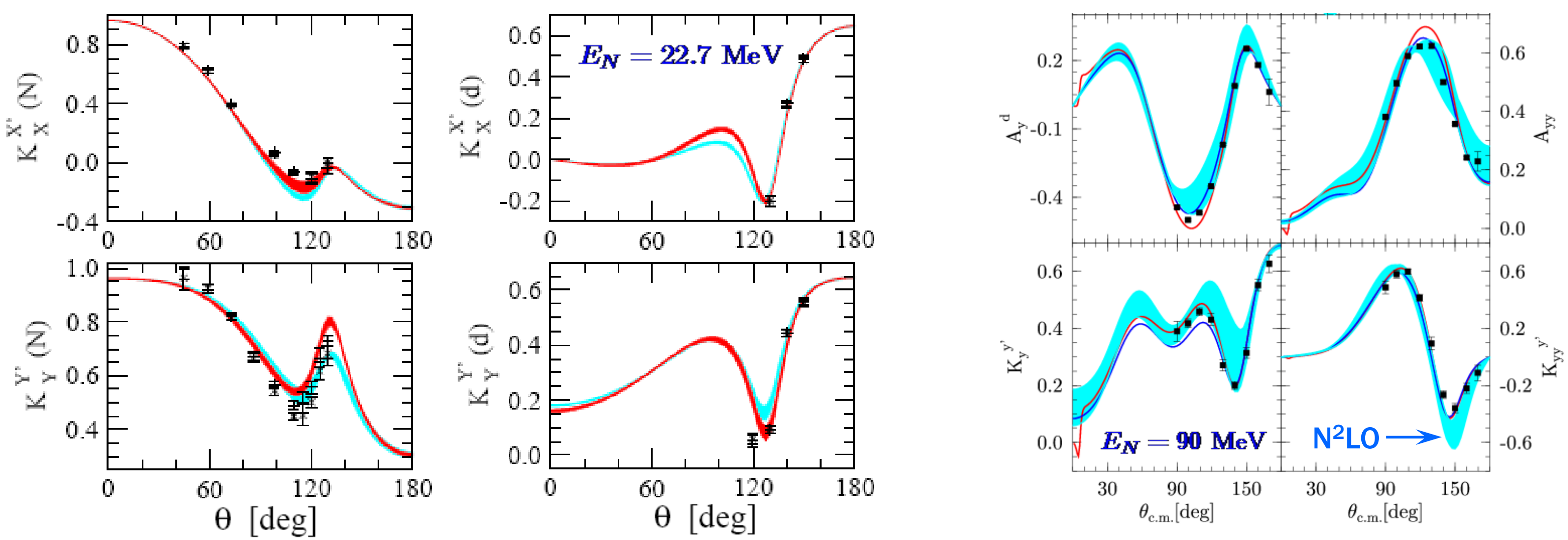
Elastic Nd scattering up to N^{2}LO

E.E. et al.'02; Kistryn et al.'05; Witala et al.'06; Ley et al.'06; Stephan et al.'07; ...

Differential cross section in elastic Nd scattering

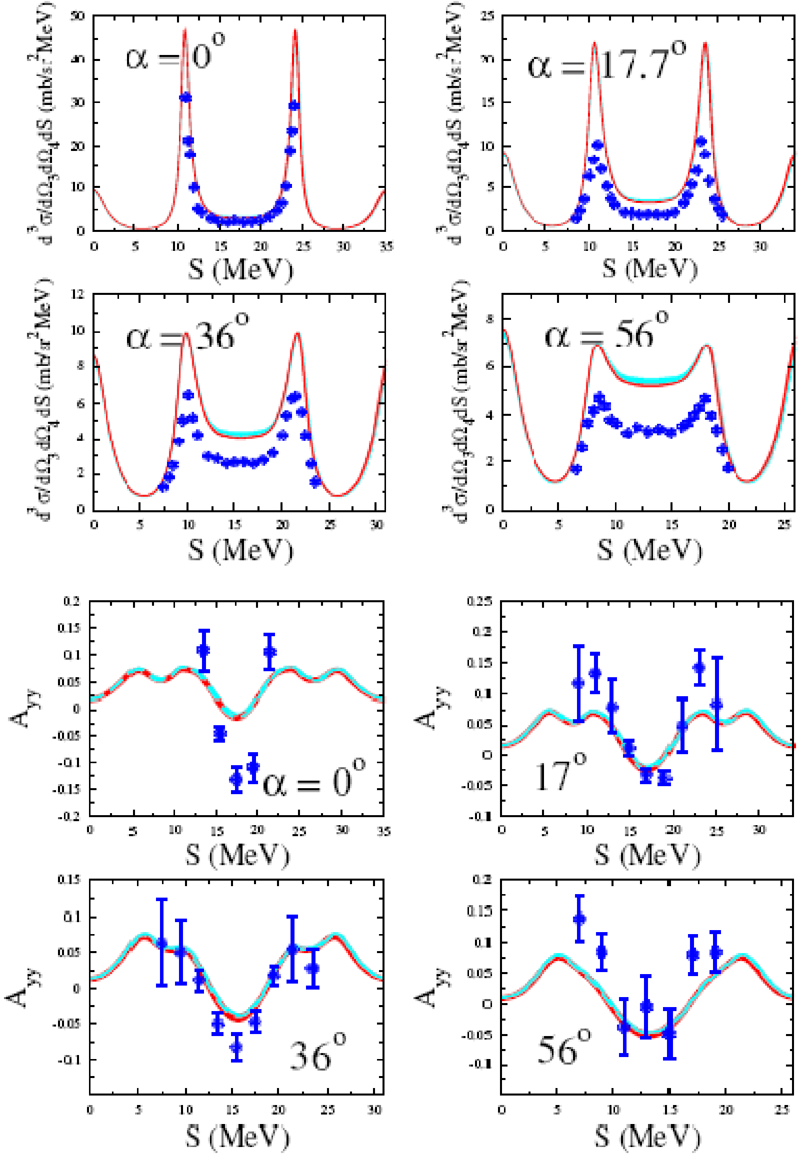


Polarization observables in elastic Nd scatering



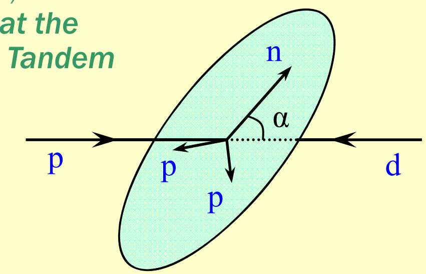
Deuteron breakup reaction

$\vec{d} + p \rightarrow p + p + n$ @ 19 MeV

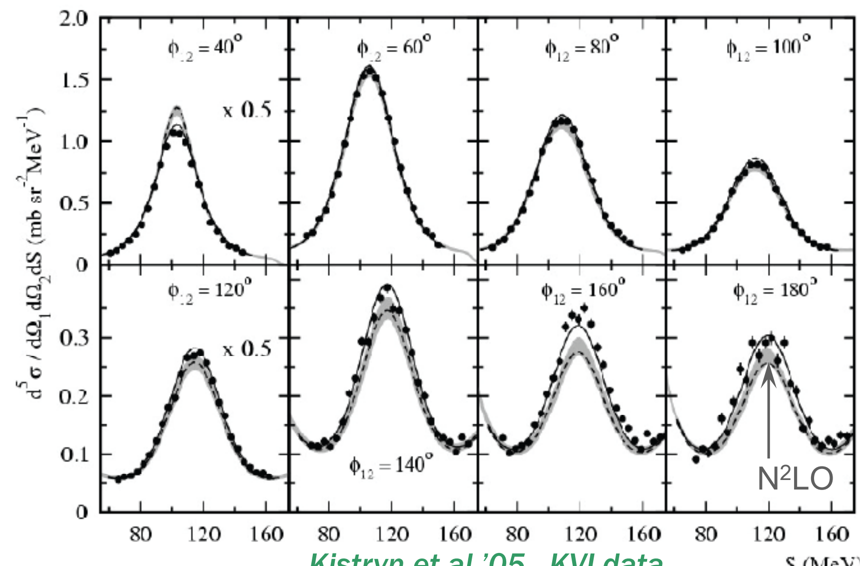


The so-called Symmetric-Constant-Energy geometry

Ley et al.'06; data taken at the Cologne FN Tandem accelerator



$d + p \rightarrow p + p + n$ @ 130 MeV



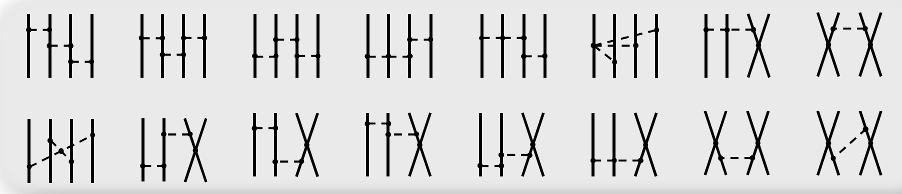
Kistryn et al.'05, KVI data

Four and more nucleons

The 4N force first appears at N^3LO

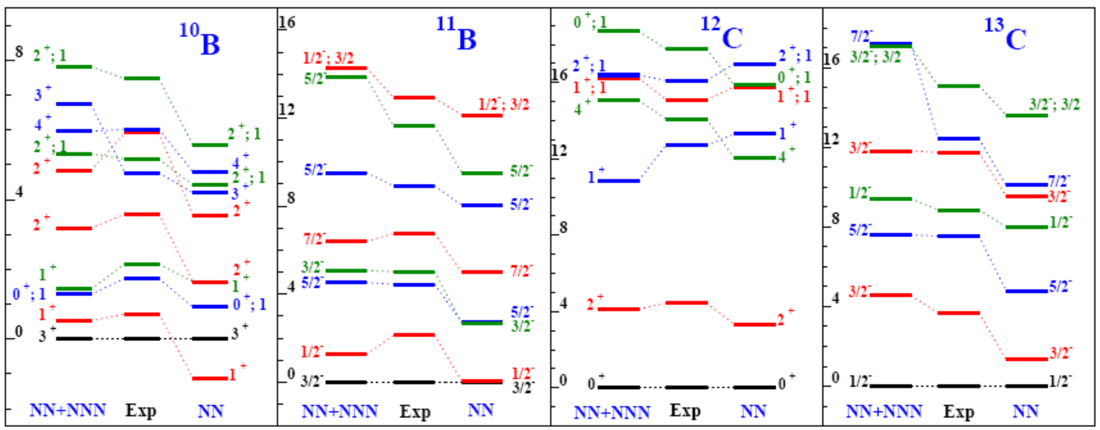
- — no free parameters
- — $\langle \Psi_{^4He} | \, V_{4N} \, | \Psi_{^4He} \rangle \sim$ few 100 keV

(Rozpedzik et al. '06)



E.E., PLB 639 (06) 456, EPJA 34 (07) 197

No-Core-Shell-Model results for ^{10}B, ^{11}B, ^{12}C and ^{13}C @ N^2LO



Navratil et al., PRL 99 (2007) 042501

^{4}He and ^{6}Li @ NLO and N^2LO

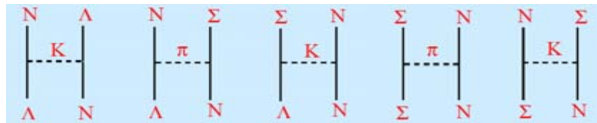
	NLO	N^2LO	Exp.
$E_{^4\text{He}}$	$-24.4 \ldots -28.8$	$-27.8 \ldots -29.1$	-28.3
$E_{^6\text{Li}}$	$-30.6 \ldots -34.2$	$-31.4 \ldots -33.2$	-32.0
$\Delta E_{^6\text{Li}}$	$1.7 \ldots 2.0$	$2.2 \ldots 2.4$	2.2

Nogga et al., NPA 737 (2004) 236

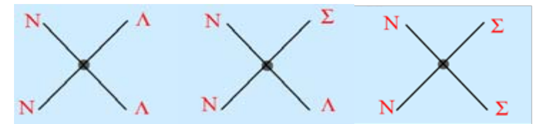
Hyperon-nucleon interactions

Polinder, Haidenbauer & Meißner '06, '07

Leading order:

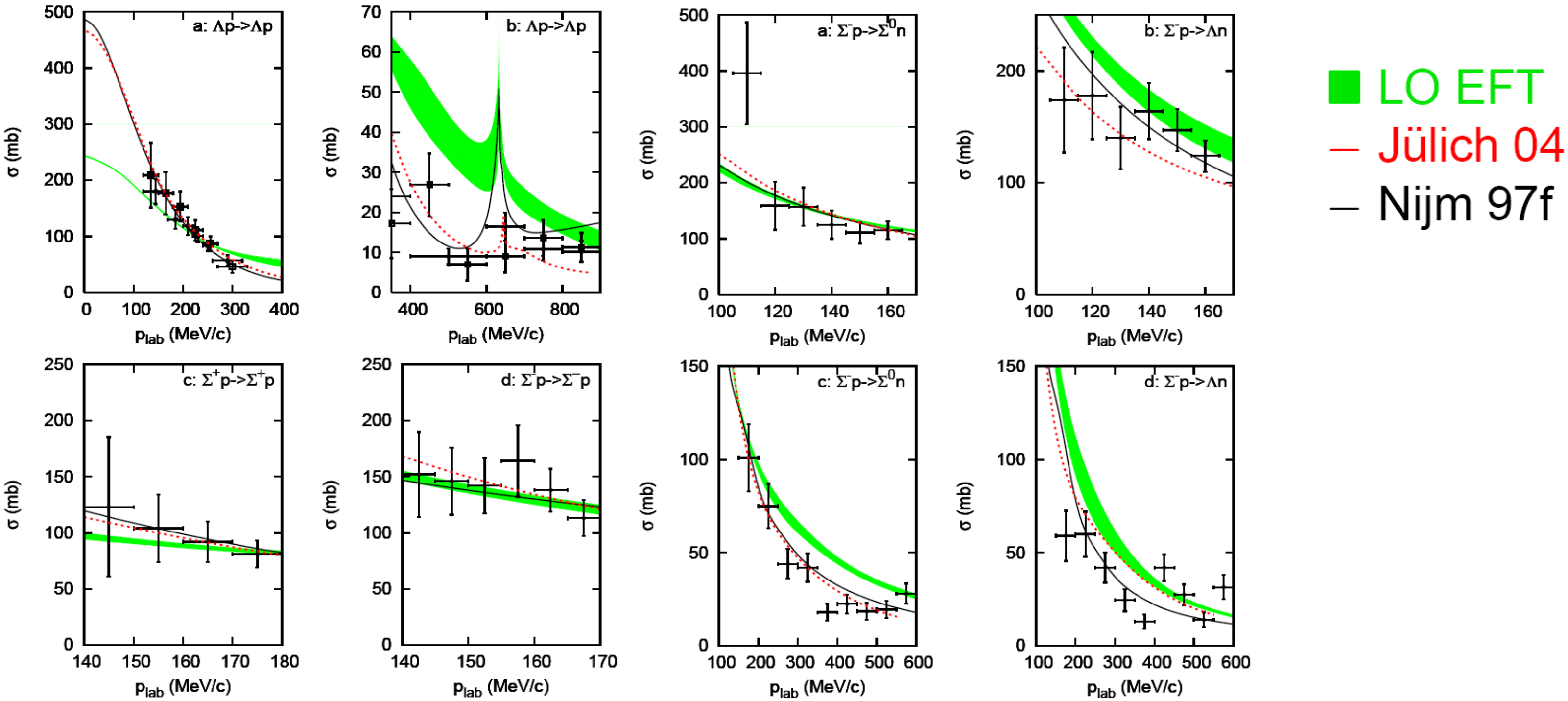


2 LECs fixed from g_A + SU(6)



5 LECs fixed from 35 YN data points

Total cross section as a function of p_{lab}



LO results for S=-2 available; extension to NLO in progress…

Further reading

Nucleon-nucleon potential at N^3LO

- *Entem, Machleidt, Phys. Rev. C68 (03) 041001*
- *E.E., Glöckle, Meißner, Nucl. Phys. A747 (05) 362*

Three-nucleon force and Nd scattering (selected papers)

- *van Kolck, Phys. Rev. C49 (94) 2932*
- *E.E. et al., Phys. Rev. Lett. 86 (01) 4787; Phys. Rev. C66 (02) 064001*
- *Ishikawa, Robilotta, Phys. Rev. C76 (07) 014006*
- *Bernard, E.E., Krebs, Meißner, Phys. Rev. C77 (08) 064004*

Chiral forces and light nuclei (incomplete)

- *Nogga et al., Nucl. Phys. A737 (04) 237; Phys. Rev. C73 (06) 064002*
- *Navratil et al., Phys. Rev. Lett. 99 (07) 042501*

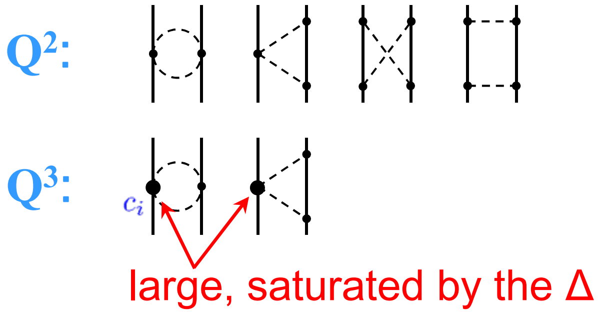
Review articles

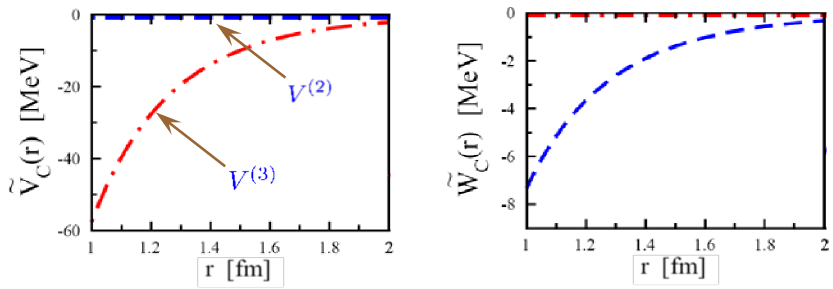
- *E.E., Prog. Part. Nucl. Phys. 57 (06) 654*
- *E.E., Hammer, Meißner, arXiv:0811.1338, Rev. Mod. Phys., in print*

Nuclear forces and the Δ isobar

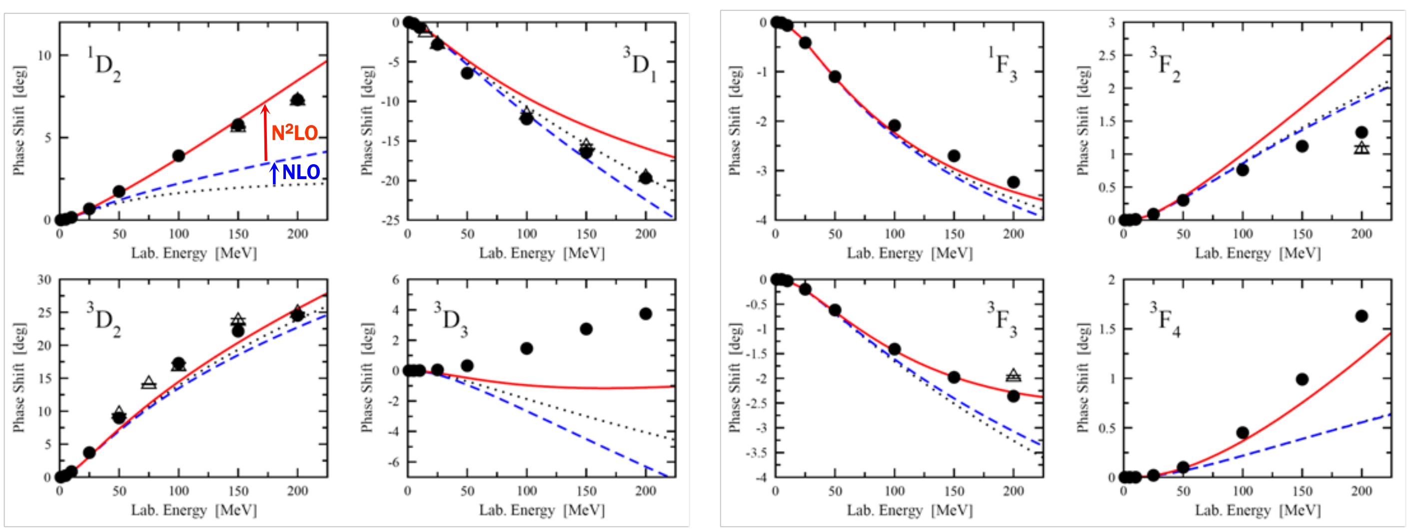
$$V_{1\pi} = V_{1\pi}^{(0)} + \underbrace{V_{1\pi}^{(2)} + V_{1\pi}^{(3)}}_{\textit{renormalize LECs}} + \ldots\ ; \quad V_{2\pi} = V_{2\pi}^{(2)} + V_{2\pi}^{(3)} + \ldots\ ; \quad V_{\rm cont} = \underbrace{V_{\rm cont}^{(0)} + V_{\rm cont}^{(2)}}_{\textit{contribute to S- and P-waves}} + \ldots$$

The subleading contributions to the 2π-exchange potential is unnaturally large:

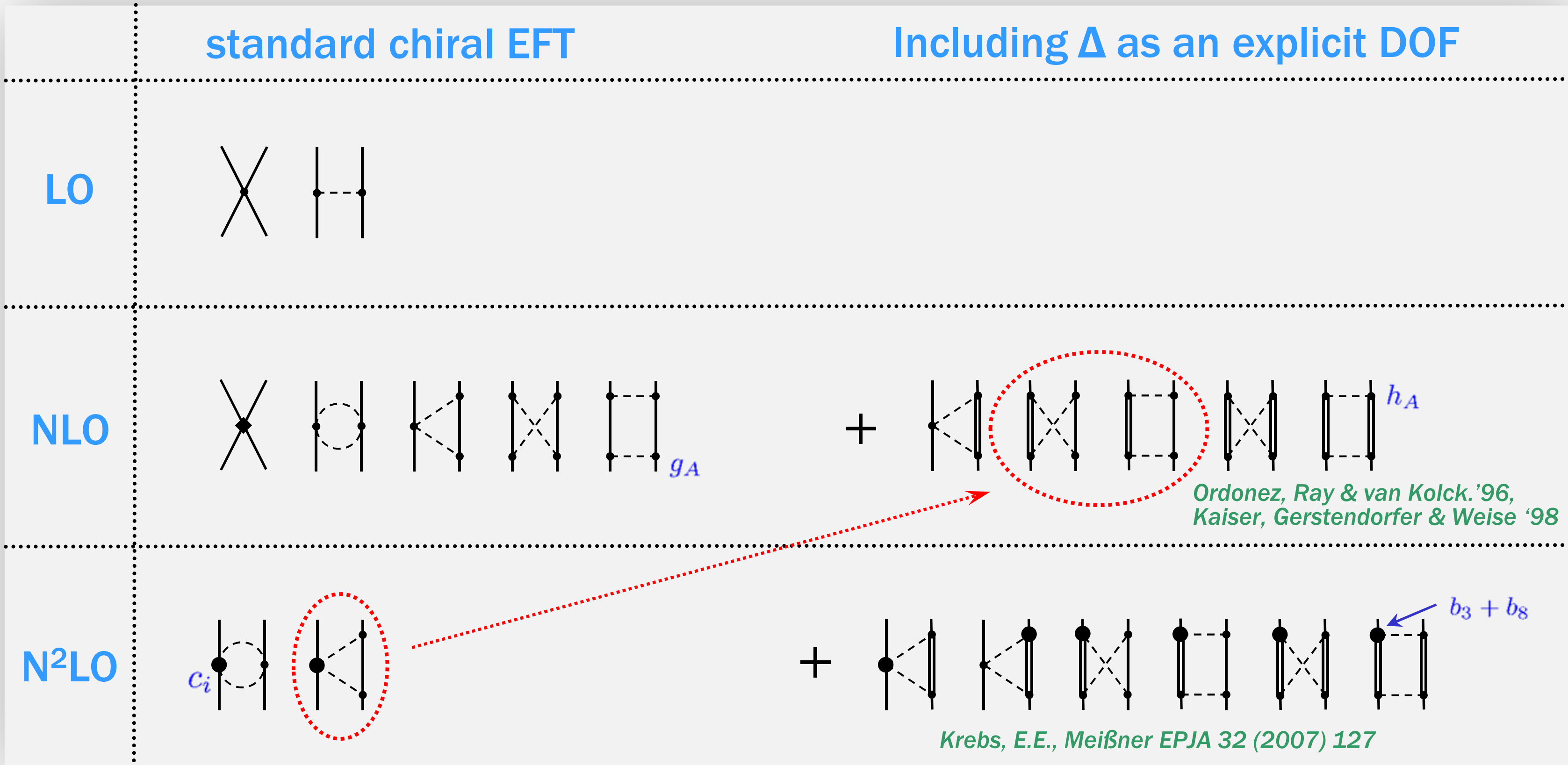




This is also visible in peripheral NN partial waves:

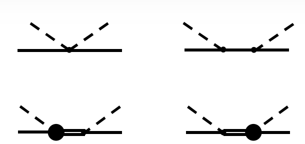


2NF in EFT with & without Δ



- The LECs $c_{2,3,4}$ strongly reduced after inclusion of the Δ (determined from πN threshold parameters)

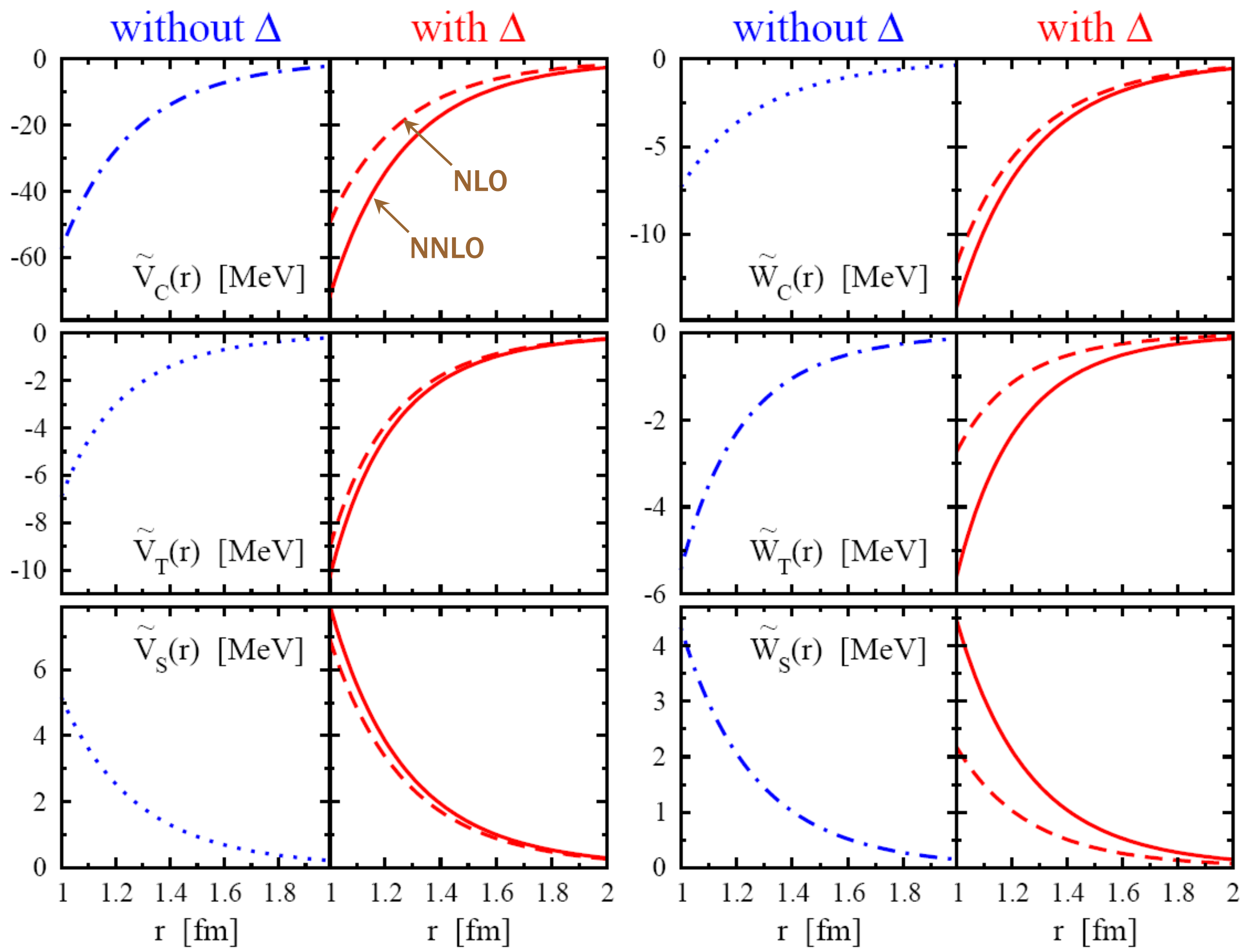
  $c_{2,3,4} \simeq 2.8,\ -3.9,\ 2.9$ ⇒ $c_{2,3,4} \simeq -0.3,\ -0.8,\ 1.3$



- Δ-contributions to 1π-exchange and contact interactions only lead to shifts in LECs

2NF in EFT with & without Δ

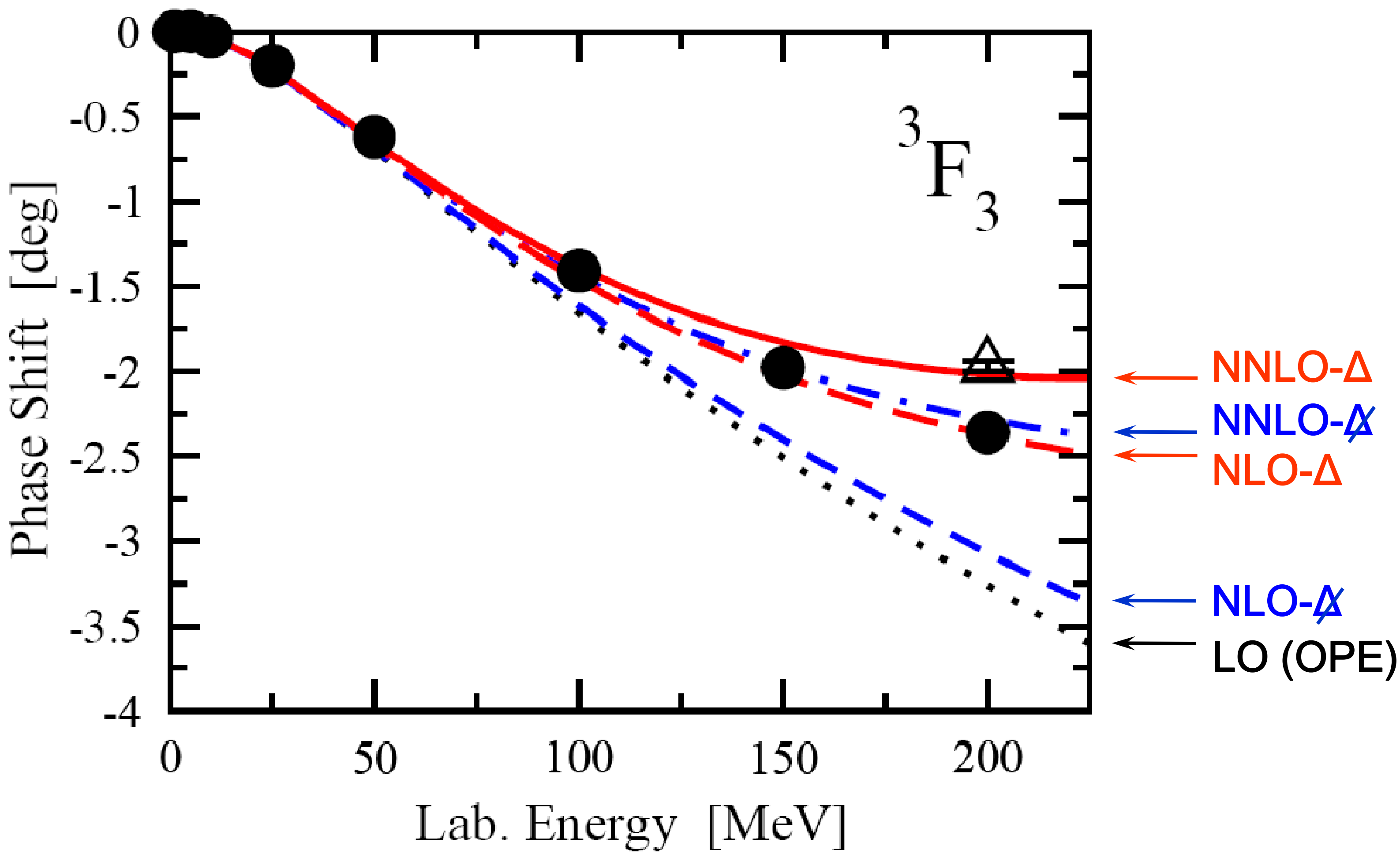
Chiral 2π-exchange up to N²LO with and without explicit Δ



Δ-contributions to the 2π–exchange potential are remarkably well reproduced in Δ-less EFT by resonance saturation of c_i's

2NF in EFT with & without Δ

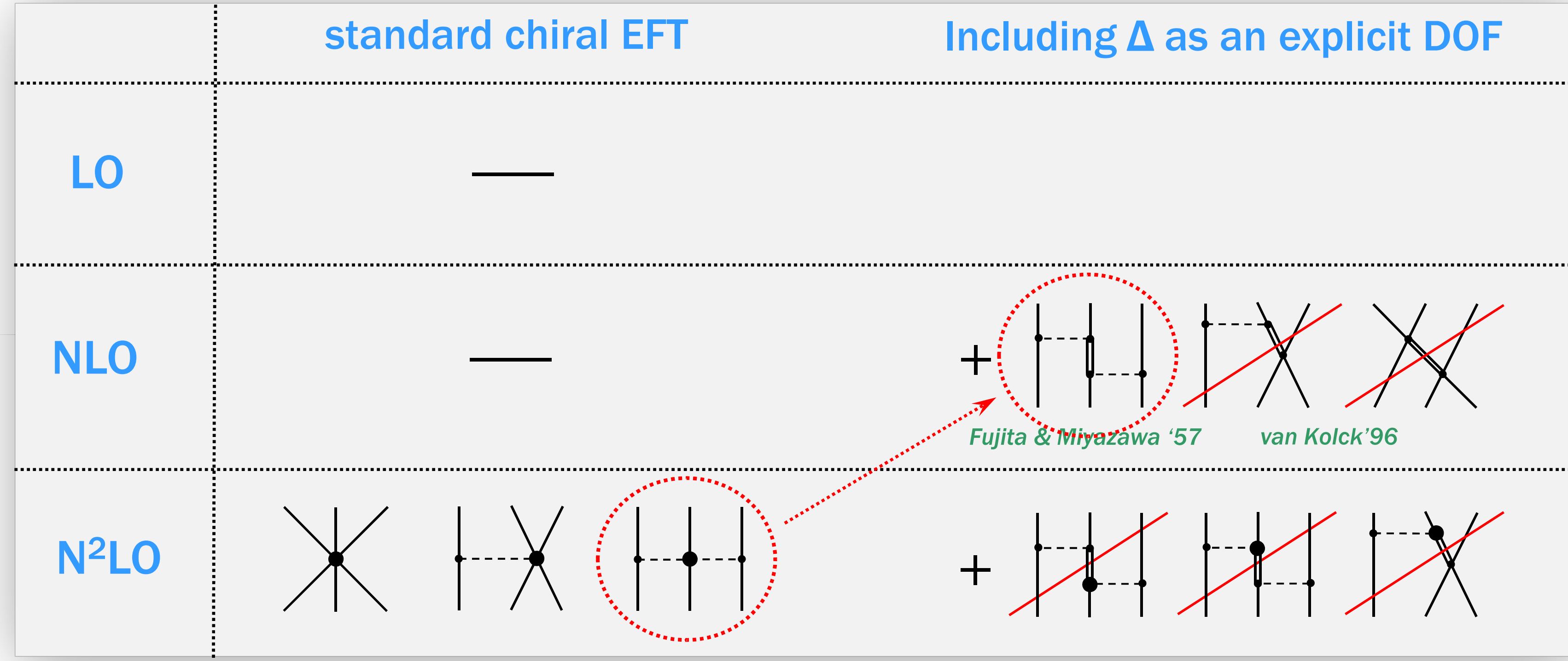
$^3F_3$ partial waves up to NNLO with and without Δ



(calculated in the first Born approximation)

3NF in EFT with & without Δ

E.E., Krebs, Meißner, Nucl. Phys. A806 (2008) 65



- The LO NNNΔ contact interaction $\bar{T}_i^{\mu} N \bar{N} S_{\mu} \tau^i N + \text{h.c.}$ vanishes due to the Pauli principle
  ⇒ the LECs D and E are not saturated by the delta.
- No contributions from subleading 2π–exchange due to ∂^0 at the $b_3 + b_8$ vertex.
- The entire effect of the Δ is given by a partial shift of the N^2LO TPE 3NF to NLO…
- Presumably, significant effects at N^3LO (similar to the 3π–exchange 2NF, *Kaiser '00*).

Further reading

- *Ordonez, Ray, van Kolck, Phys. Rev. C53 (96) 2086*
- *Kaiser, Gerstendorfer, Weise Nucl. Phys. A637 (98) 395*
- *Krebs, E.E., Meißner, Eur. Phys. J. A32 (07) 127;*
- *E.E., Krebs, Meißner, Nucl. Phys. A806 (08) 65; Phys. Rev C77 (08) 034006*

Summary & outlook

- Chiral effective field theory for nuclear forces
  - provides a model-independent and systematically improvable theoretical framework to derive nuclear forces in a consistent way
  - qualitative & quantitative understanding of nuclear forces and few-N dynamics, promising results for hyperon-nucleon scattering
  - many applications available (isospin-breaking effects, chiral extrapolations, reactions with external probes, …)
- In the future:
  - hypernuclei, electroweak reactions, heavier systems, higher precision, …