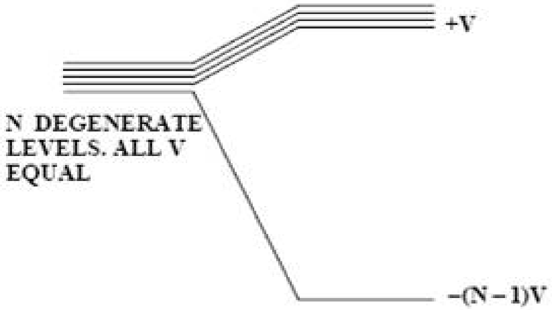
Development of collective behavior in nuclei

- Results primarily from correlations among valence nucleons.

- Instead of pure “shell model” configurations, the wave functions are mixed – linear combinations of many components.

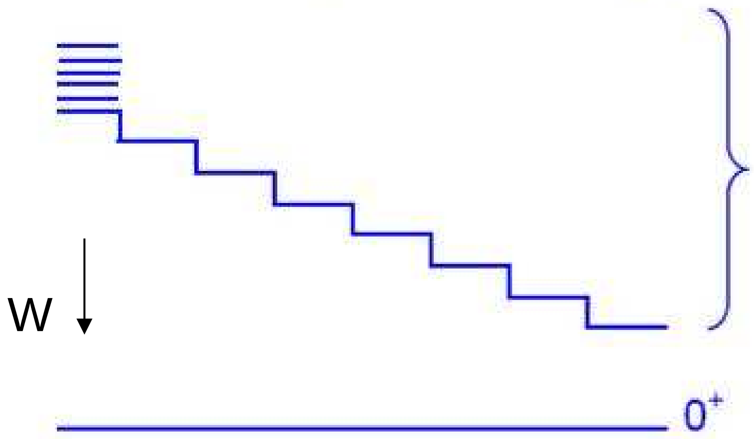
- Leads to a lowering of the collective states and to enhanced [illegible] eristic signatures.



$$\Psi_{LOWEST} = \frac{1}{\sqrt{N}}\left[\phi_1 + \phi_2 + \ldots + \phi_N\right]$$

Coherence and Transition Rates

Consider simple case of N degenerate levels: 2^+



$$\Delta E = (N-1)V$$

$$\Psi = a\varphi_1 + a\varphi_2 + \cdots a\varphi_N$$

$$\text{where } a = \frac{1}{\sqrt{N}}$$

$$\left(\sum_{i=1}^{N} a^2 = \frac{N}{N} = 1 \right)$$

Consider transition rate from $2_1^+ \rightarrow 0_1^+$

$$B(E2;\, 2_1^+ \rightarrow 0_1^+) = \frac{1}{2J_i + 1} \left\langle 0_1^+ \| E2 \| 2_1^+ \right\rangle^2$$

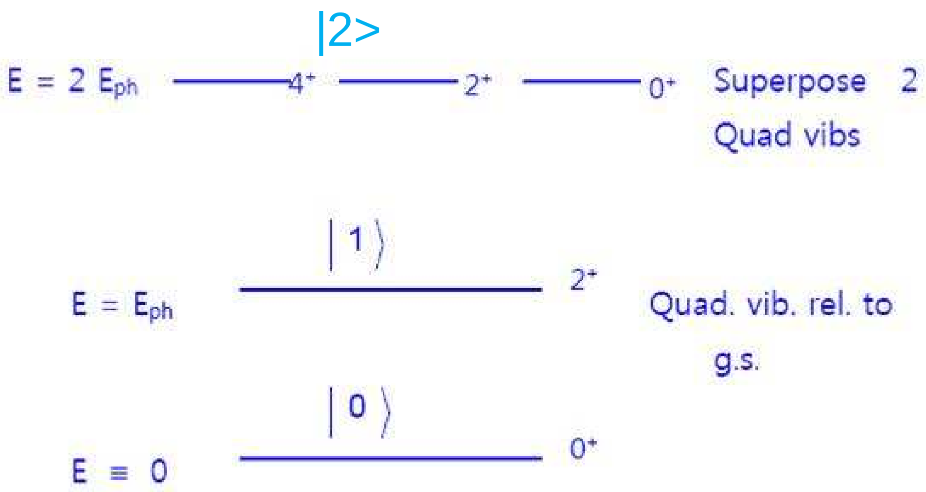
$$\left\langle 0_1^+ \| E2 \| 2_1^+ \right\rangle = \left\langle 0_1^+ \| E2 \| \Psi \right\rangle = a \sum_{i=1}^{N} \left\langle 0_1^+ \| E2 \| \varphi_i \right\rangle$$

The more configurations that mix, the stronger the B(E2) value and the lower the energy of the collective state.
Fundamental property of collective states.

Low Lying → **Quadrupole Vibrations**

Multiphonon States

Angular Momentum 2^+



Phonon creation and destruction operators

Quadrupole case

$b_{2\mu}$, $b^{\dagger}_{2\mu}$ (drop "2μ")

$\left| n_b \right\rangle \equiv$ state with n_b phonons

$$b \left| n_b \right\rangle = \sqrt{n_b} \left| n_b - 1 \right\rangle$$

$$b^{\dagger} \left| n_b \right\rangle = \sqrt{n_b + 1} \left| n_b + 1 \right\rangle$$

$$b \left| 0 \right\rangle = 0$$

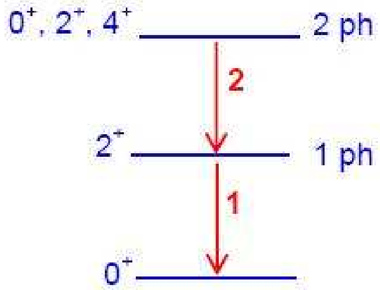
$$b^{\dagger} \left| 0 \right\rangle = \left| n_b = 1 \right\rangle = \Psi_{1\ \text{phonon}}$$

$b^{\dagger} b$ = number operator—counts n_b:

$$b^{\dagger} b \left| n_b \right\rangle = b^{\dagger} \sqrt{n_b} \left| n_b - 1 \right\rangle = \sqrt{n_b} \sqrt{(n_b - 1) + 1} \left| n_b \right\rangle$$

$$b^{\dagger} b \left| n_b \right\rangle = n_b \left| n_b \right\rangle$$

Electromagnetic Transitions in the phonon model



$E2$ operator is proportional to the annihilation operator, b, for a phonon.

$$\langle n_f | b | n_i \rangle = \langle n_f | \sqrt{n_i} | n_i - 1 \rangle$$
$$= \sqrt{n_i} \langle n_f | n_i - 1 \rangle$$
$$= \sqrt{n_i}\, \delta_{n_f, n_{i-1}}$$

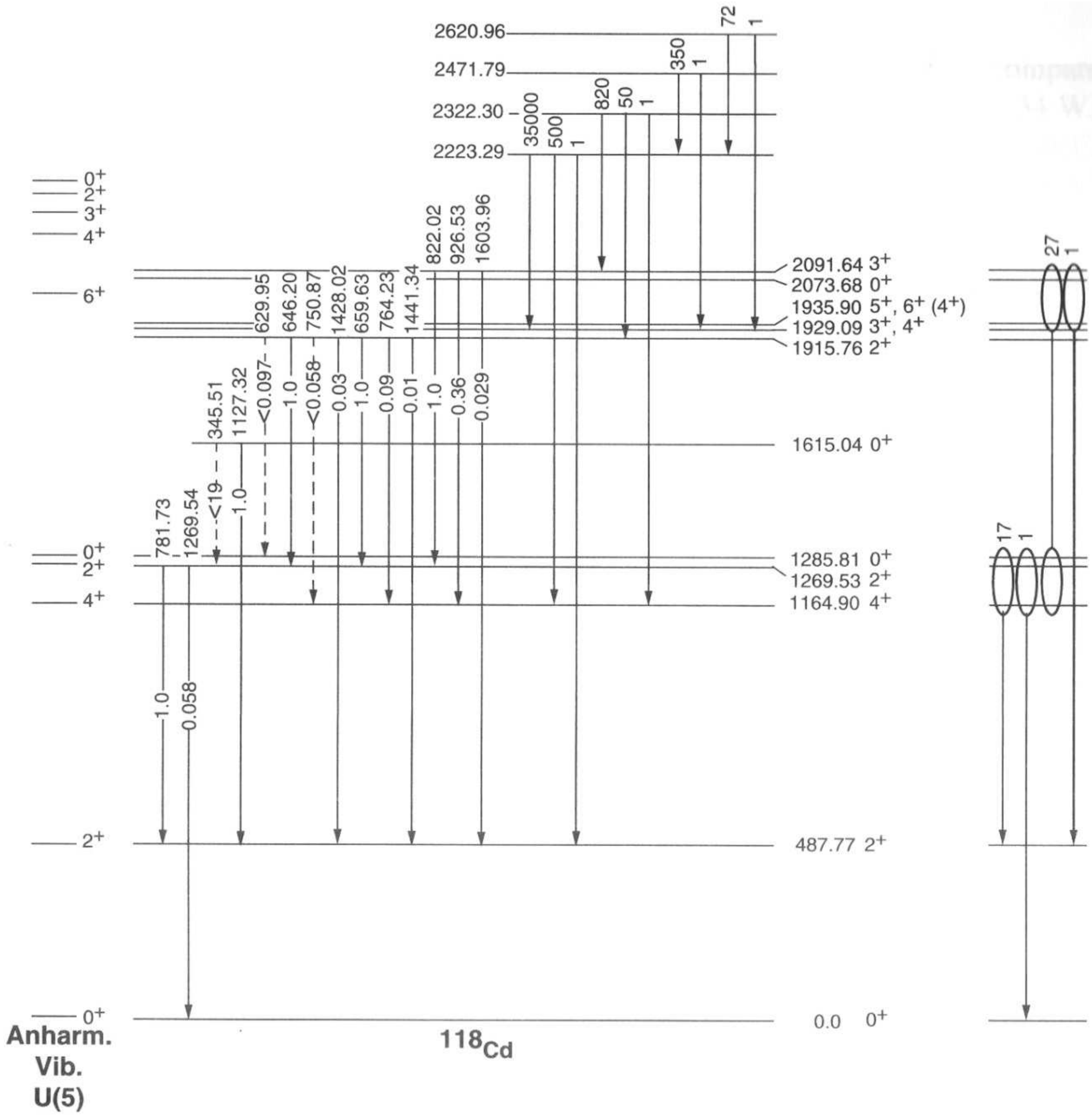
a) $E2$ transition probability

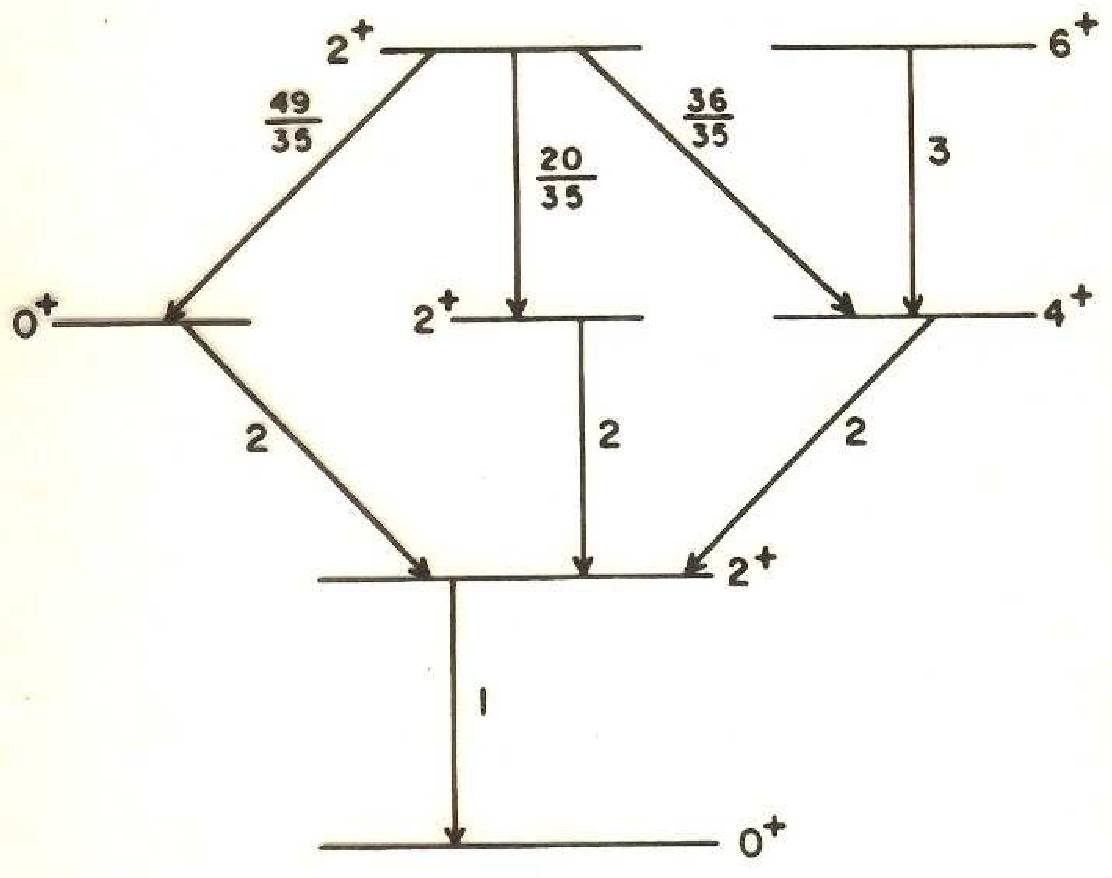
$$[\propto \langle \; | \; | \; \rangle^2] \propto \boxed{n_i}$$

b) Selection rule $\boxed{\Delta n = 1}$

c) Branching ratio $\dfrac{B(E2; n = 2 \rightarrow n = 1)}{B(E2; n = 1 \rightarrow n = 0)} = 2$

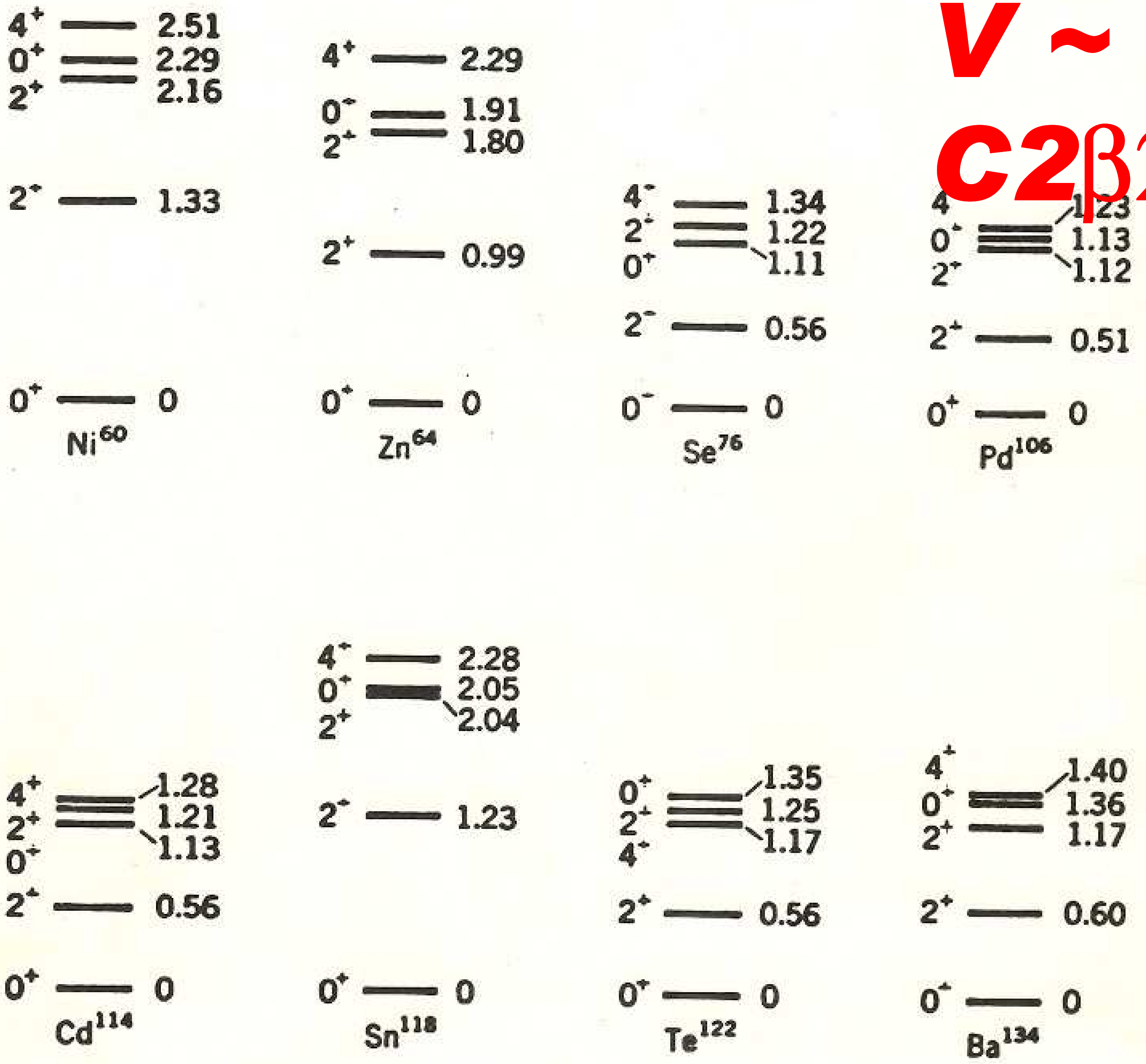
d) $B(E2; n = 2 \rightarrow 0^+ \text{ g.s.}) = 0$ --- forbidden


2620.96
2471.79
2322.30
2223.29
72
1
350
1
820
50
1
35000
500
1
822.02
926.53
1603.96
629.95
646.20
750.87
1428.02
659.63
764.23
1441.34
2091.64 3+
2073.68 0+
1935.90 5+, 6+ (4+)
1929.09 3+, 4+
1915.76 2+
345.51
1127.32
<0.097
1.0
<0.058
0.03
1.0
0.09
0.01
1.0
0.36
0.029
1615.04 0+
781.73
1269.54
<19
1.0
1285.81 0+
1269.53 2+
1164.90 4+
1.0
0.058
487.77 2+
0.0 0+
0+
2+
3+
4+
6+
0+
2+
4+
2+
0+
27
1
17
1
Anharm.
Vib.
U(5)
118Cd



B(E2) VALUES FOR DECAY OF
MULTI-PHONON STATES

$v \sim c^2\beta^2$



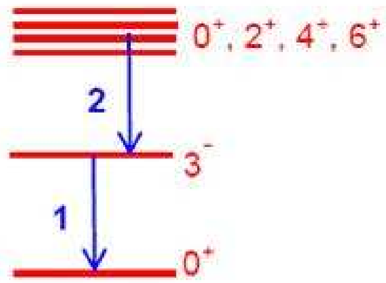
Octupole Vibrations

3^-

2-phonon $\quad 3^- \otimes 3^- \quad \Rightarrow \; J = 0^+, 2^+, 4^+, 6^+$

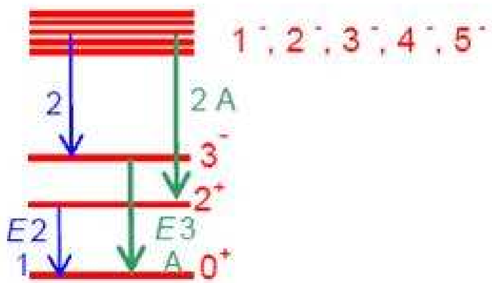
A few examples beginning to be known

${}^{96}_{40}Zr$, $\quad {}^{146}_{64}Gd$



Multi-phonon $\qquad$ Octupole – Quadrupole

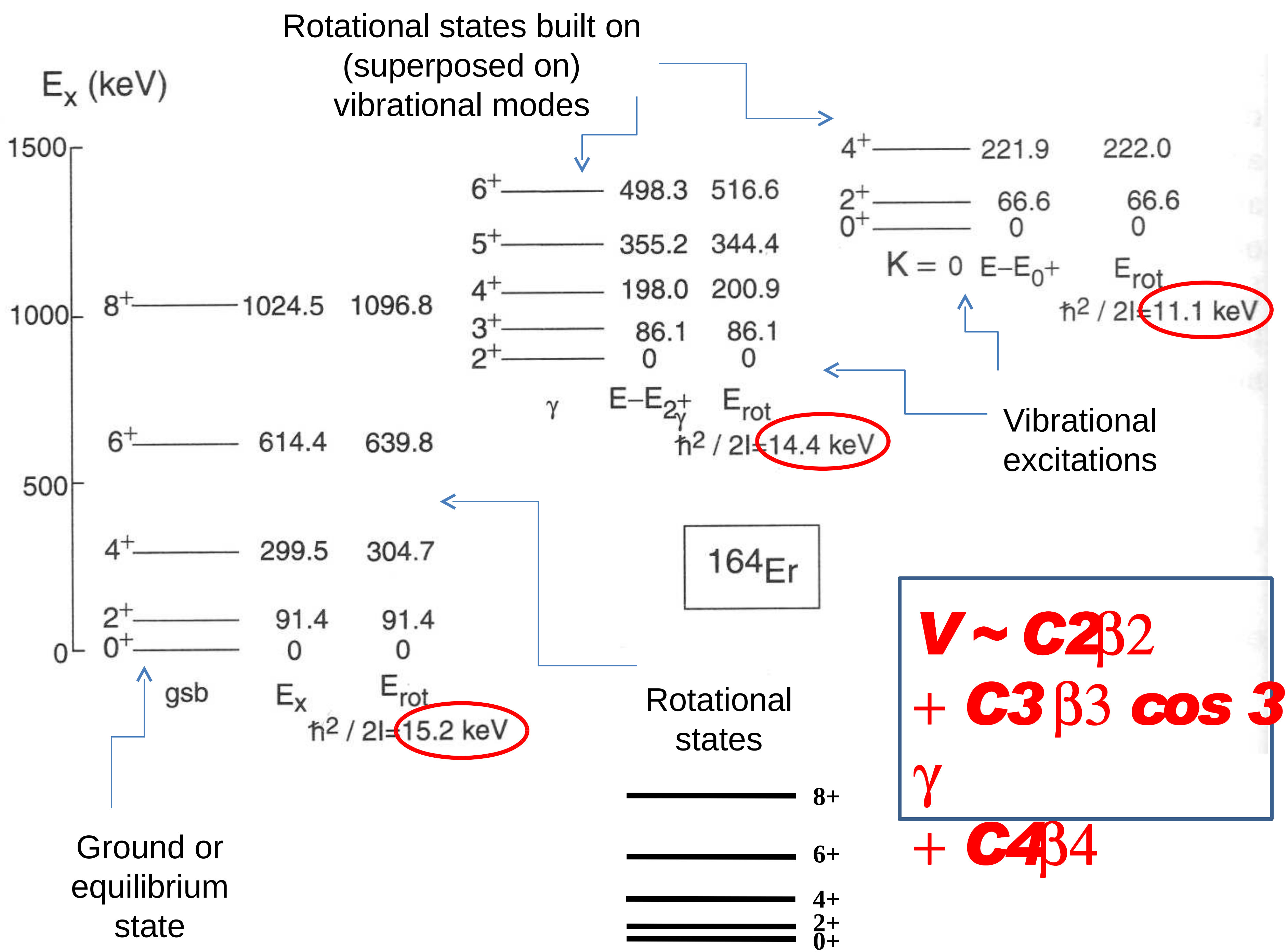
$3^- \otimes 2^+$



Deformed, ellipsoidal, rotational nuclei

Lets look at a typical example and see the various aspects of structure it shows

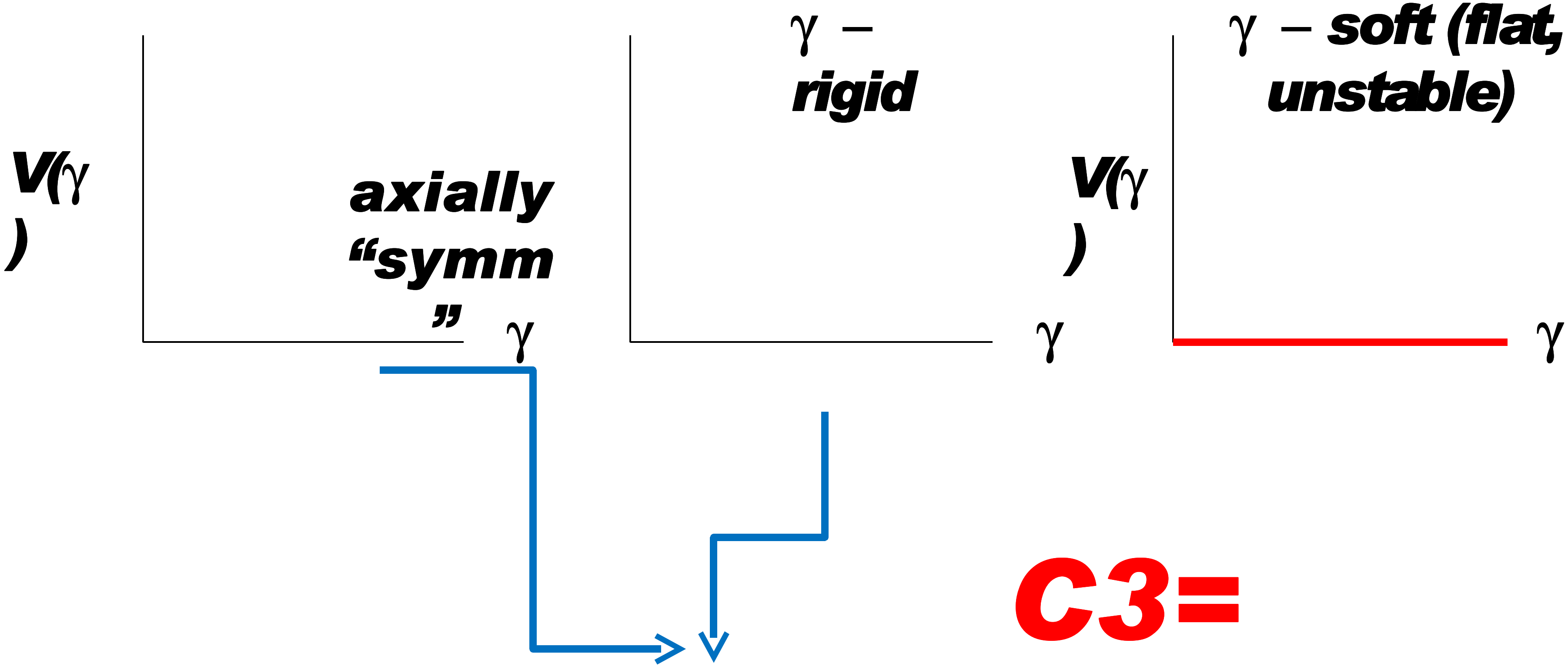
Axially symmetric case
Axial asymmetry



$$V \sim C_2\beta^2 + C_3\beta^3 \cos 3\gamma + C_4\beta^4$$

Axial asymmetry (Triaxiality)

(Specified in terms of the coordinate γ (in degrees), either from 0 –> 60 or from -30 –> +30 degrees – zero degrees is axially symmetric)



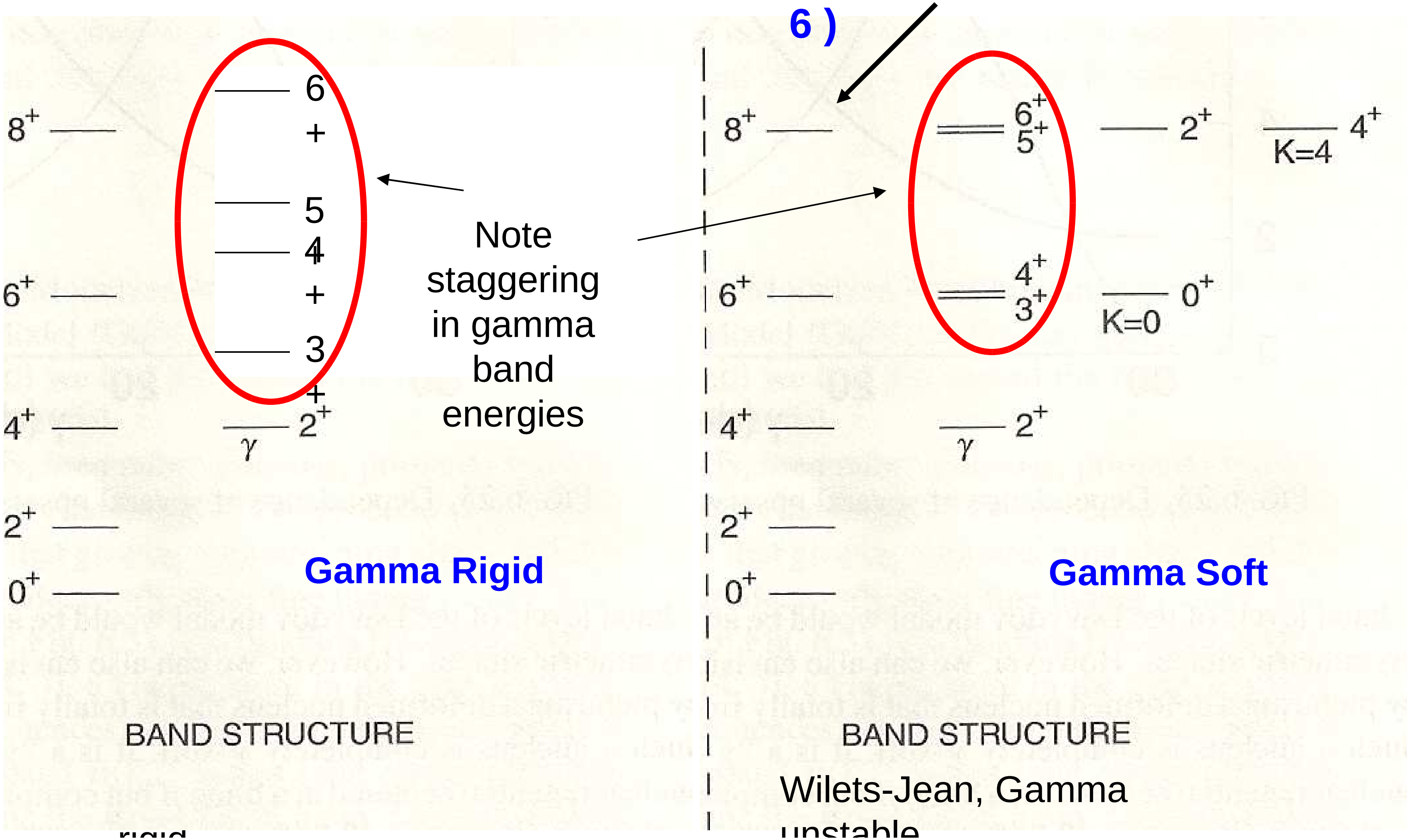
C3= 0

$$V \sim \mathbf{C2}\beta^2 + \mathbf{C3}\cos 3\gamma\, \beta^3 + \mathbf{C4}\beta^4$$

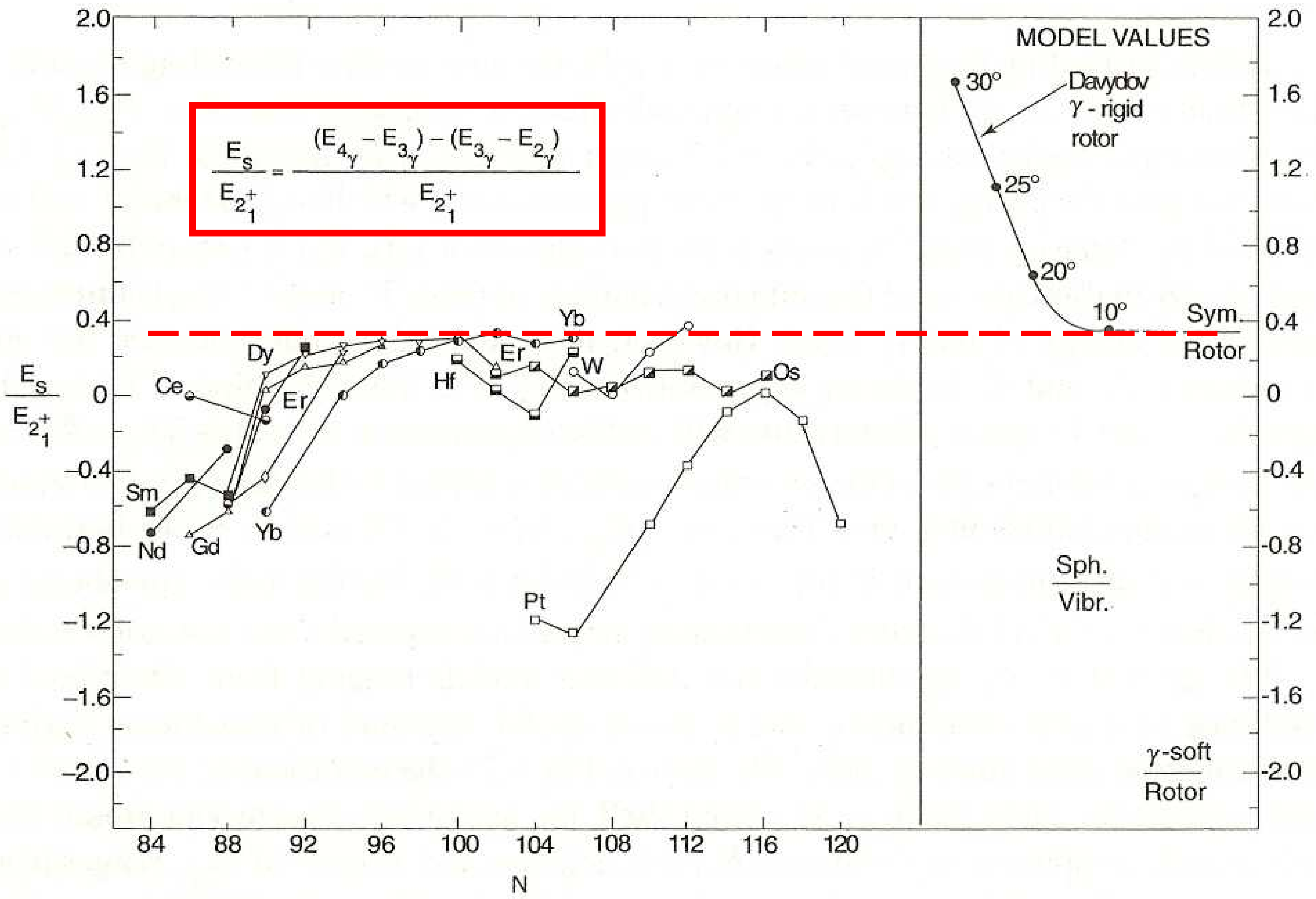
Note: for axially symm. deformed nuclei, MUST have

Axial Asymmetry in Nuclei – two types

E ~ Λ (Λ + 3) ~ J (J + 6)

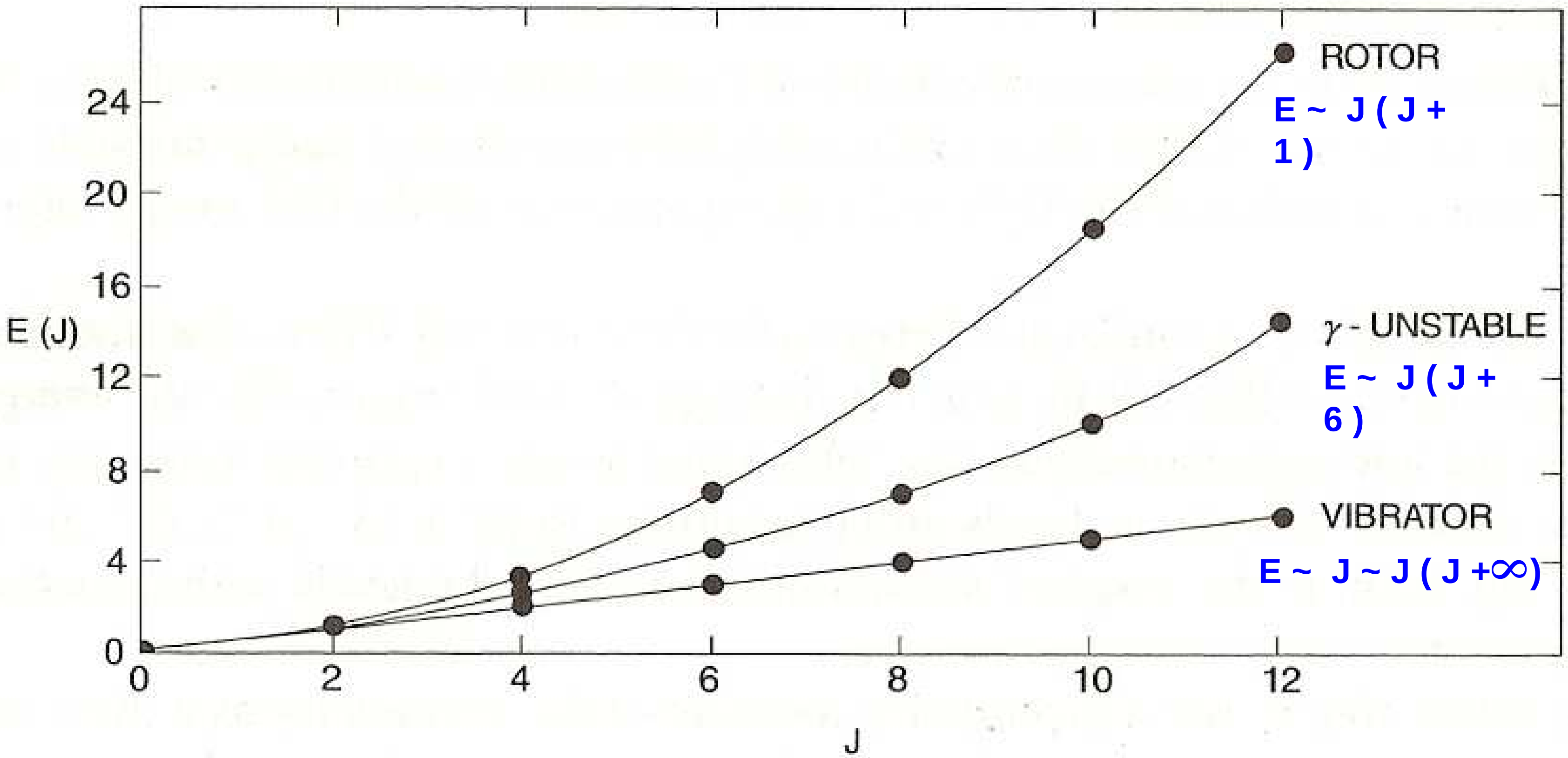


Use staggering in gamma band energies as signature for the kind of axial asymmetry



Overview of yrast energies

Can express energies as E ~ J (J + X)



Now that we know some simple models of atomic nuclei, how do we know where each of these structures will appear? How does structure vary with Z and N? What do we know?

- **Near closed shells nuclei are spherical and can be described in terms of a few shell model configurations.**

- **As valence nucleons are added, configuration mixing, collectivity and, eventually, deformation develop. Nuclei near mid-shell are collective and deformed.**

- **The driver of this evolution is a competition between the pairing force and the p-n interaction, both primarily acting on the valence nucleons.**

Estimating the properties of nuclei

We know that 134Te (52, 82) is spherical and non-collective.

We know that 170Dy (66, 104) is doubly mid-shell and very collective.

What about:

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156Te (52, 104)   156Gd (64, 92)   184Pt (78, 106) ???

All have 24 valence nucleons. What are their relative structures ???

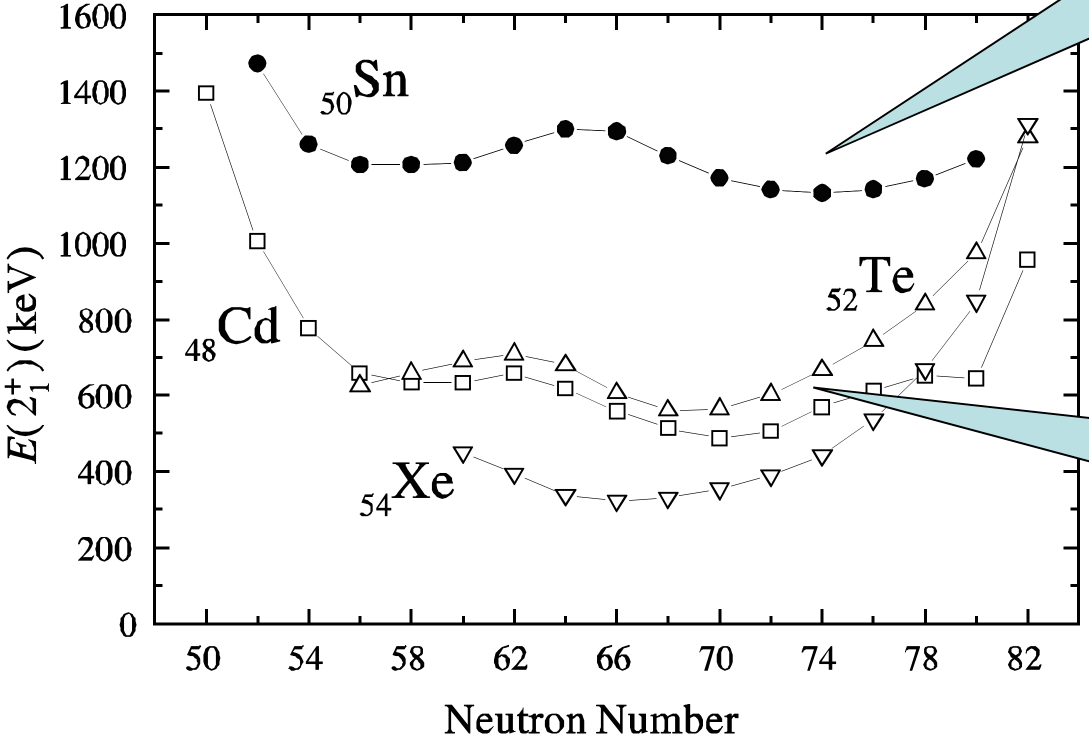
Valence Proton-Neutron Interaction

Development of configuration mixing, collectivity and deformation – competition with pairing

Changes in single particle energies and magic numbers

Partial history: Goldhaber and de Shalit (1953); Talmi (1962); Federman and Pittel (late 1970's); Casten et al (1981); Heyde et al (1980's); Nazarewicz, Dobacewski et al (1980's); Otsuka et al(2000's); Cakirli et al (2000's); and many others.

The idea of "both" types of nucleons – the p-n interaction



Sn – Magic: no valence p-n interactions

Both valence protons and neutrons

If p-n interactions drive configuration mixing, collectivity and deformation, perhaps they can be exploited to understand the evolution of structure.

Lets assume, just to play with an idea, that all p-n interactions have the same strength. This is not realistic since the interaction strength depends on the orbits the particles occupy, but, maybe, on average, it might be OK.

How many valence p-n interactions are there? Np x Nn

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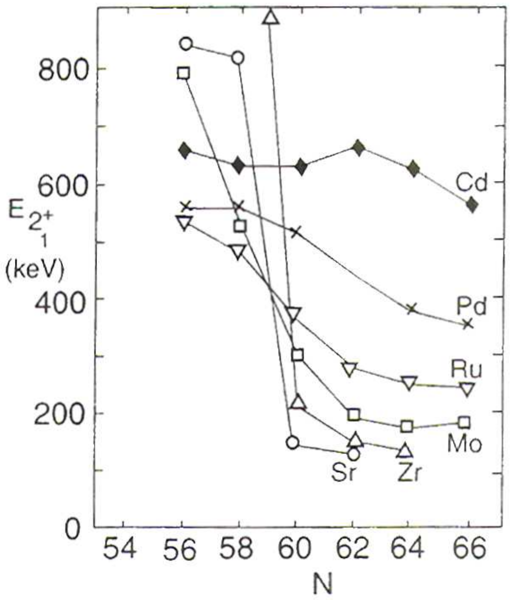
If all are equal then the integrated p-n strength should scale with Np x Nn

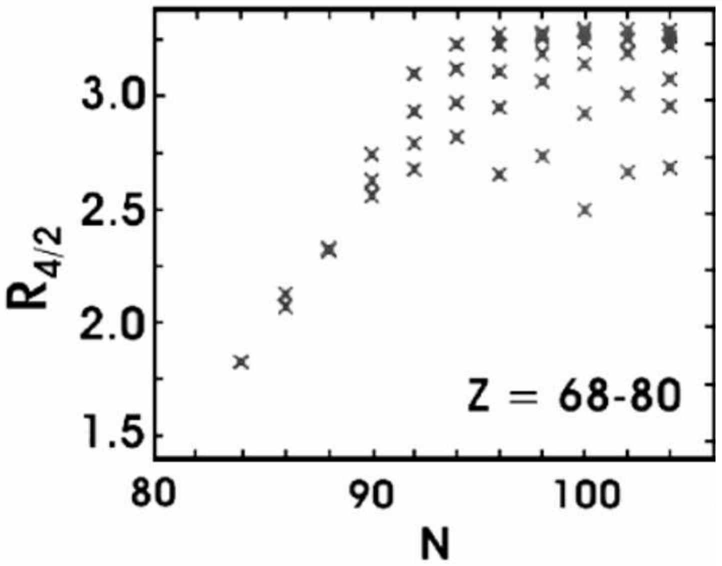
The NpNn Scheme

Valence Proton-Neutron Interactions

Correlations, collectivity, deformation. Sensitive to magic numbers.

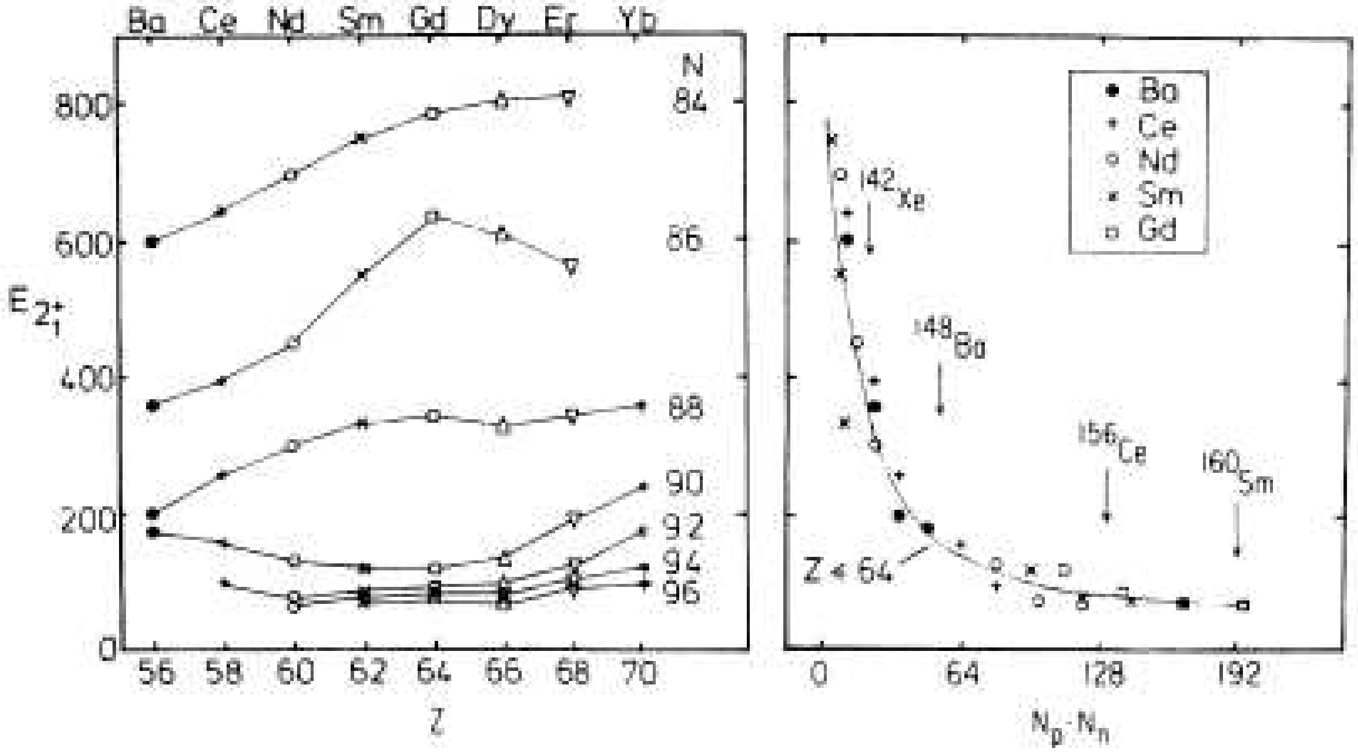


NpNn Scheme

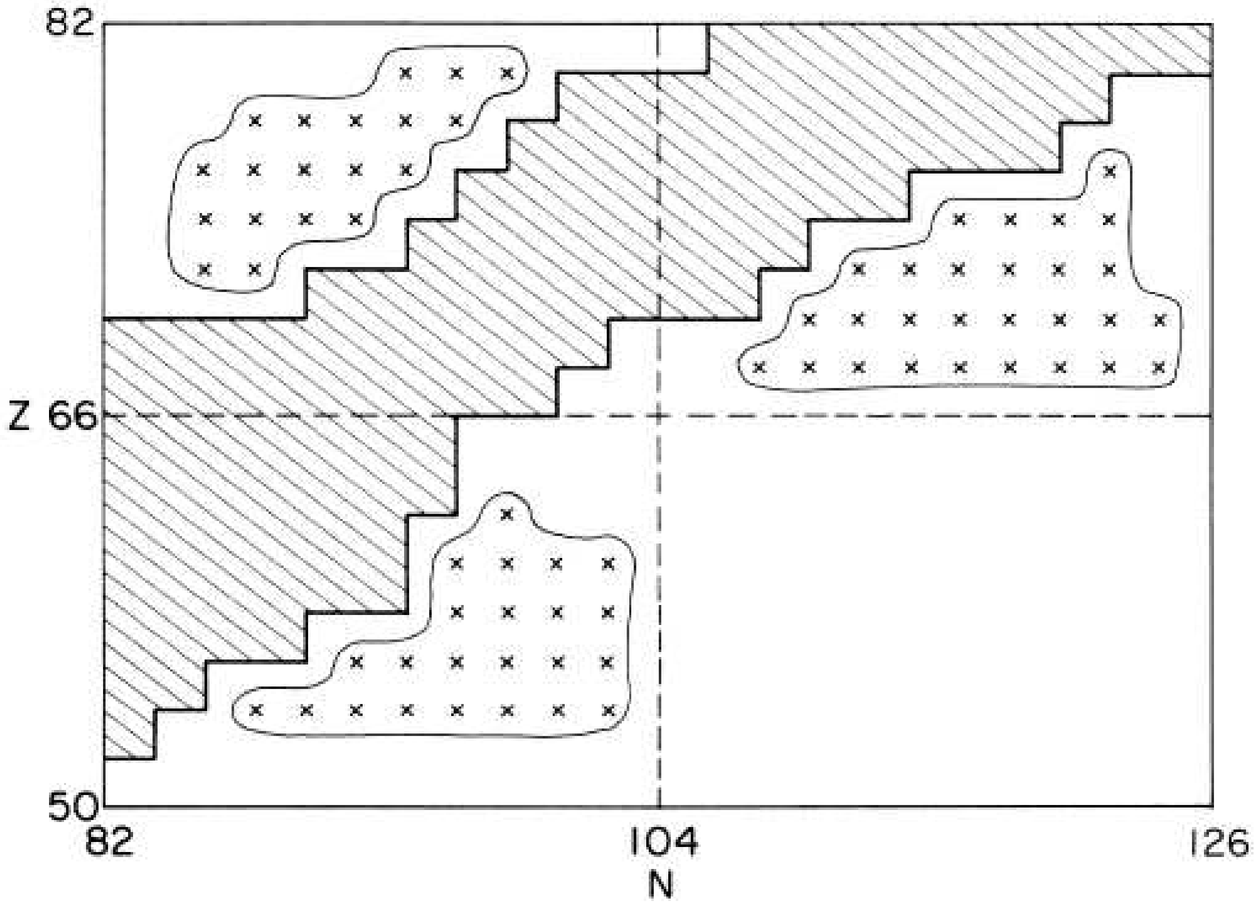


P = NpNn/ (Np+Nn)
p-n interactions per pairing interaction

The NpNn scheme: Interpolation vs. Extrapolation



Predicting new nuclei with the NpNn Scheme



All the nuclei marked with x's can be predicted by INTERpolation

Competition between pairing and the p-n interactions

A simple microscopic guide to the evolution of structure

(The next slides allow you to estimate the structure of any nucleus by multiplying and dividing two numbers each less than 30)

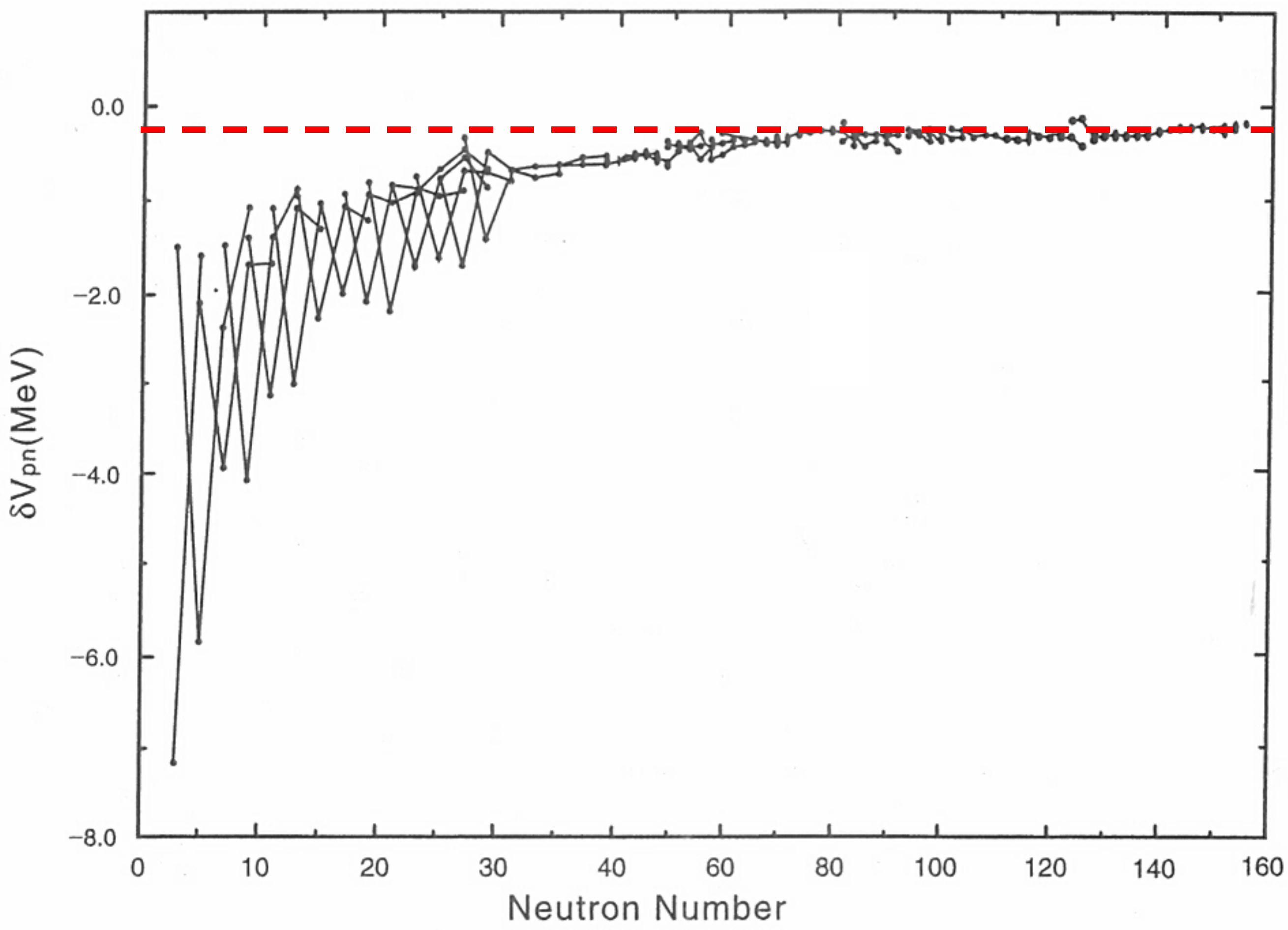
(or, if you prefer, you can get the same result from 10 hours of supercomputer time)

Valence p-n interaction: Can we measure it?

Empirical interactions of the last proton with the last neutron

$\delta V_{pn}(Z, N) = -\frac{1}{4}\{[B(Z, N) - B(Z, N-2)]$

$- [B(Z-2, N) - B(Z-2, N-2)]\}$



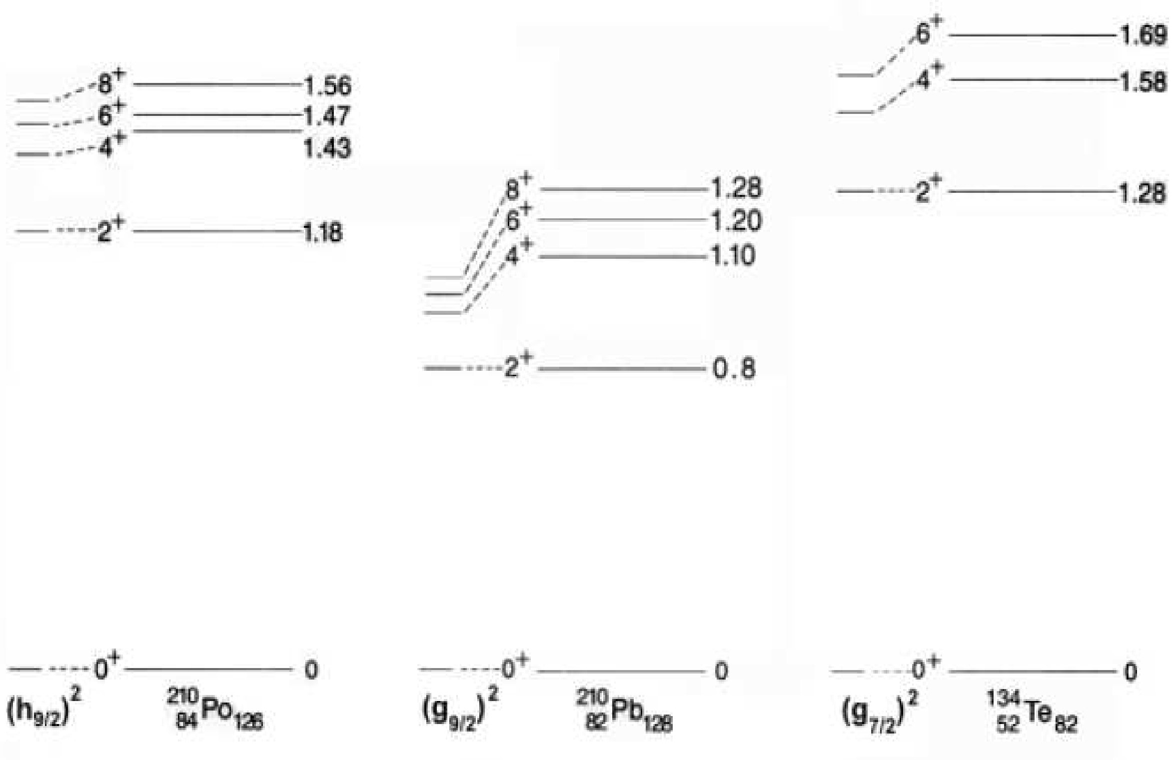
p-n / pairing

$$P = \frac{N_p N_n}{N_p + N_n} \approx \frac{p-n}{pairing}$$

p-n interactions per pairing interaction

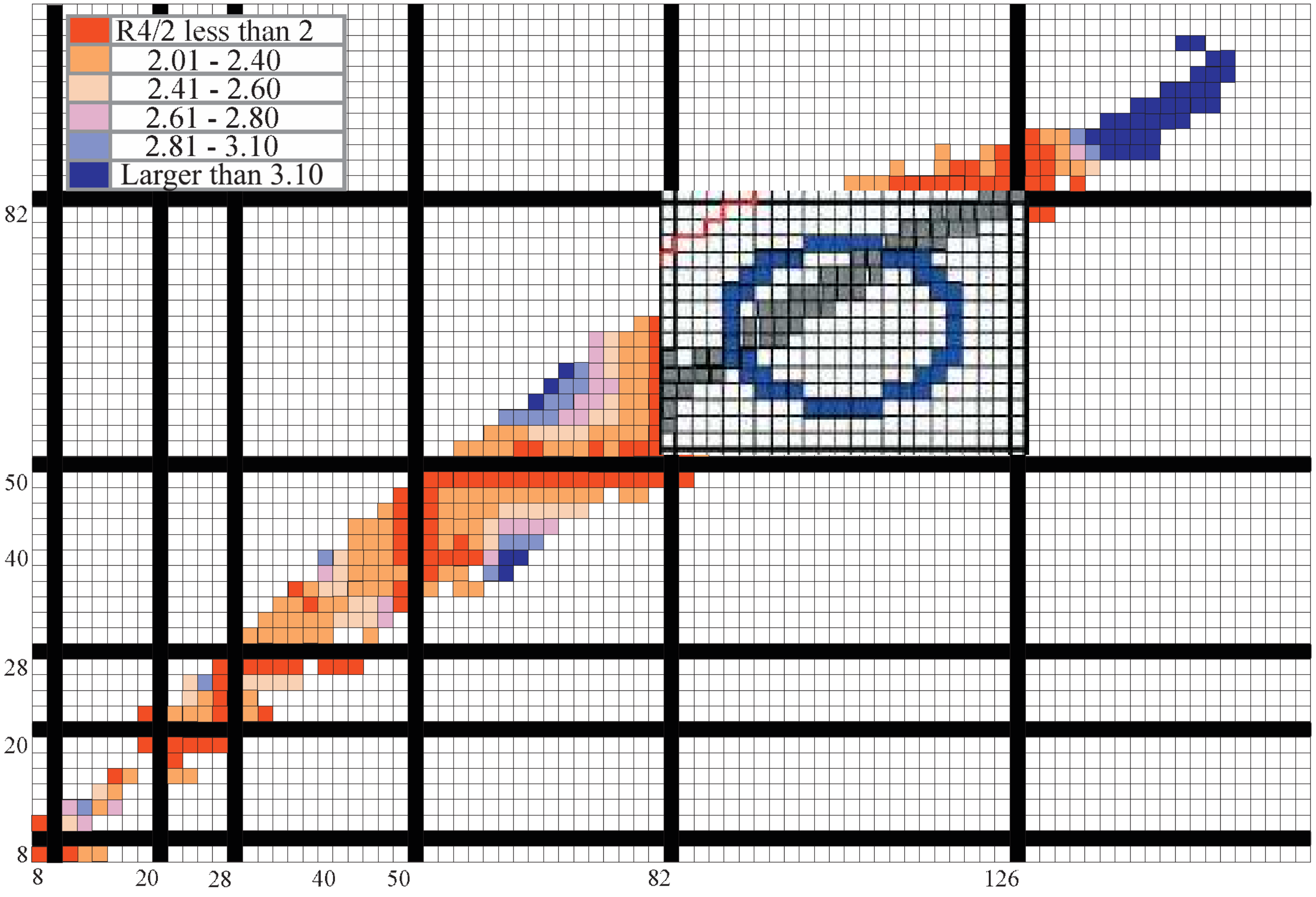
Pairing int. ~ 1 MeV, p-n ~ 200 keV

Hence takes ~ 5 p-n int. to compete with one pairing int.



P~5

Comparison with the data



The IBA

The Interacting Boson Approximation Model

A very simple phenomenological model, that can be extremely parameter-efficient, for collective structure

- Why the IBA
- Basic ideas about the IBA, including a primer on its Group Theory basis
- The Dynamical Symmetries of the IBA
- Practical calculations with the IBA

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IBA – A Review and Practical Tutorial

F. Iachello and A. Arima

Drastic simplification of shell model

- Valence nucleons
- Only certain configurations
- Simple Hamiltonian – interactions

"Boson" model because it treats nucleons in pairs

2 fermions → boson

Why do we need to simplify – why not just calculate with the Shell Model????

The Need for Simplification in Multiparticle Spectra

Example: How many 2^+ states?

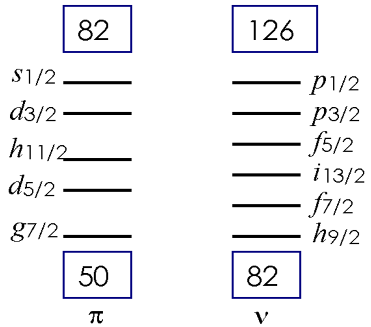
# nucl.			
2	$d^2_{5/2}$	1	
4	$d_{5/2}\, g_{7/2}$	≥ 7	$\left\| d^2_{5/2}\, J=2,\, g^2_{7/2}\, J=0 \right\rangle$, $\left\| d^2_{5/2}\, J=0,\, g^2_{7/2}\, J=2 \right\rangle$
			$\left\| d^2_{5/2}\, J=4,\, g^2_{7/2}\, J=2;\, J=2 \right\rangle$,
			$\left\| d^2_{5/2}\, J=2,\, g^2_{7/2}\, J=4;\, J=2 \right\rangle$,
			$\left\| d^2_{5/2}\, J=4,\, g^2_{7/2}\, J=6;\, J=2 \right\rangle$,
			$\left\| d_{5/2}\, g_{7/2}\, J=1,\, d_{5/2}\, g_{7/2}\, J=1;\, J=2 \right\rangle$,
			$\left\| d^2_{5/2}\, J=4,\, g^2_{7/2}\, J=4;\, J=2 \right\rangle$.

$^{154}_{62}Sm_{92}$

cl. sh. 50 82

$N_p = 12$ $N_n = 10$

12 val. π in 50 – 82
10 val. ν in 82 – 126



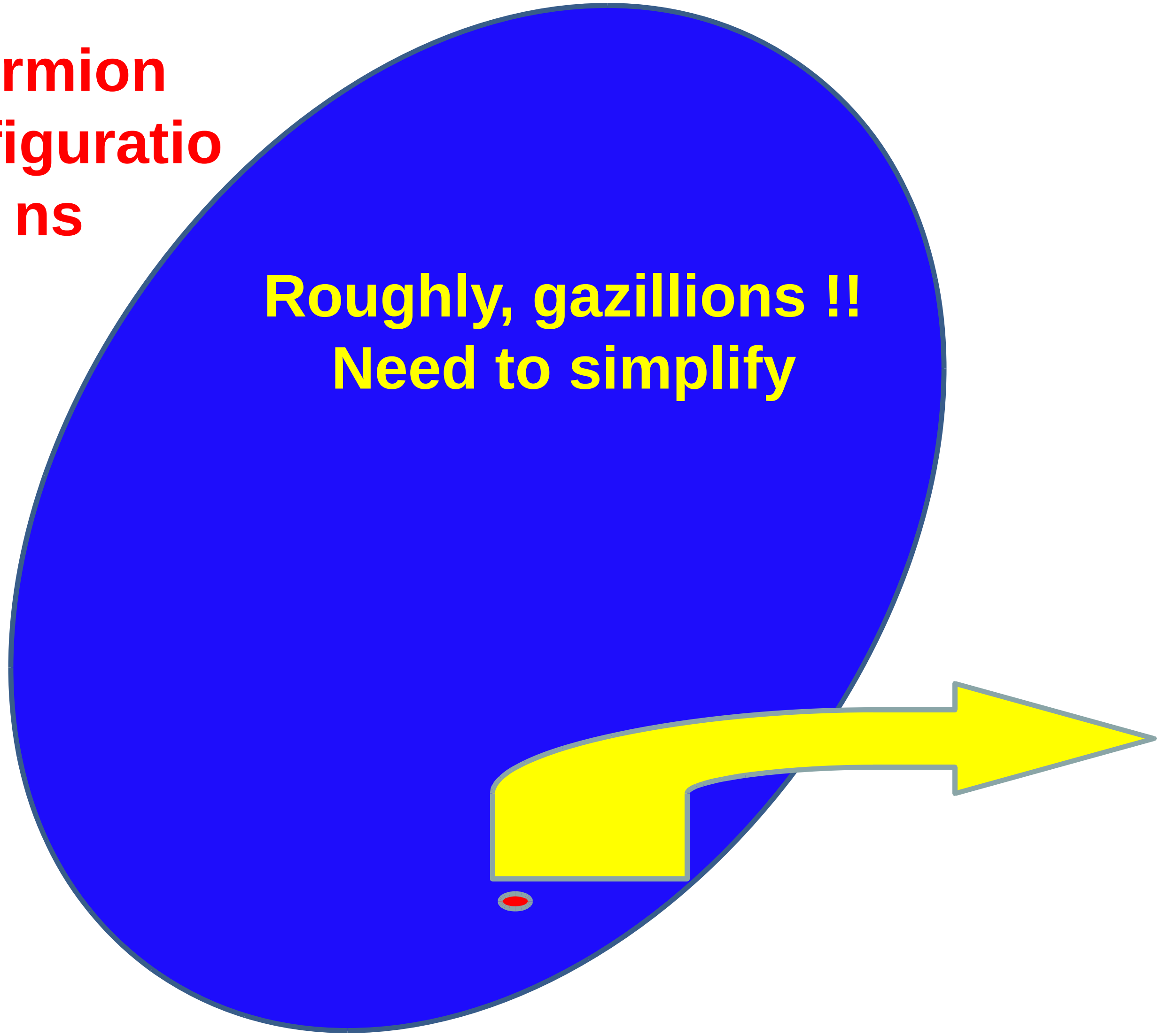
How many 2^+ states subject to Pauli Principle limits?

3×10^{14} !!!

^{154}Sm 2^+ states within the valence shell space

Shell Model Configurations

Fermion configurations



The IBA

Boson configurations

(by considering only configurations of pairs of fermions with J = 0 or 2.)

IBM Assume valence fermions couple in pairs to bosons of spins 0+ and 2+

0+ *s*-boson

2+ *d*-boson

s boson is like a Cooper pair

d boson is like a generalized pair

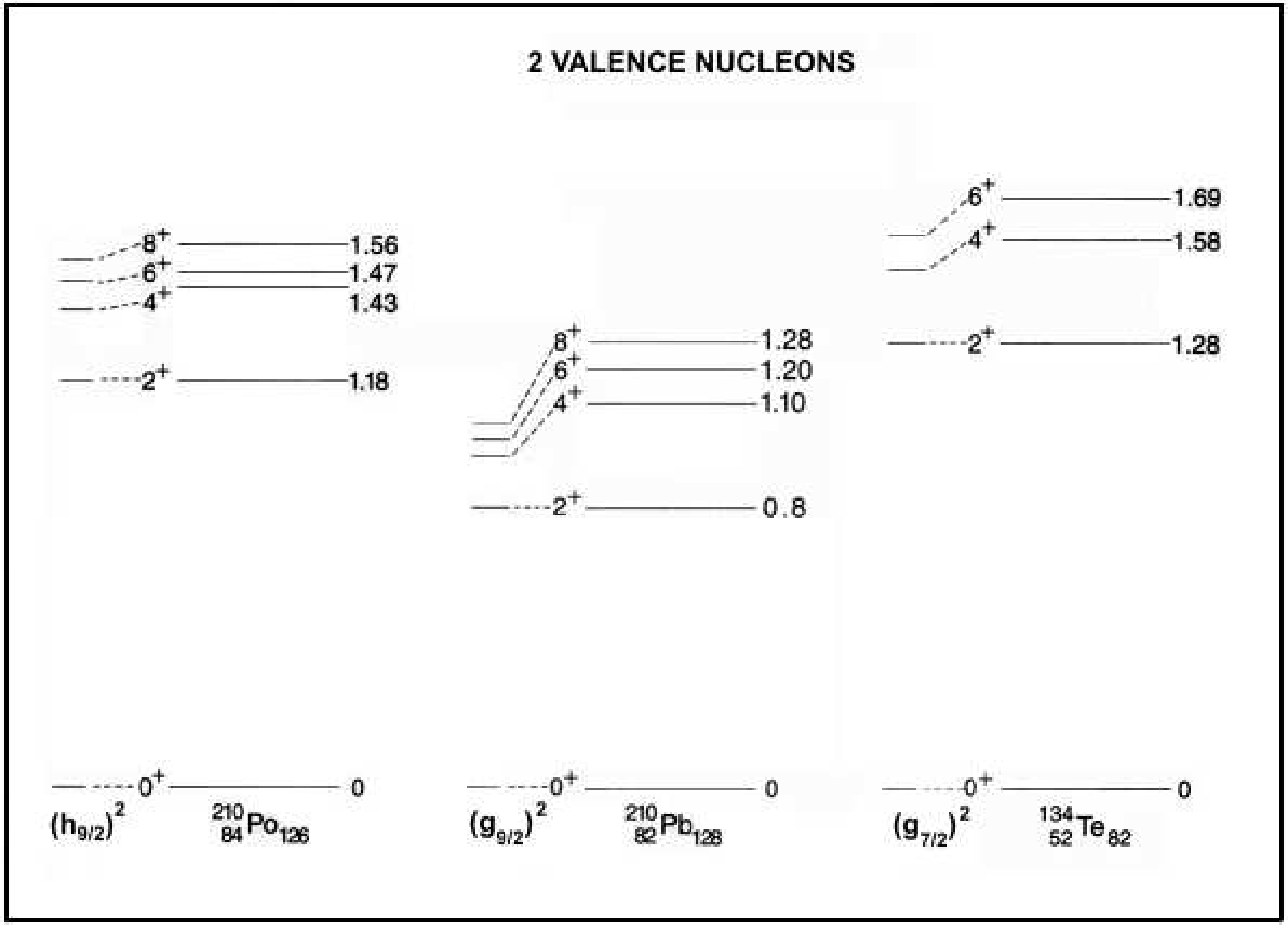
- Valence nucleons only
- *s*, *d* bosons – creation and destruction operators

$H = H_s + H_d + H_{interactions}$

Number of bosons fixed: $N = n_s + n_d$

= ½ # of val. protons + ½ # val. neutrons

Why s, d bosons?



Lowes

non-magic

s r

always 2+

□ - fct gives 0+ ground state

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□ - fct gives 2+ next above 0+

Modeling a Nucleus

Why the IBA is the best thing since baseball, a jacket potato, aceto balsamico, Mt. Blanc, raclette, pfannekuchen, baklava, ….

154Sm ► Shell model ► 3 x 1014 2+ states

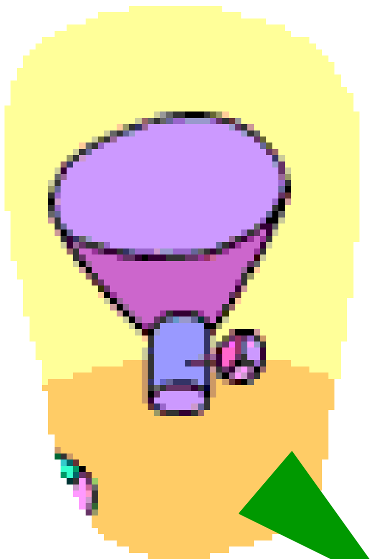
Need to truncate

IBA assumptions

1. ***Only valence nucleons***
2. ***Fermions → bosons***

J = 0 (s bosons)

J = 2 (d bosons)



Is it conceivable that these 26 basis states are correctly chosen to account for the properties of the low lying collective states?

IBA: 26 2+ states

Why the IBA ?????

- Why a model with such a drastic simplification – Oversimplification ???

- Answer: Because it works ***!!!!!***
- ***By far the most successful general nuclear collective model for nuclei***
- ***Extremely parameter-economic***

Note key point:

□

Bosons in IBA are pairs of fermions in valence shell

Number of bosons for a given nucleus is ***a fixed*** number

$^{154}_{62}Sm_{92}$ ***N□ = 6 5 = N□***

□ ***NB = 11***

Basically the IBA is a Hamiltonian written in terms of s and d bosons and their interactions. It is written in terms of boson creation and destruction operators.

Where the IBA fits in the pantheon of nuclear models

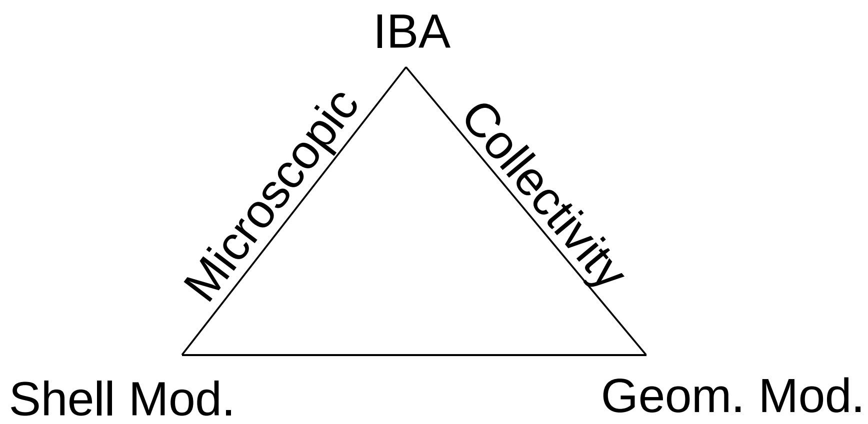
- Shell Model $\begin{pmatrix}\text{Sph.}\\ \text{Def.}\end{pmatrix}$ - (Microscopic)
- Geometric – (Macroscopic)
- Third approach — “Algebraic”

↓

Group Theoretical

Dynamical Symmetries

→ Phonon-like model with microscopic basis explicit from the start.



IBA has a deep relation to Group theory

That relation is based on the operators that create, destroy *s* and *d* bosons

$s^\dagger$, s, $d^\dagger$, d operators

Ang. Mom. 2

$d^\dagger_\mu$, d_μ μ = 2, 1, 0, -1, -2

Hamiltonian is written in terms of *s, d* operators

Since boson number is conserved for a given nucleus, *H* can only contain “bilinear” terms: 36 of them.

$s^\dagger s$, $s^\dagger d$, $d^\dagger s$, $d^\dagger d$

Gr. Theor. classification of Hamiltonian

Group is called

U(6)

Brief, simple, trip into the Group Theory of the IBA

DON'T BE SCARED

You do not need to understand all the details but try to get the idea of the relation of groups to degeneracies of levels and quantum numbers

A more intuitive name for this application of Group Theory is

"Spectrum Generating Algebras"

Review of phonon creation and destruction operators

$$\mathbf{b}|n_b\rangle = \sqrt{n_b}\,|n_b - 1\rangle$$

$$\mathbf{b}^\dagger|n_b\rangle = \sqrt{(n_b + 1)}\,|n_b + 1\rangle$$

What is a creation operator? Why useful?

A) Bookkeeping – makes calculations very simple.

B) “Ignorance operator”: We don’t know the structure of a phonon but, for many predictions, we don’t need to know its microscopic basis.

$$\mathbf{b}^\dagger\mathbf{b}|n_b\rangle = \mathbf{b}^\dagger\sqrt{n_b}\,\Big|n_b - 1\rangle = \sqrt{n_b}\,\sqrt{(n_b - 1) + 1}|n_b\rangle = n_b|n_b\rangle$$

$\mathbf{b}^\dagger\mathbf{b}$ is a **b-**phonon number operator.

For the IBA a boson is the same as a phonon – think of it as a collective excitation with ang. mom. 0 (s) or 2 (d).

Concepts of group theory

First, some fancy words with simple meanings:

Generators, Casimirs, Representations, conserved quantum numbers, degeneracy splitting

Generators of a group: Set of operators , *Oi* that close on commutation.

[*Oi* , *Oj*] = *Oi Oj* - *Oj Oi* = *Ok* *i.e.*, their commutator gives back 0 or a member of the set

For IBA, the 36 operators **s†s, d†s, s†d, d†d** are generators of the group U(6)

ex:

$$[d^\dagger s, s^\dagger s]\,|n_d n_s\rangle = (d^\dagger s s^\dagger s - s^\dagger s d^\dagger s)\,|n_d n_s\rangle$$

$$= d^\dagger s n_s\,|n_d n_s\rangle - s^\dagger s d^\dagger s\,|n_d n_s\rangle$$

$$= (n_s - s^\dagger s)\, d^\dagger s\,|n_d n_s\rangle$$

$$= (n_s - s^\dagger s)\sqrt{n_d+1}\sqrt{n_s}\,|n_d+1, n_s-1\rangle$$

$$= \sqrt{n_d+1}\sqrt{n_s}\,[n_s-(n_s-1)]\,|n_d+1, n_s-1\rangle$$

$$= \sqrt{n_d+1}\sqrt{n_s}\,|n_d+1, n_s-1\rangle$$

$$= d^\dagger s\,|n_d n_s\rangle$$

e.g:

$$[N, s^\dagger\tilde{d}]\Psi = [N(s^\dagger\tilde{d}) - s^\dagger\tilde{d}N]\Psi$$

$$= Ns^\dagger\tilde{d}\Psi - s^\dagger\tilde{d}N\Psi$$

$$= Ns^\dagger\tilde{d}\Psi - Ns^\dagger\tilde{d}\Psi = 0$$

or:

$$[d^\dagger s, s^\dagger s] = d^\dagger s$$

A **Hamiltonian** written solely in terms of Casimirs can be solved analytically

Sub-groups:

Subsets of generators that commute among themselves.

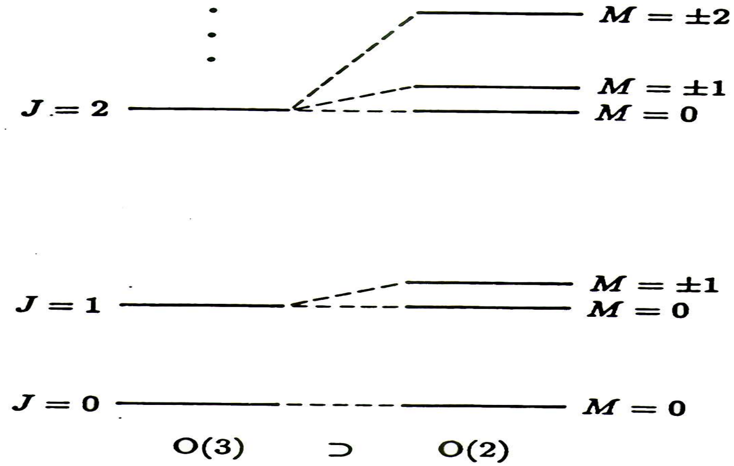
e.g: $d^\dagger d$ 25 generators—span U(5)

They conserve *nd* (# *d* bosons)

Set of states with same *nd* are the representations of the group [U(5)]

Simple example of dynamical symmetries, group chain, degeneracies

Simple example of Dyn. Sym, Group Chain, Degeneracies



$$E_{JM} = 2a\,J(J+1) \quad + \quad 2b\,M^2$$

$[H, J^2] = [H, J_z] = 0$ J, M constants of motion

$[H, J2] = [H, JZ] = 0$ J, M constants of motion

Let's illustrate group chains and degeneracy-breaking.

Consider a Hamiltonian that is a function ONLY of:
$s^\dagger s + d^\dagger d$

That is: $H = a(s^\dagger s + d^\dagger d) = a\,(n_s + n_d)$

H’ = H + b … : aN … /

Now, add a term to this Hamiltonian:

Now the energies depend not only on N but also on nd

States of a given nd are now degenerate. They are “representations” of the group U(5). States with different nd are not degenerate

H’ = aN d = a

b nd

2a N + 2

a N + 1

Etc. with further terms

0 N

E

U(6)

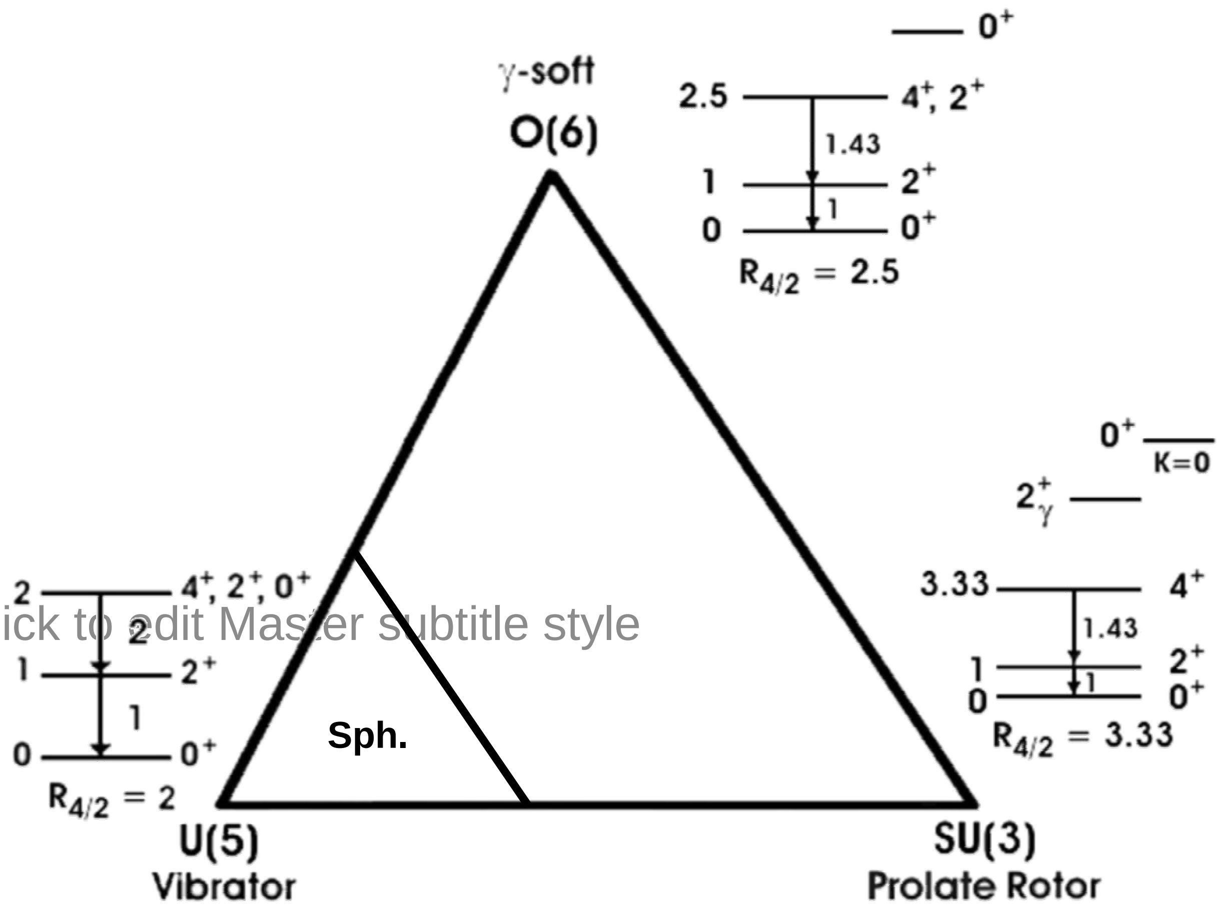
H’ = aN

OK, here's the key point :

Concept of a Dynamical Symmetry

Spectrum generating algebra !!

Next time



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Classifying Structure -- The Symmetry Triangle