École Joliot-Curie

27 September - 3 October 2009

Lacanau - France

Strong interaction in the nuclear medium: new trends

Effective interactions and energy functionals: applications to nuclear systems – II

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Overview of the second lecture

- **1. Mean-field based models for the ground state. From HF to HFB (pairing included)**

- **2. Mean-field based models to describe excited states. RPA derived as small amplitude approximation from time-dependent HF equations. Pairing -> QRPA**

- **3. The interaction (and then the functional) in the pairing channel**

- **4. Isospin effects. Exotic nuclei. New phenomena… stronger and/or different correlations?**

- **5. Correlations. Going beyond mean field**

- **6. Nuclear matter and its properties. The EoS (case of Skyrme)**

- **7. Asymmetric matter, neutron matter. Isospin effects**

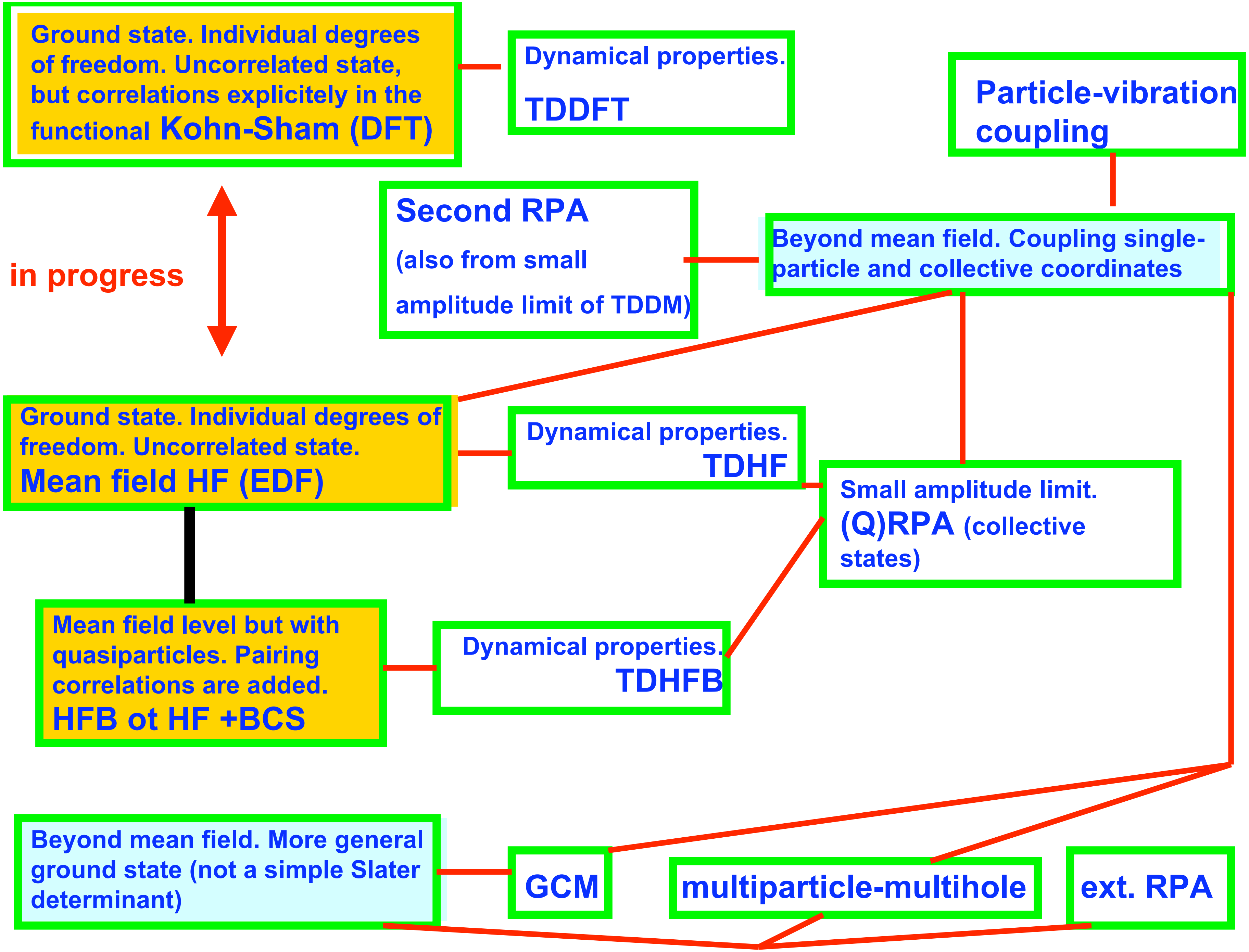
- **8. Spin instabilities of nuclear matter and the Skyrme interaction**

1. Mean-field based models for the ground state

- From HF to HFB (pairing included)

- **In HF mean field the ground state is a Slater determinant (completely uncorrelated single-particle wave functions)**

- **Pairing correlations may be included at the mean field level by introducing the concept of quasiparticles (the number of particles is not conserved: symmetry breaking)**


Ground state. Individual degrees of freedom. Uncorrelated state, but correlations explicitely in the functional Kohn-Sham (DFT)
Dynamical properties. TDDFT
Particle-vibration coupling
in progress
Second RPA (also from small amplitude limit of TDDM)
Beyond mean field. Coupling single-particle and collective coordinates
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Beyond mean field. More general ground state (not a simple Slater determinant)
GCM
multiparticle-multihole
ext. RPA

Odd-even effect

No isotopes (Z=102)

Duguet et al. arXiv:nucl-th/0005040v1

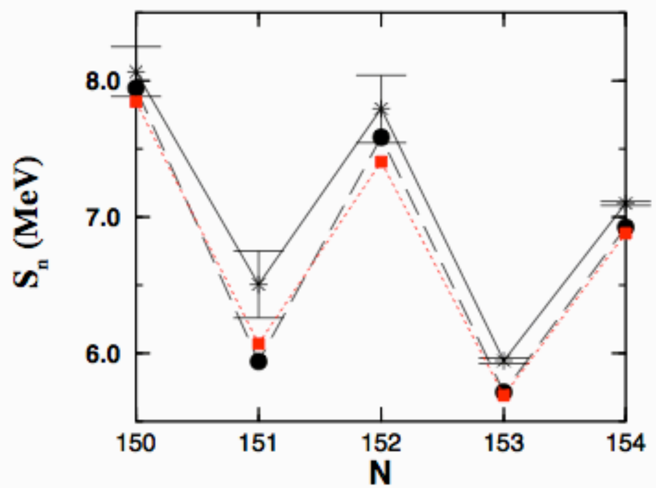


Figure 2: One neutron separation energy S_n for five ^{254}No isotopes. Stars with full line are for experiment with error bars, circles with long-dashed line for pairing 1 and squares with dotted line for pairing 2.

Sn = E (Z,N)-E(Z,N-1)

The Hartree-Fock-Bogoliubov (HFB) or Bogoliubov-de Gennes equations

We can derive them in a very elegant way in the simple case of:

- spherical system
- two spin states ↑ and ↓
- zero-range interaction acting only between states with opposite spin

P. de Gennes, Superconductivity of metals and alloys

The Hamiltonian is composed by a 1-body term and a 2-body term (zero-range interaction of intensity V)

$$H_1 = \int d^3\vec{r} \sum_{\alpha} \Psi^+(\vec{r}\alpha) U_1 \Psi(\vec{r}\alpha),$$

$$H_2 = \frac{1}{2} V \int d^3\vec{r} \sum_{\alpha\beta} \Psi^+(\vec{r}\alpha)\Psi^+(\vec{r}\beta)\Psi(\vec{r}\beta)\Psi(\vec{r}\alpha)$$

The mean field approximation leads to a 1-body effective term that approximates H_2

$$H_{eff} = \int d^3\vec{r} \left\{ \sum_{\alpha} \Psi^+(\vec{r}\alpha) H_e(\vec{r}) \Psi(\vec{r}\alpha) + W(\vec{r})\Psi^+(\vec{r}\alpha)\Psi(\vec{r}\alpha) + \Delta(\vec{r})\Psi^+(\vec{r}\uparrow)\Psi^+(\vec{r}\downarrow) + \Delta^*(\vec{r})\Psi(\vec{r}\downarrow)\Psi(\vec{r}\uparrow) \right\}$$

$$H_1 - \lambda N = \sum_{\alpha} \int d^3\vec{r} \Psi^+(\vec{r}\alpha) H_e \Psi(\vec{r}\alpha)$$

$$N \equiv \sum_{\alpha} \int d^3\vec{r} \Psi^+(\vec{r}\alpha)\Psi(\vec{r}\alpha)$$

Since H_{eff} is a quadratic form in Ψ and Ψ^+, it can be diagonalized by unitary transformations like the Bogoliubov transformations

$$\Psi(\vec{r}\uparrow) = \sum_n \left(u_n(\vec{r})\gamma_{n\uparrow} - v_n^*(\vec{r})\gamma_{n\downarrow}^+\right),$$
$$\Psi(\vec{r}\downarrow) = \sum_n \left(u_n(\vec{r})\gamma_{n\downarrow} + v_n^*(\vec{r})\gamma_{n\uparrow}^+\right),$$

The Bogoliubov transformations diagonalize $H_{eff.}$ It means:

$$H_{eff} = \varepsilon_0 + \sum_{n\alpha} \varepsilon_n \gamma_{n\alpha}^+ \gamma_{n\alpha}$$

$$\left[H_{eff}, \gamma_{n\alpha}\right] = -\varepsilon_n \gamma_{n\alpha},$$

$$\left[H_{eff}, \gamma_{n\alpha}^+\right] = \varepsilon_n \gamma_{n\alpha}^+.$$

One calculates:

$$\left[\Psi(\vec{r}\uparrow), H_{eff}\right] = (H_e + W(\vec{r}))\Psi(\vec{r}\uparrow) + \Delta(\vec{r})\Psi^+(\vec{r}\downarrow),$$
$$\left[\Psi(\vec{r}\downarrow), H_{eff}\right] = (H_e + W(\vec{r}))\Psi(\vec{r}\downarrow) - \Delta^*(\vec{r})\Psi^+(\vec{r}\uparrow).$$

Using the Bogoliubov transformations one gets the HFB equations. The solutions are the two-component wave functions (u,v) and the associated quasiparticle energies ε:

$$\varepsilon u(\vec{r}) = [H_e + W(\vec{r})]u(\vec{r}) + \Delta(\vec{r})v(\vec{r}),$$
$$\varepsilon v(\vec{r}) = -[H^*_e + W(\vec{r})]v(\vec{r}) + \Delta^*(\vec{r})u(\vec{r}).$$

To derive the expressions for the mean field W and the pairing field Δ, we impose that the free energy F is stationary

$$0 = \delta F = \delta\langle H\rangle - T\delta S,$$

where the mean value of H is defined as:

$$\langle H\rangle \equiv \frac{\sum_\phi \langle\phi|H|\phi\rangle \exp(-\beta E_\phi)}{\sum_\phi \exp(-\beta E_\phi)}, \quad \beta = \frac{1}{K_B T}.$$

← **Temperature dependence**

It can be shown that F is stationary if:

$$W(\vec{r}) = V\langle\Psi^+(\vec{r}\uparrow)\Psi(\vec{r}\uparrow)\rangle = V\langle\Psi^+(\vec{r}\downarrow)\Psi(\vec{r}\downarrow)\rangle,$$
$$\Delta(\vec{r}) = V\langle\Psi(\vec{r}\downarrow)\Psi(\vec{r}\uparrow)\rangle = -V\langle\Psi(\vec{r}\uparrow)\Psi(\vec{r}\downarrow)\rangle.$$

Density

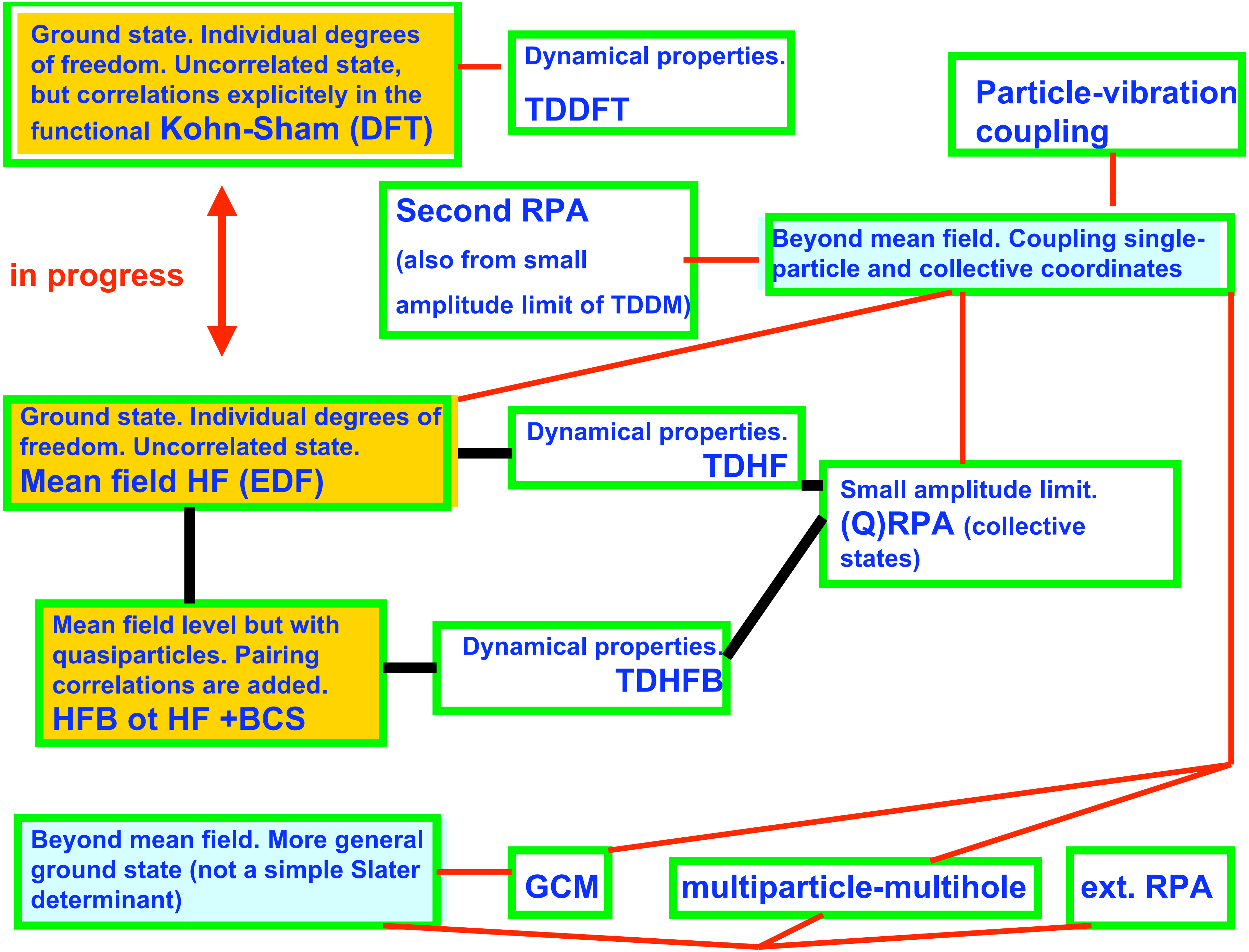
$$\rho(\vec{r}) \equiv \langle\Psi^+(\vec{r}\alpha)\Psi(r\alpha)\rangle = \sum_n \left[(1-f_n)v_n^2(\vec{r}) + f_n u_n^2(\vec{r})\right],$$

Pairing density

$$\tilde{\rho}(\vec{r}) \equiv \langle\Psi(\vec{r}\downarrow)\Psi(r\uparrow)\rangle = -\sum_n \left[(1-2f_n)v_n^*(\vec{r})u_n(\vec{r})\right],$$

The temperature dependence is contained in a Fermi factor → $f_n = \dfrac{1}{\exp(\beta\varepsilon_n)+1}$

2. Mean-field based models to describe excited states. RPA derived as small amplitude approximation from time-dependent HF equations. Pairing -> QRPA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Excitation modes (small amplitudes)

Quadrupole mode QRPA in <u>particle-hole channel</u>

Response function for ^{22}O

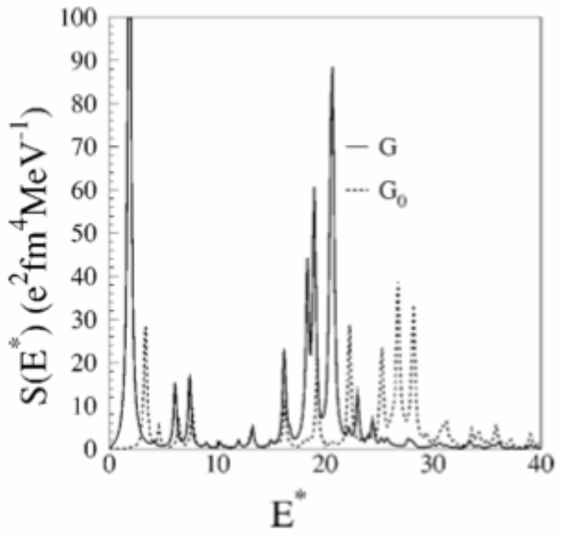
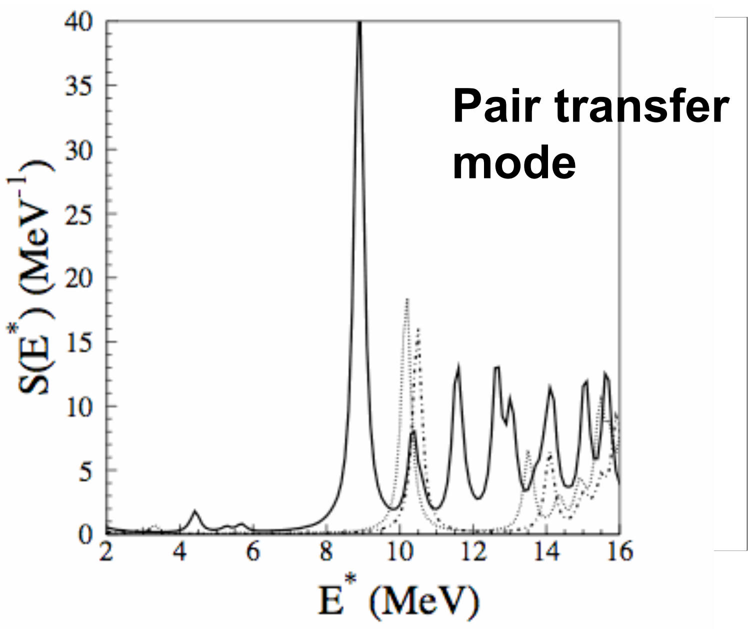


FIG. 2. Isoscalar quadrupole strength function calculated in continuum QRPA for the ^{22}O nucleus. The unperturbed strength (dashed line) is also shown.

Khan, Sandulescu, Grasso, and Van Giai, PRC 66, 024309 (2002)

Two-neutron 0^+ addition mode QRPA in <u>particle-particle channel</u>

Response function for ^{124}Sn



Khan, Grasso, Margueron, submitted PRC

FIG. 5: QRPA response function for ^{124}Sn in the two neutrons 0^+ addition mode. The pure surface mode is in solid line, the η=0.65 mode is in dotted line, and the η=0.35 mode in dashed-dotted lines.

The RPA equations can be introduced in different ways

- 1) equations of motion method (Rowe. 1968) (starting from the stationary Schroedinger equation)
- 2) Green's function techniques
- 3) From the small amplitude limit of time-dependent Hartree-Fock (TDHF) equations
- Both 2) and 3) are based on the linear response theory

Linear Response Theory

- Action of a time-dependent external field: response of the system

- Limit of a weak perturbing field: linear response

RPA from TDHF equations

External perturbation:

$$F(t) = Fe^{-i\omega t} + Fe^{i\omega t}$$

$$F(t) = \sum_{\alpha\beta} f_{\alpha\beta}(t) a^{\dagger}_{\alpha} a_{\beta}$$

We impose that $\rho(t)$ corresponds to a Slater determinant. It means: $\rho^2=\rho$. The density satisfies the following equations of motion:

$$i\hbar \frac{d\rho}{dt} = [h[\rho] + f(t), \rho]$$

We introduce a variation of the density (linear with the external perturbation)

$$\rho(t) = \rho_0 + \delta\rho(t)$$

Inserting $\rho(t) = \rho_0 + \delta\rho(t)$ **in the equations of**

motion and keeping only linear terms one finds the RPA equations (solving for f(t)=0). The matrices A and B are written as:

$$\begin{vmatrix} A & B \\ -B & -A \end{vmatrix} \begin{vmatrix} X^{(\alpha)} \\ Y^{(\alpha)} \end{vmatrix} = \omega_\alpha \begin{vmatrix} X^{(\alpha)} \\ Y^{(\alpha)} \end{vmatrix}$$

$$A_{php'h'} = (\epsilon_p - \epsilon_h)\delta_{pp'}\delta_{hh'} + \frac{\partial h_{ph}}{\partial \rho_{p'h'}}$$

$$B_{php'h'} = \frac{\partial h_{ph}}{\partial \rho_{h'p'}}$$

One recognizes the residual interaction (different from V if V is density dependent ⇨ rearrangement term):

$$V_{res} = \frac{\partial h_{ab}}{\partial \rho_{cd}} = \frac{\partial^2 E}{\partial \rho_{ab} \partial \rho_{cd}}$$

E is the energy density functional

Pairing included: linear approximation from TDHFB

Khan et al. PRC 66, 024309 (2002)

From time-depedent Hartree-Fock-Bogoliubov

$$i\hbar \frac{\partial \mathcal{R}}{\partial t} = [\mathcal{H}(t) + \mathcal{F}(t), \mathcal{R}(t)],$$

where $\mathcal{R}$ and $\mathcal{H}$ are the time-dependent generalized density and HFB Hamiltonian. The external periodic field $\mathcal{F}$ is given by

$$\mathcal{F} = F e^{-i\omega t} + \text{H.c.},$$

where F includes both particle-hole and two-particle transfer operators.

Variations

$$\mathcal{R}(t) = \mathcal{R}^0 + \mathcal{R}' e^{-i\omega t} + \text{H.c.}, \tag{2.4}$$

$$\mathcal{H}(t) = \mathcal{H}^0 + \mathcal{H}' e^{-i\omega t} + \text{H.c.} \tag{2.5}$$

the TDHFB equation (2.1) becomes

$$\hbar\omega \mathcal{R}' = [\mathcal{H}', \mathcal{R}^0] + [\mathcal{H}^0, \mathcal{R}'] + [F, \mathcal{R}^0]. \tag{2.6}$$

The generalized density variation has the form

$$\mathcal{R}'_{ij} = \begin{pmatrix} \rho'_{ij} & \kappa'_{ij} \\ \bar{\kappa}'_{ij} & -\rho'_{ji} \end{pmatrix}, \tag{2.7}$$

Variation of the particle density

Variation of the pairing tensor

After manipulations the Bethe-Salpeter equation can be found

$$\mathbf{G} = (1 - \mathbf{G}_0 \mathbf{V})^{-1} \mathbf{G}_0 = \mathbf{G}_0 + \mathbf{G}_0 \mathbf{V} \mathbf{G}.$$

In the case of transitions from the ground state to excited states within the same nucleus, only the (ph,ph) component of **G** is acting. If the interaction does not depend on spin variables the strength function is thus given by

$$S(\omega) = -\frac{1}{\pi} \mathrm{Im} \int F^{11*}(\mathbf{r}) \mathbf{G}^{11}(\mathbf{r}, \mathbf{r}'; \omega) F^{11}(\mathbf{r}') d\mathbf{r} \; d\mathbf{r}'.$$

3. The interaction (and then the functional) in the pairing channel

In mean field calculations with Gogny

(Almost) the same interaction as in the particle-hole (mean field) channel

Pairing in Skyrme-HFB mean field framework

- Non empirical pairing energy density functional derived at lowest order in the two-nucleon vacuum interaction including Coulomb (many-body perturbation theory) (Lesinski, et al., arXiv:0809.2895 [nucl-th]

- Derived from a microscopic nucleon-nucleon interaction. If LDA is valid: fit to reproduce the gap in symmetric and neutron matter (Margueron, et al. Phys. Rev. C 77, 054309 (2008))
- Also dependence on the isovector density (Margueron)

- Empirical pairing energy density functional with constraints from nuclei: odd-even mass staggering, separation energies

- Also dependence on the isovector density (Yamagami, et al., arXiv:0812.3197 [nucl-th])

- In the context of empirical pairing functional: New constraints? Pairing vibrations?

  Matsuo, Proceedings of COMEX3, to be published

  Grasso, et al. Proceedings of NSD09, to be published

  Khan et al., 2009

Model: Skyrme-HFB with zero-range pairing interaction and dependence on the isoscalar density

$$V(\vec{r}_1 - \vec{r}_2) = V_0\left[1 - x\left(\frac{\rho(r)}{\rho_0}\right)^{\gamma}\right]\delta(\vec{r}_1 - \vec{r}_2)$$

ρ_0 **is currently chosen equal to the saturation density 0.16 fm^{-3}**

Ultraviolet divergence. Renormalization of the HFB equations

VOLUME 88, NUMBER 4 PHYSICAL REVIEW LETTERS 28 JANUARY 2002

Renormalization of the Hartree-Fock-Bogoliubov Equations in the Case of a Zero Range Pairing Interaction

Aurel Bulgac and Yongle Yu
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(Received 26 June 2001; published 14 January 2002)

the diagonal part of the anomalous density matrix $\nu(\boldsymbol{r},\boldsymbol{r})$ diverges, since when $|\boldsymbol{r}_1 - \boldsymbol{r}_2| \to 0$ the anomalous density $\nu(\boldsymbol{r}_1,\boldsymbol{r}_2)$ has the singular behavior,

$$\nu(\boldsymbol{r}_1,\boldsymbol{r}_2) = \sum_i v_i^*(\boldsymbol{r}_1)u_i(\boldsymbol{r}_2) \propto \frac{1}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|}$$

Thomas-Fermi approximation for the calculation of the regular part of the Green's function

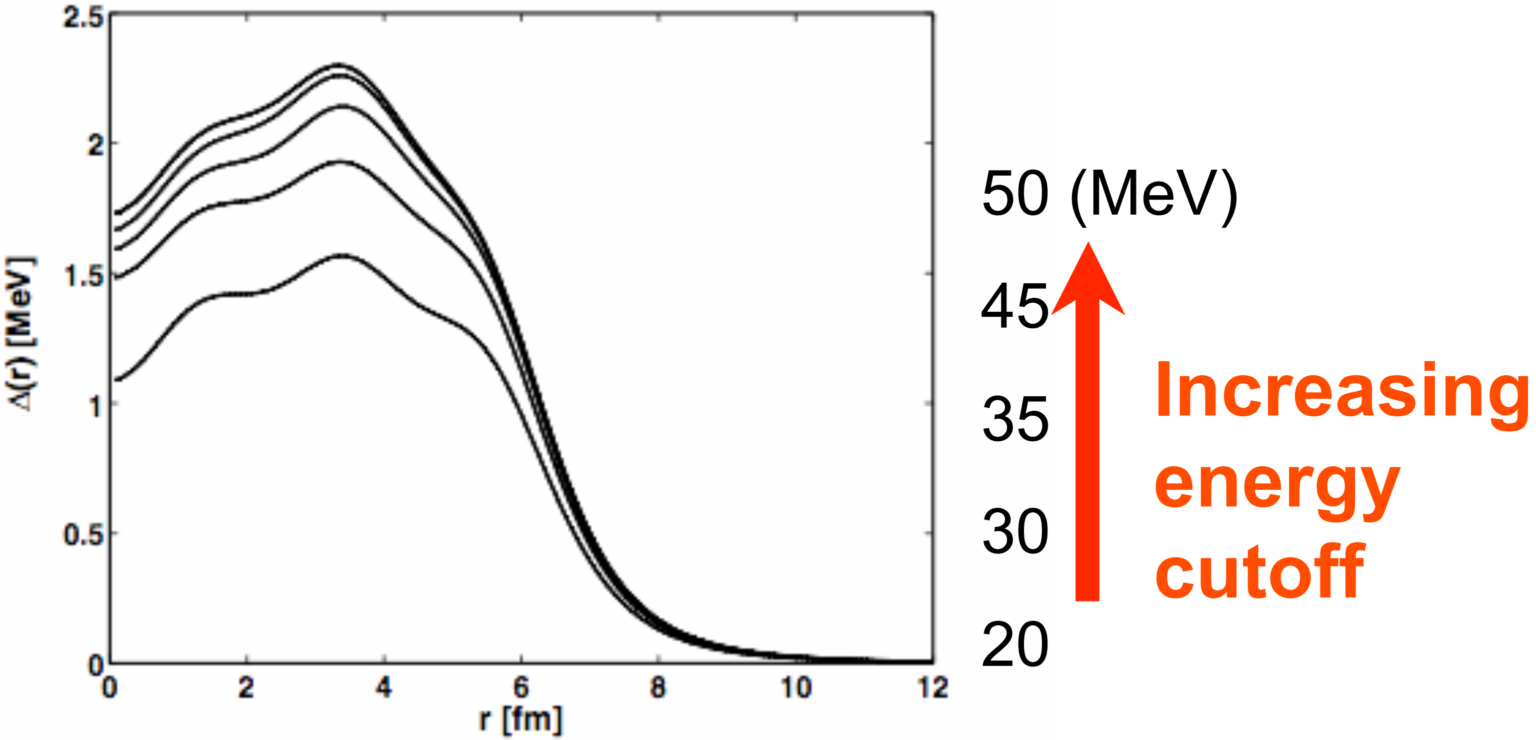
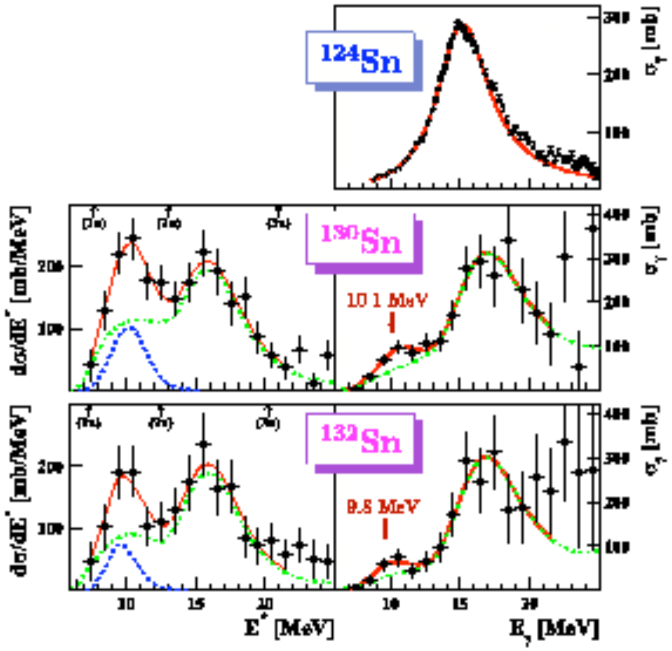


FIG. 1. The neutron pairing field (17) as a function of the radial coordinate and the cutoff energy E_c. Upward various curves correspond to $E_c = 20, 30, 35, 40, 45$, and 50 MeV, respectively. On the scale of the figure the last two curves are indistinguishable.

Bulgac and Yu, 2002

4. Isospin effects. Exotic nuclei. New phenomena… stronger and /or different correlations?

Evolution of excitation modes in exotic nuclei. One example.



Coulomb cross section

Deduced photo-neutron cross section

Coulomb excitation, GSI

PYGMY DIPOLE RESONANCE

Adrich et al., PRL 95, 132501 (2005)

Change of magic numbers. Importance of a better treatment of single-particle states

PRL **99,** 022503 (2007) PHYSICAL REVIEW LETTERS week ending 13 JULY 2007

Collapse of the $N = 28$ Shell Closure in ^{42}Si

B. Bastin,[2] S. Grévy,[1,*] D. Sohler,[3] O. Sorlin,[1,4] Zs. Dombrádi,[3] N. L. Achouri,[2] J. C. Angélique,[2] F. Azaiez,[4] D. Baiborodin,[5] R. Borcea,[6] C. Bourgeois,[4] A. Buta,[6] A. Bürger,[7,8] R. Chapman,[9] J. C. Dalouzy,[1] Z. Dlouhy,[5] A. Drouard,[7] Z. Elekes,[3] S. Franchoo,[4] S. Iacob,[6] B. Laurent,[2] M. Lazar,[6] X. Liang,[9] E. Liénard,[2] J. Mrazek,[5] L. Nalpas,[7] F. Negoita,[6] N. A. Orr,[2] Y. Penionzhkevich,[10] Zs. Podolyák,[11] F. Pougheon,[4] P. Roussel-Chomaz,[1] M. G. Saint-Laurent,[1] M. Stanoiu,[4,6] and I. Stefan[1]

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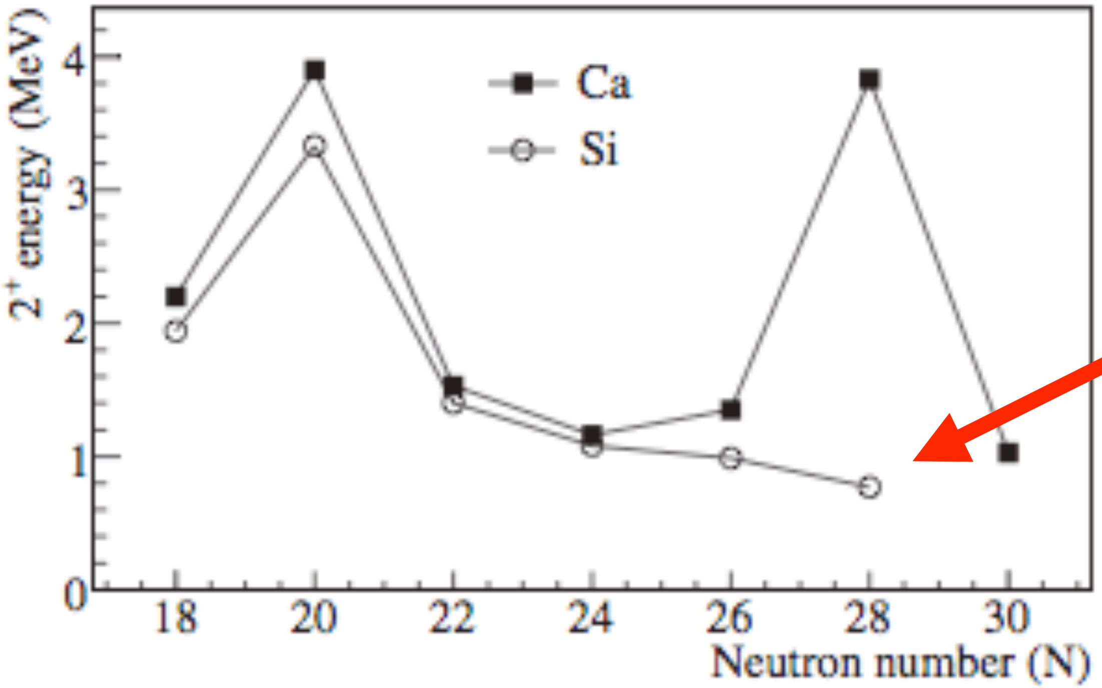
(Received 20 December 2006; published 12 July 2007)

Bastin, et al. 2002



Low energy. Collapse of the N=28 shell closure at Z=14

FIG. 3. Energies of the 2^+ states measured in the Ca and Si isotopes. Present result for $^{40,42}Si$—770(19) keV—brings evidence for the collapse of the $N = 28$ shell closure at $Z = 14$.

Not beyond mean field, but towards a more complete formalism

Tensor force

- Shell model : T. Otsuka, et al., PRL 95, 232502 (2005)

- Relativistic HFB (one needs exchange contribution so that the pion can be active) : W. Long, et al., PLB 640, 150 (2006)

- Non relativistic mean field:
- Skyrme : G. Colò, et al., PLB 646, 227 (2007)
- Fit: Lesinski, et al., PRC 76, 014312 (2007)
- Gogny : T. Otsuka, et al., PRL 97, 162501 (2006) aggiornare riferimenti

Variation of the energy density (contributions depending on *J*)

$$\Delta H = \frac{1}{2}\alpha\left(J_n^2 + J_p^2\right) + \beta J_n J_p$$

J -> spin-orbit density

$$J_q(r) = \frac{1}{4\pi r^3}\sum_i (2j_i + 1)\left[j_i(j_i + 1) - l_i(l_i + 1) - \frac{3}{4}\right]v_i^2(r)$$

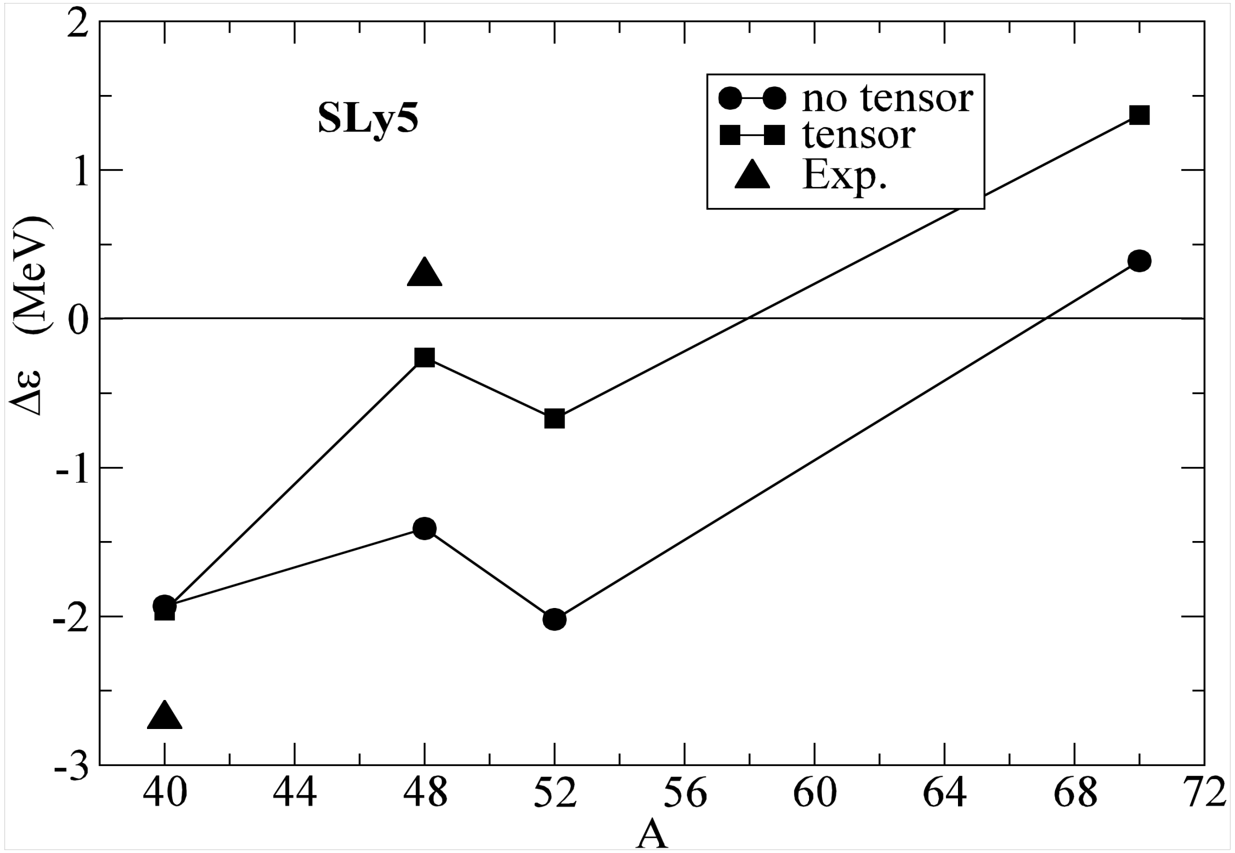
Spin–orbit potential is modified by these contributions

$$U_{SO}^q = \frac{W_0}{2}\left(2\frac{d\rho_q}{dr} + \frac{d\rho_{q'}}{dr}\right) + \left(\alpha J_q + \beta J_{q'}\right)$$

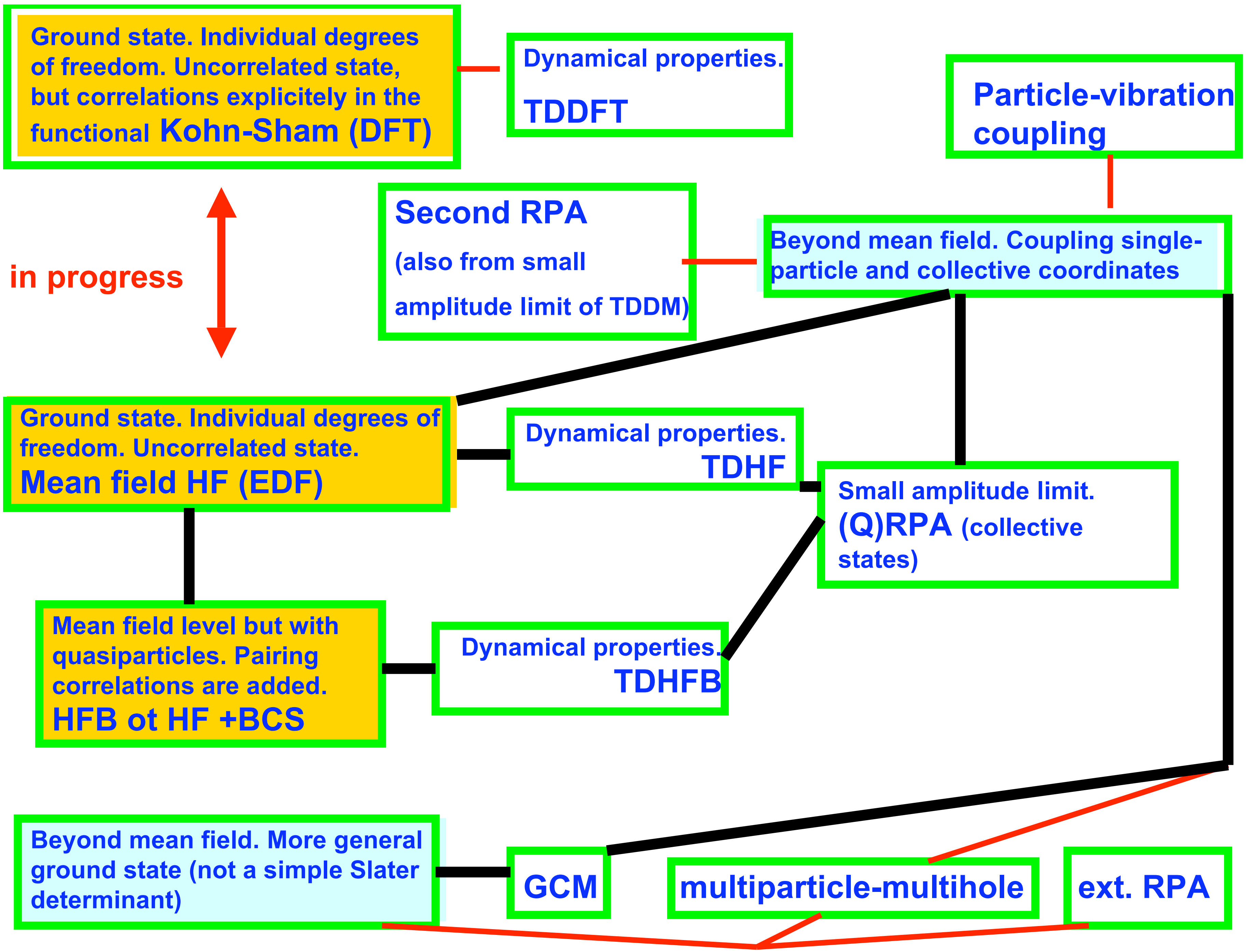
$$\alpha = \alpha_C + \alpha_T \qquad \beta = \beta_C + \beta_T$$

$$\alpha_C = \frac{1}{8}(t_1 - t_2) - \frac{1}{8}(t_1 x_1 + t_2 x_2) \qquad \beta_C = -\frac{1}{8}(t_1 x_1 + t_2 x_2)$$

Effect due to the tensor contribution with SLy5. **Energy difference between proton states 2s1/2 and 1d3/2 in Ca isotopes**



5. Correlations.
Going beyond mean field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Beyond mean field. Adding correlations by using more general wave functions than Slater determinants

Generator Coordinate Method (GCM)

For the formalism: Bonche, et al., Nucl. Phys. A 510 (1990), 466

-Put together collective and single-particle degrees of freedom in a single quantum formulation (collective phenomena, dynamics of large amplitude deformations)

-Variational method that extends configuration mixing to the case of a continuous collective variable

-Projection methods (used to restore broken symmetries) are special forms of GCM (where the wave functions are known a priori)

- **Given a family of N-body wave functions $|\phi(q)\rangle$ depending on a collective variable q the GCM determines approximate states of the Hamiltonian of the form:**

$$|\psi_k> = \int dq f_k(q) |\Phi(q)>$$

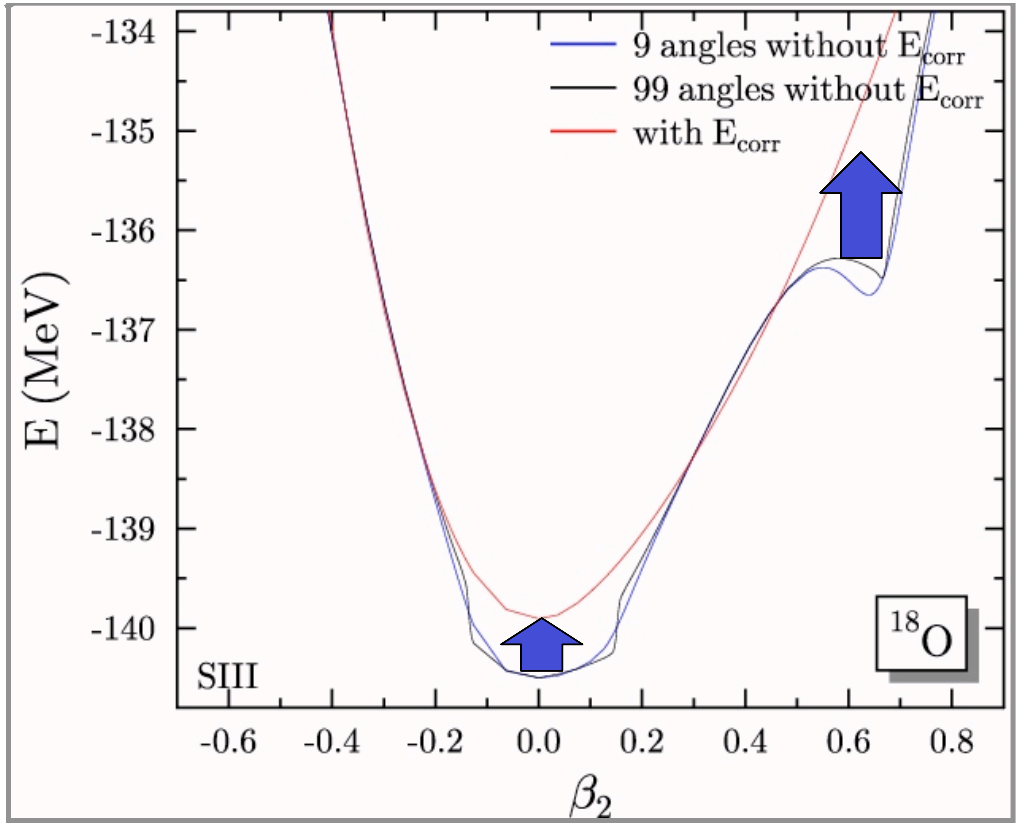
- **The coefficients f_k are found by imposing that**

$$E_k = \frac{<\psi_k|H|\psi_k>}{<\psi_k|\psi_k>}$$

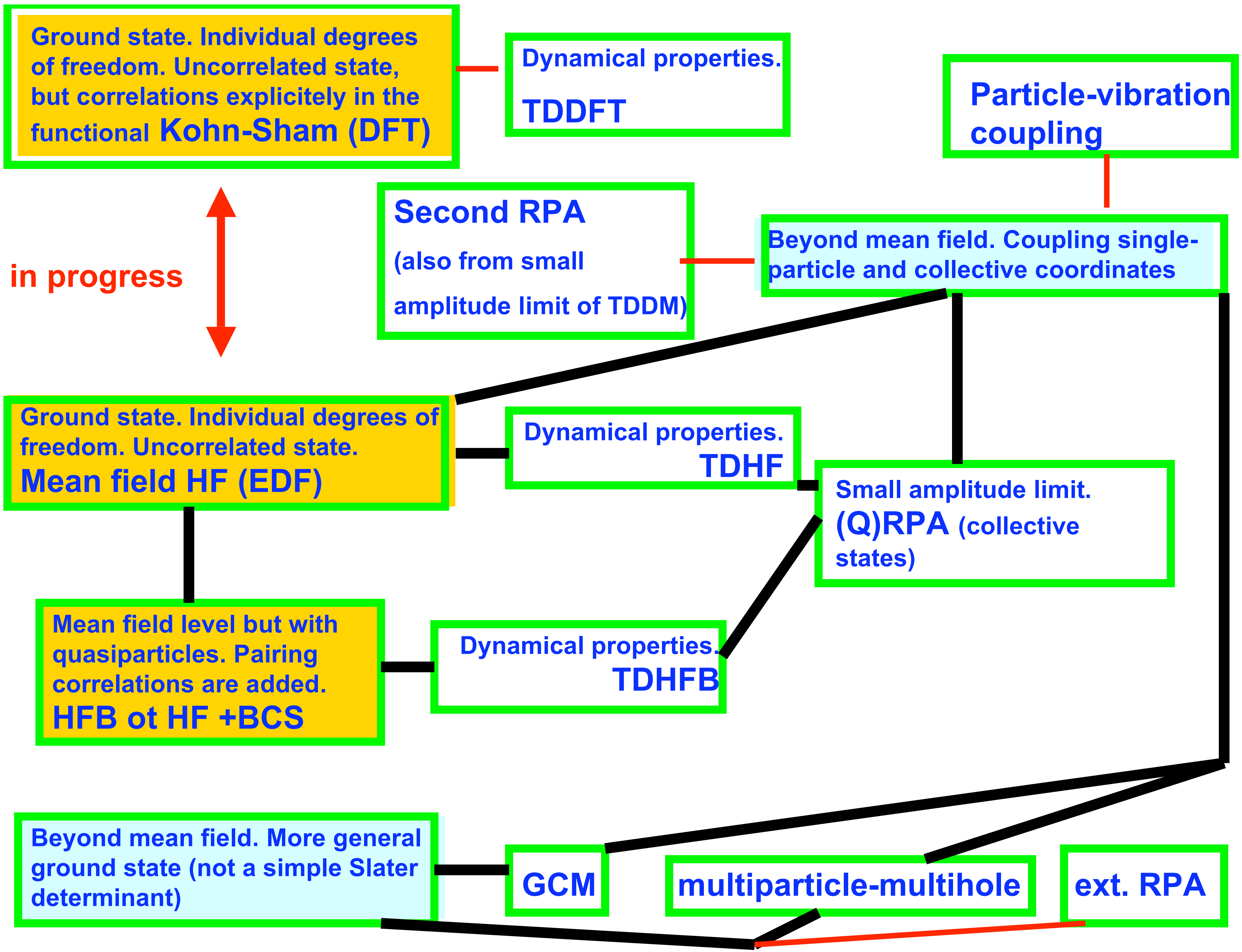
is stationary with respect to variations δf_k

Some technical problems are known and studied: existence of jumps and divergences (related to the functionals)

Solutions proposed in Bender et al., PRC 79 (2009), Duguet et al., PRC 79 (2009), Lacroix et al., PRC 79 (2009)



From D. Lacroix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Correlations. Multiparticle-multihole configurations. The ground state is a superposition of Slater determinants

- **Higher Tamm Dancoff approximation**

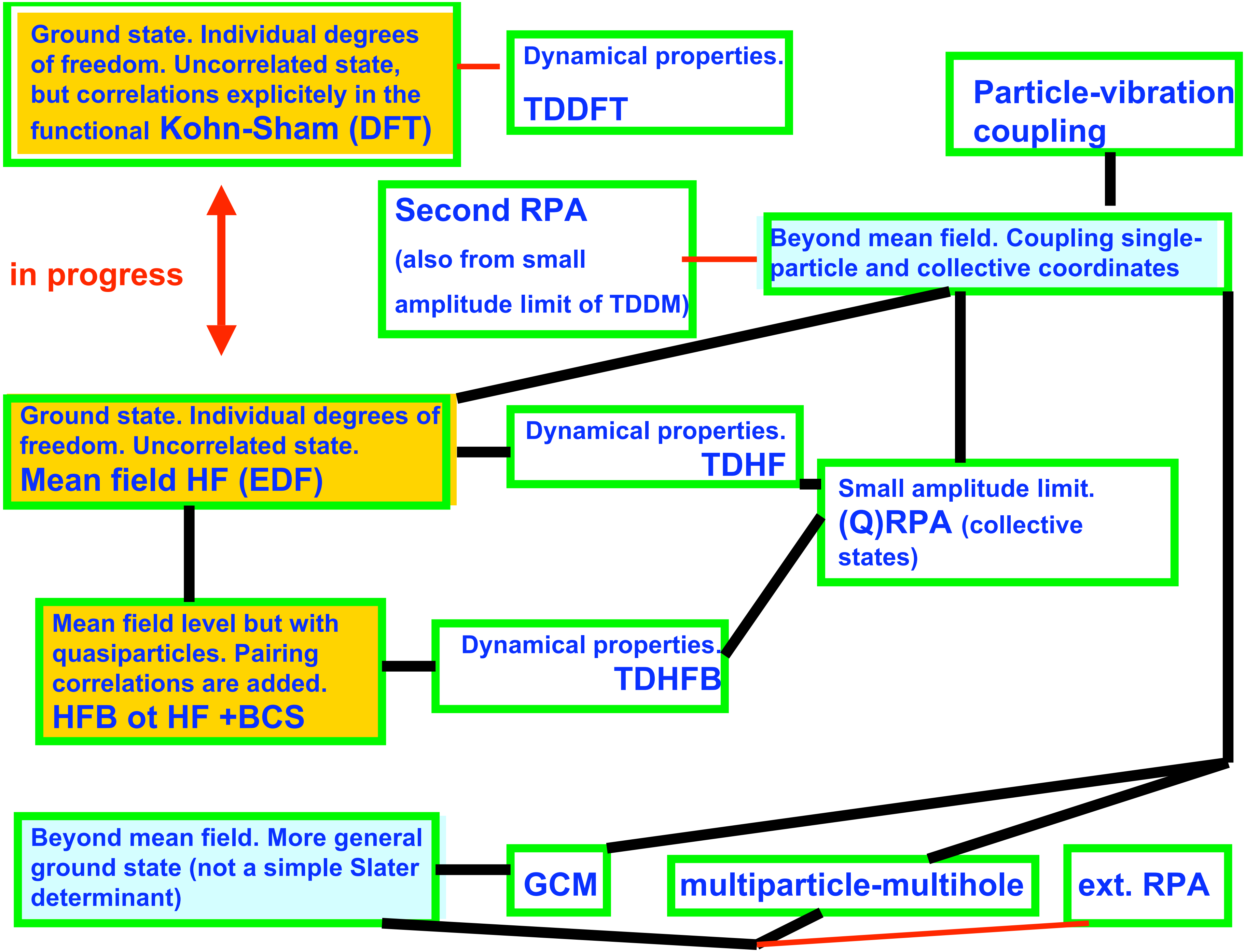
[17] N. Pillet, P. Quentin, and J. Libert, Nucl. Phys. **A697**, 141 (2002).
[18] P. Quentin, H. Laftchiev, D. Samsoen, I. N. Mikhailov, and J. Libert, Nucl. Phys. **A734**, 477 (2004); K. Sieja, T. L. Ha, P. Quentin, and A. Baran, Int. J. Mod. Phys. E **16**, 289 (2007); L. Bonneau, P. Quentin, and K. Sieja, Phys. Rev. C **76**, 014304 (2007).

- **More refined method:**

PHYSICAL REVIEW C **78**, 024305 (2008)

Variational multiparticle-multihole configuration mixing method applied to pairing correlations in nuclei

N. Pillet,[1] J.-F. Berger,[1] and E. Caurier[2]


Ground state. Individual degrees of freedom. Uncorrelated state, but correlations explicitely in the functional Kohn-Sham (DFT)
Dynamical properties.
TDDFT
Particle-vibration coupling
in progress
Second RPA
(also from small amplitude limit of TDDM)
Beyond mean field. Coupling single-particle and collective coordinates
Ground state. Individual degrees of freedom. Uncorrelated state.
Mean field HF (EDF)
Dynamical properties.
TDHF
Small amplitude limit.
(Q)RPA (collective states)
Mean field level but with quasiparticles. Pairing correlations are added.
HFB ot HF +BCS
Dynamical properties.
TDHFB
Beyond mean field. More general ground state (not a simple Slater determinant)
GCM
multiparticle-multihole
ext. RPA

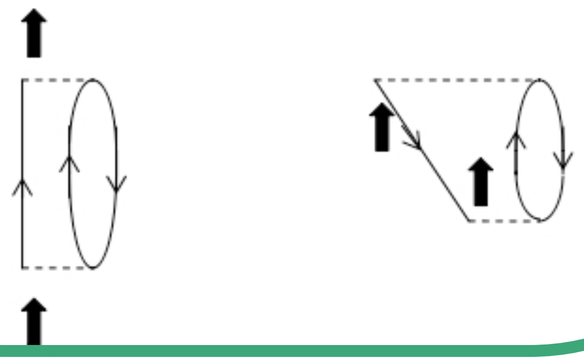
Beyond mean field. Particle-vibration coupling: coupling between single-particle and collective degrees of freedom

In RPA framework (taking into account ph excitations) the HF mass operator is modified by:

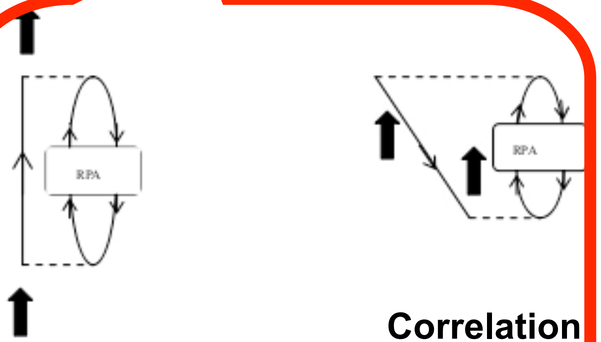
$$M(E) = M^{HF} + \Sigma(E) \; ,$$

with:

$$\Sigma(E) = \Sigma^{RPA}(E) - \Sigma^{(2)}(E) \; .$$



These diagrams have to be subtracted to reduce the double counting related to the use of RPA in the description of particle-vibration coupling

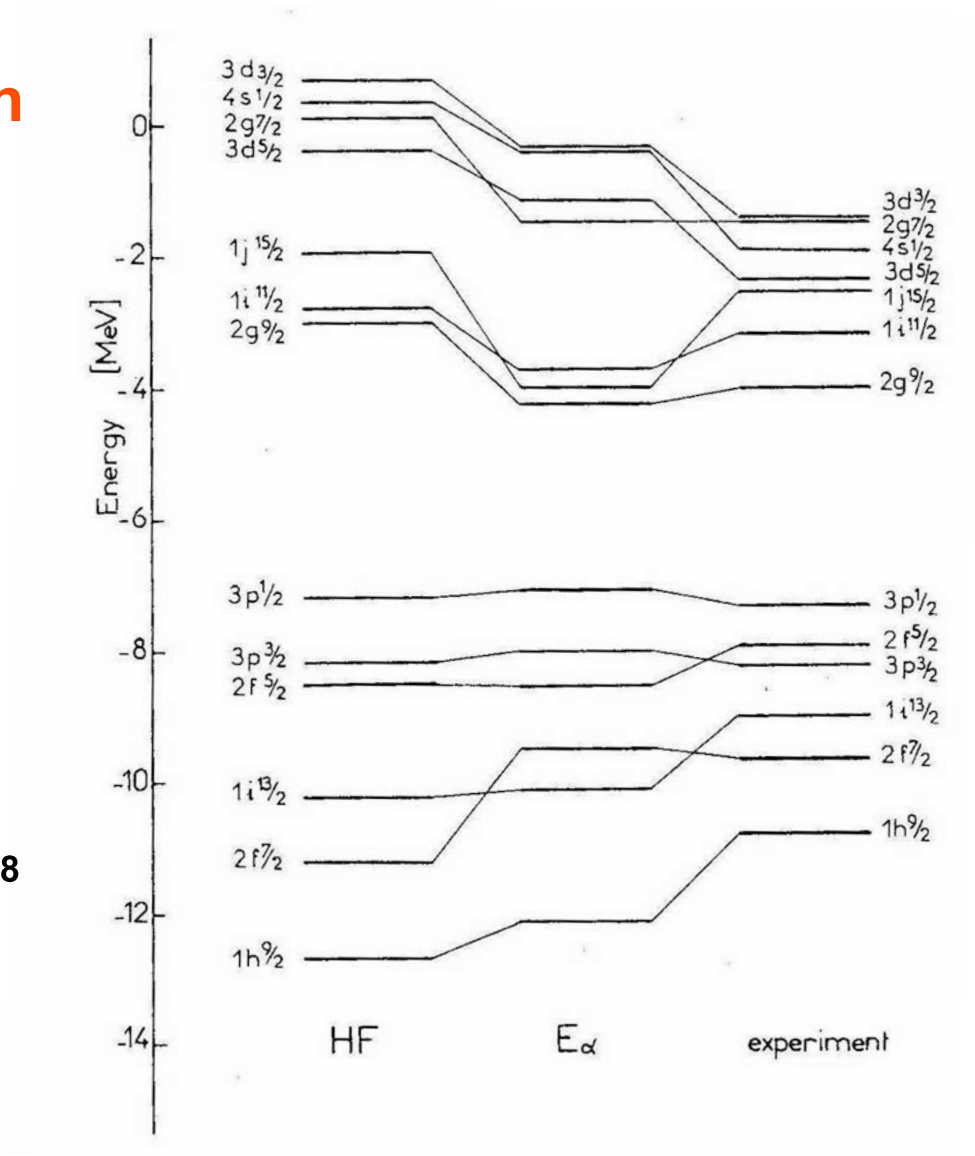


Polarization diagram

Correlation diagram

These correlations modify the single particle spectrum (occupation numbers different from 1 and 0 -> we can evaluate the spectroscopic factors; the single-particle energies are shifted) and the low-lying excited states

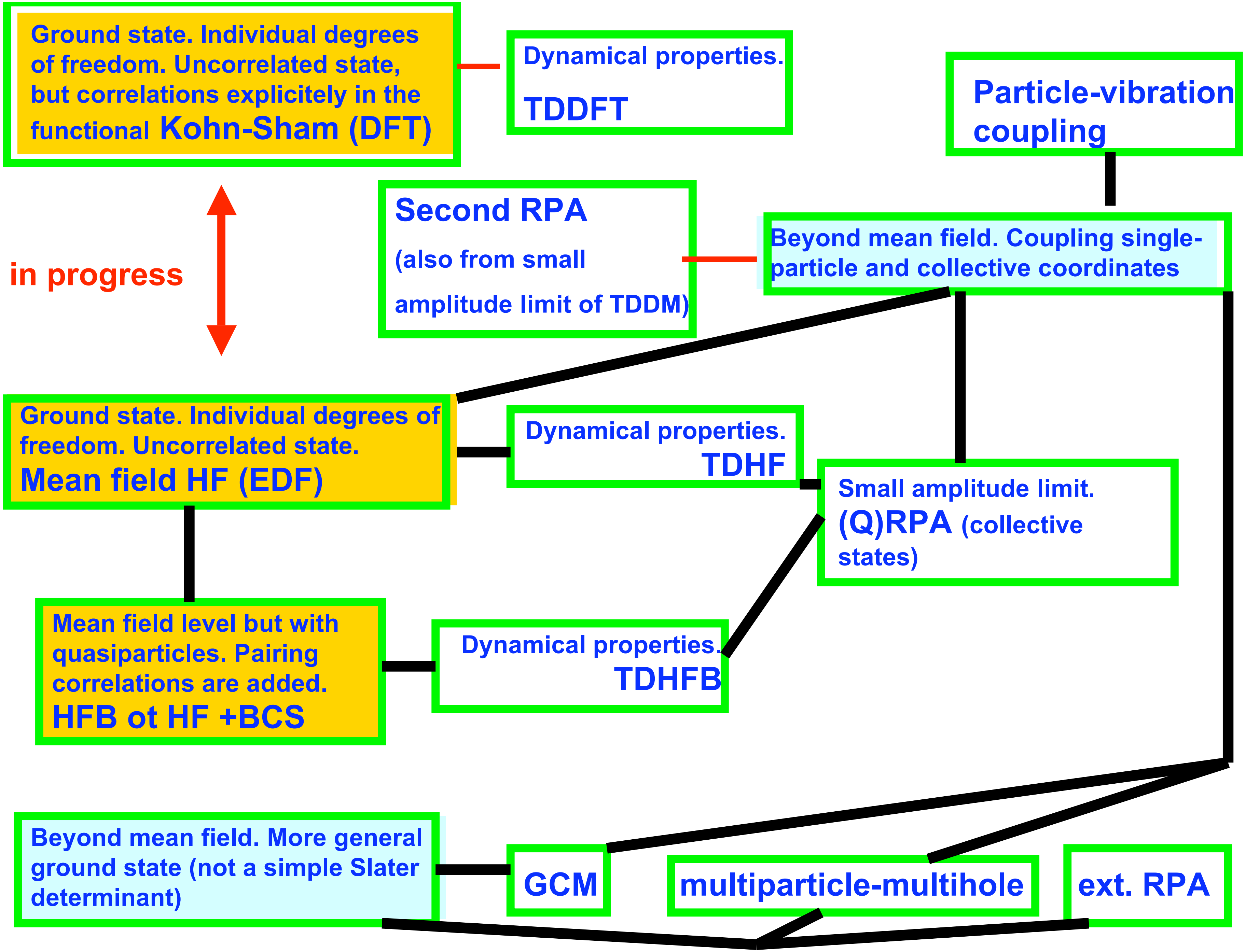
Effects of particle-vibration coupling on the single-particle spectrum



Neutron states in ^{208}Pb

Bernard, Van Giai, Nucl. Phys. A 348 (1980), 75

<u>**These correlations also affect the excited states …**</u>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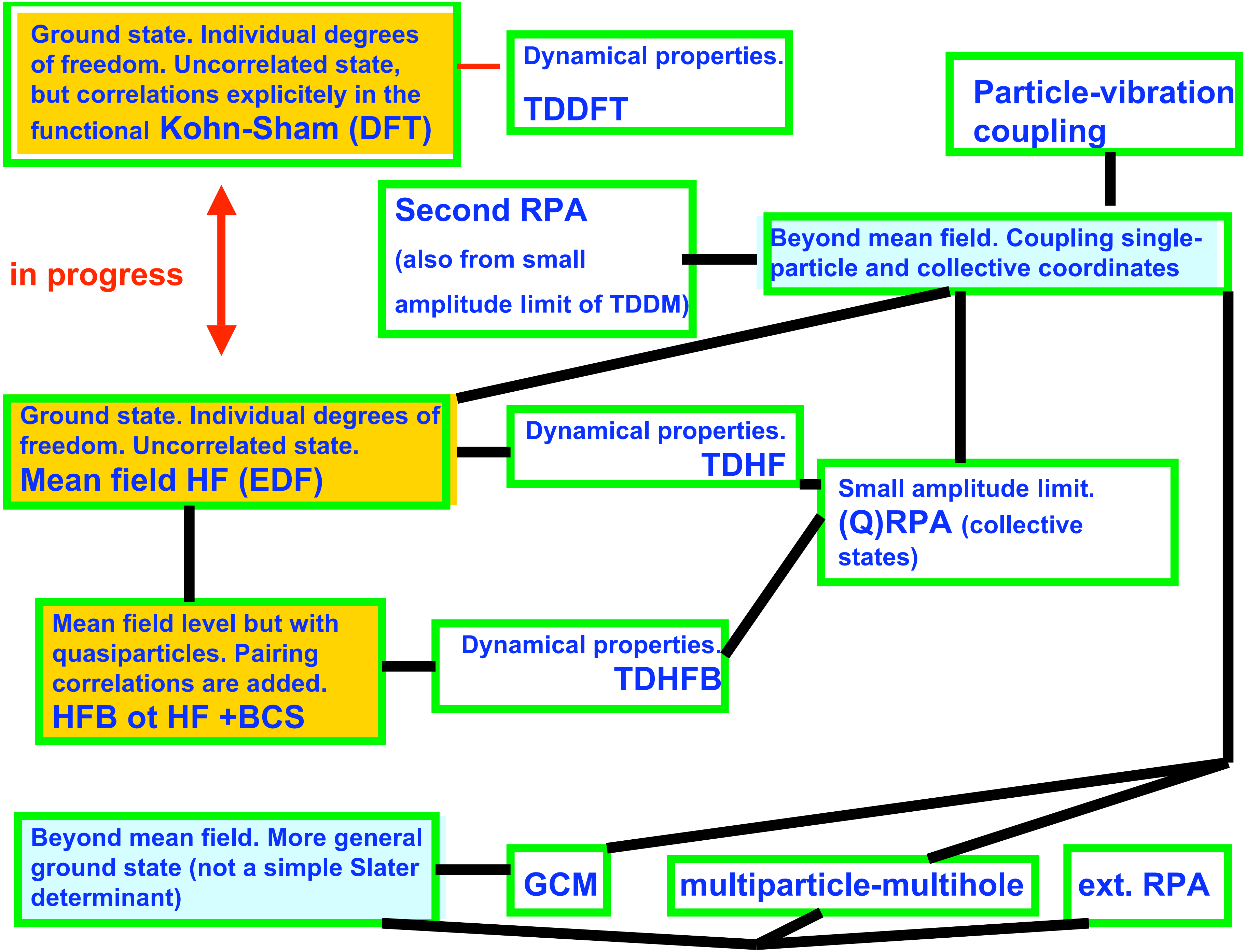
Extensions of RPA. Problem of the Quasiboson Approximation (violation of the Pauli principle)

Gambacurta, et al. PRC 80, 014303 (2009) and references therein

Another possible way to explicitly introduce correlations in the ground state consists in constructing a RPA-like formalism where the violations of the Pauli principle related to the use of the QBA are cured. Two main lines have been developed in the past decades using either boson expansion methods [10] or extensions in the fermionic space [11]. Many of these approaches are based on the so-called renormalized RPA (RRPA) method starting from the early works of Hara and Rowe in the 1960s [12]. In all these RRPA models: (i) the ground state is explicitly correlated with occupation numbers different from 1 and 0 appearing in some renormalization factors; (ii) the Pauli principle is satisfied because the QBA is not adopted.



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GCM
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ext. RPA

Second RPA

C. Yannouleas, Phys. Rev. C 35 (1986), 1159

$$Q_\nu^+ \equiv \sum_{ph} \left[Y_{ph}(\omega_\nu) a_p^+ a_h - Z_{ph}(\omega_\nu) a_h^+ a_p \right]$$
$$+ \sum_{p<p',h<h'} \left[Y_{pp'hh'}(\omega_\nu) a_p^+ a_{p'}^+ a_{h'} a_h - Z_{pp'hh'}(\omega_\nu) a_h^+ a_{h'}^+ a_{p'} a_p \right]$$

Excitation operator:

1p1h +

2p2h

It can be derived by a linearization of Time Dependent Density Matrix (TDDM)

Tohyama and Gong, Z. Phys. A - Atomic Nuclei 332, 269 (1989)

Lacroix, Ayik, Chomaz, Prog. Part. Nucl. Phys. 52, 497 (2004)

Diagonalization of:

$$\begin{pmatrix} \Phi & \Gamma \\ -\Gamma^* & -\Phi^* \end{pmatrix} \begin{pmatrix} \theta(\omega_\nu) \\ \tau(\omega_\nu) \end{pmatrix} = \hbar\omega_\nu \begin{pmatrix} \theta(\omega_\nu) \\ \tau(\omega_\nu) \end{pmatrix},$$

where:

$$\Phi = \begin{pmatrix} A_{ph,p'h'} & A_{ph,p'p''h'h''} \\ A_{p\bar{p}h\bar{h},p'h'} & A_{p\bar{p}h\bar{h},p'p''h'h''} \end{pmatrix},$$

$$\Gamma = \begin{pmatrix} B_{ph,p'h'} & B_{ph,p'p''h'h''} \\ B_{p\bar{p}h\bar{h},p'h'} & B_{p\bar{p}h\bar{h},p'p''h'h''} \end{pmatrix},$$

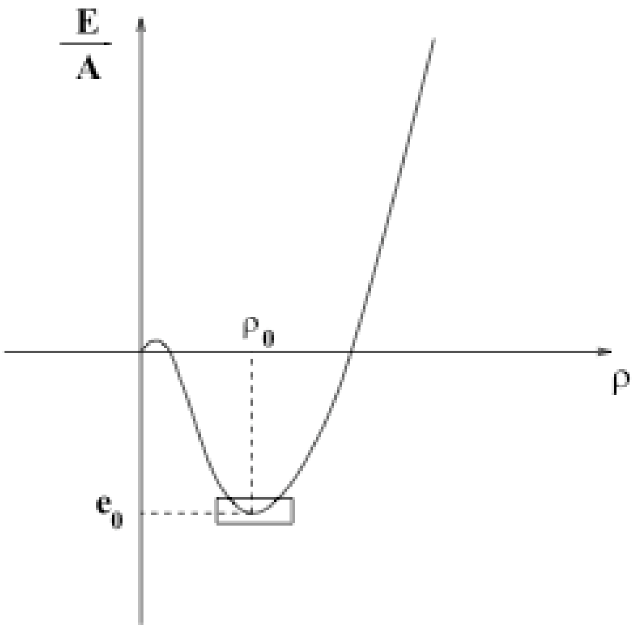
$$\theta(\omega_\nu) = \begin{pmatrix} Y_{ph}(\omega_\nu) \\ Y_{p\bar{p}h\bar{h}}(\omega_\nu) \end{pmatrix} \quad \tau(\omega_\nu) = \begin{pmatrix} Z_{ph}(\omega_\nu) \\ Z_{p\bar{p}h\bar{h}}(\omega_\nu) \end{pmatrix}.$$

Matrix elements $A_{php'p''h'h''}$ couple 1p1h with 2p2h

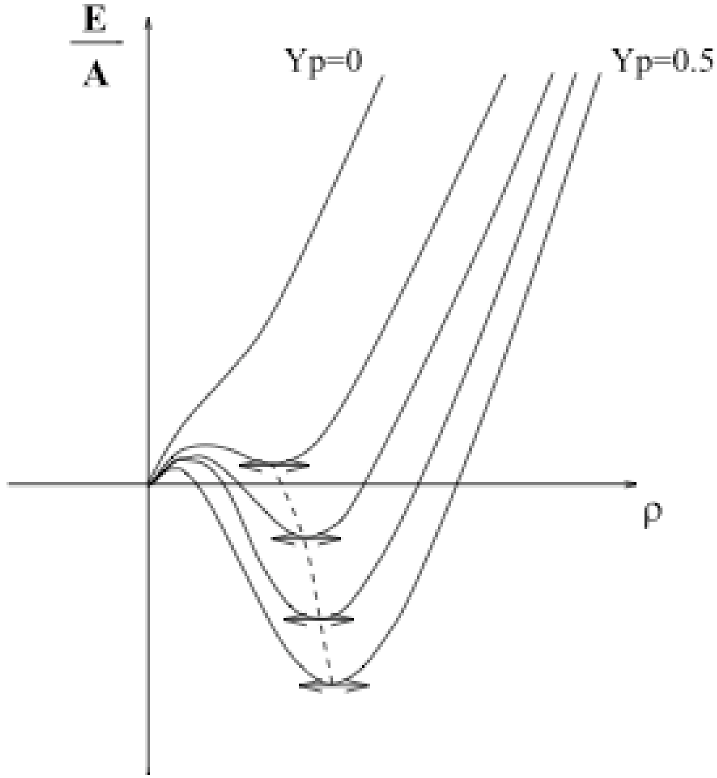
(particle-phonon coupling is thus included in a complete way)

6. Nuclear matter and its properties. The EoS (case of Skyrme)

With an EDF model (for instance from the Skyrme density functional) we can calculate the Equation of State (EoS) of nuclear matter



Symmetric



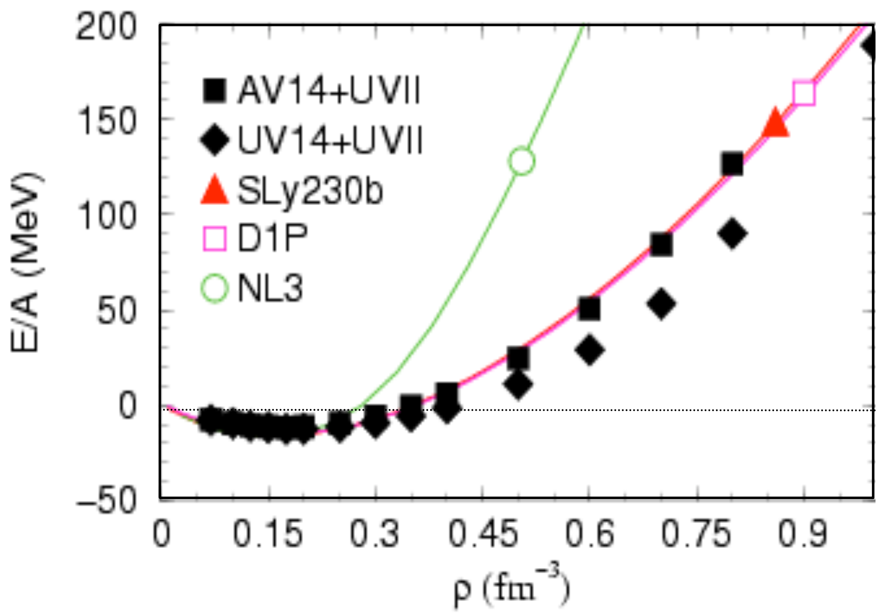
From symmetric to neutron matter. Y_p=Z/A

Isospin effects

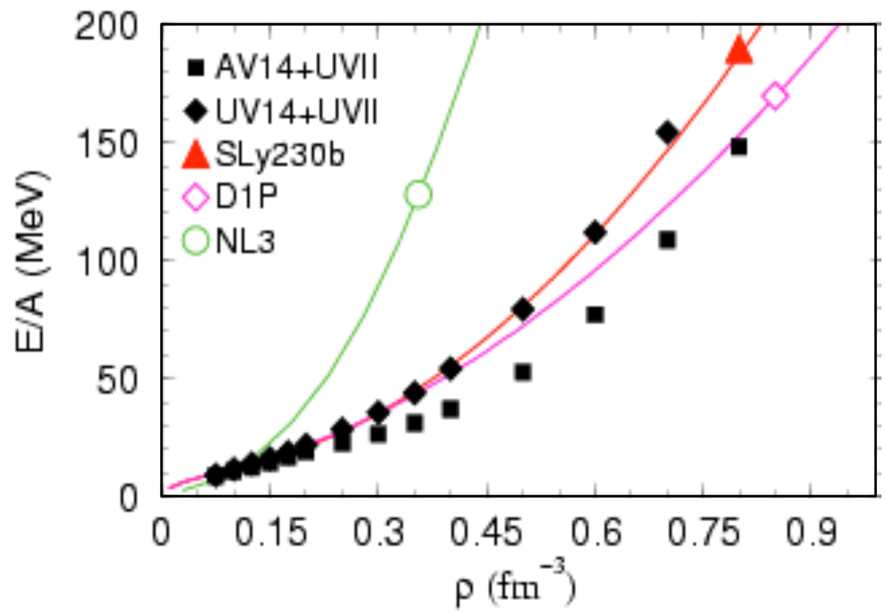
Differences between symmetric nuclear matter EoSs and neutron-rich matter EoSs (density dependence of symmetry energy)

Equation of State

Symmetric nuclear matter



Pure neutron matter



From Sagawa, Nuclear Bulk Properties, MSU 2008

Saturation density

Compressibility

Symmetry energy

Force	SIII [15]	Ska [106]	SkM [116]	SGII [124]	SkM* [119]	RATP [122]
ρ_0 (fm^{-3})	0,145	0,155	0,160	0,158	0,160	0,160
k_F (fm^{-1})	1,291	1,320	1,334	1,328	1,333	1,333
r_0 (fm)	1,180	1,154	1,142	1,147	1,143	1,143
a_v (MeV)	–15,851	–15,991	–15,570	–15,794	–15,770	–16,046
K_∞ (MeV)	355,4	263,10	216,6	214,6	216,6	239,51
$(m^*/m)_s$	0,76	0,61	0,79	0,79	0,79	0,67
a_I (MeV)	28,16	32,91	30,03	26,83	30,03	29,26
κ_v (E1 ; $T = 1$)	0,53	0,94	0,53	0,49	0,53	0,78

Tableau 19. Propriétés de la matière nucléaire infinie pour les forces effectives de type Skyrme citées dans le texte (*cf.* Tab. 18 pour les commentaires).
[Properties of the infinite nuclear matter for the Skyrme effective forces used in the text (see Tab. 18).]

WITH SOME SKYRME FORCES

Force	T6 [118]	SkP [35]	SLy4	SLy5	SLy6	SLy7
ρ_0 (fm^{-3})	0,161	0,162	0,160	0,160	0,159	0,158
k_F (fm^{-1})	1,335	1,340	1,333	1,334	1,330	1,328
r_0 (fm)	1,141	1,137	1,143	1,143	1,145	1,147
a_v (MeV)	–15,963	–15,948	–15,969	–15,983	–15,920	15,894
K_∞ (MeV)	235,93	200,96	229,90	229,90	229,80	229,70
$(m^*/m)_s$	1,00	1,00	0,70	0,70	0,69	0,69
a_I (MeV)	29,97	30,00	32,00	32,03	31,96	31,99
κ_v (E1 ; $T = 1$)	0,00	0,35	0,25	0,25	0,25	0,25

J. Meyer Ann. Phys. Fr. **28** • N° 3 • 2003

How these quantities are known?

- **Saturation point.** **Extracted from electron scattering experiments: the central density of heavy nuclei is always the same, independently of the nucleus.**

  Empirical values

$$\rho_0 = 0{,}16 \pm 0{,}002 \text{ fm}^{-3}; \qquad E/A(\rho_0) = -16{,}0 \pm 0{,}2 \text{ MeV}.$$

- **From the EoS with Skyrme (eliminating surface and spin-orbit and with $\rho_n = \rho_p = \rho/2$) the volume energy is:**

$$\frac{E}{A}(\rho) = \frac{3}{10}\frac{\hbar^2}{m}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}}\rho^{\frac{2}{3}} + \frac{3}{8}t_0\rho + \frac{3}{80}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}}\Theta_s\rho^{\frac{5}{3}} + \frac{1}{16}t_3\rho^{\alpha+1}$$

$$\Theta_s = 3t_1 + t_2(5 + 4x_2).$$

- **Incompressibility** (curvature at saturation point). From studies on monopole giant resonance (theory and experiment)

  Accepted value (systematic of Blaizot (RPA))

$$\boxed{K_\infty = 210 \pm 30\ \mathrm{MeV.}}$$

From the Skyrme EoS

$$K_\infty = 9\rho_0^2 \left(\frac{\mathrm{d}^2}{\mathrm{d}\rho^2} \frac{E}{A}(\rho) \right)_{\rho=\rho_0} = -\frac{3}{5}\frac{\hbar^2}{m}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}} \rho_0^{\frac{2}{3}} + \frac{3}{8}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}} \Theta_s \rho_0^{\frac{5}{3}} + \frac{9}{16}\alpha(\alpha+1)t_3\rho_0^{\alpha+1}.$$

- **Isoscalar effective mass** (very important for level densities around Fermi energy). It can be related to the energy of the isoscalar quadrupole giant resonance

$$\varepsilon_{\mathbf{p}} = \frac{\mathbf{p}^2}{2m} + \Sigma(\mathbf{p}, \varepsilon_p) = \frac{\mathbf{p}^2}{2m^*}.$$

Empirical value:

$$\boxed{\frac{m^*}{m} = 0{,}8 \div 0{,}9.}$$

From Skyrme EoS:

$$\frac{\hbar^2}{2m_q^*}\tau_q = \frac{\hbar^2}{2m}\left(\frac{m}{m_q^*}\right)\tau_q$$

$$\left(\frac{m}{m_q^*}\right) = 1 + \frac{1}{4}\frac{m}{\hbar^2}\rho\Big[t_1(2+x_1) + t_2(2+x_2)\Big] + \frac{1}{4}\frac{m}{\hbar^2}\rho_q\Big[t_2(1+2x_2) - t_1(1+2x_1)\Big]$$

$$= 1 + \frac{1}{4}\frac{m}{\hbar^2}\rho\,\Theta_v + \frac{1}{4}\frac{m}{\hbar^2}\rho_q[\Theta_s - 2\Theta_v]$$

$$\Theta_s = 3t_1 + t_2(5+4x_2), \quad \Theta_v = t_1(2+x_1) + t_2(2+x_2)$$

7. Asymmetric matter, neutron matter. Isospin effects

The EoS (Y_p=Z/A, I=(N-Z)/A))

$$\frac{E}{A}\left(Y_\mathrm{p}\ \mathrm{ou}\ I;\rho\right) = \frac{3}{10}\frac{\hbar^2}{m}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}}\rho^{\frac{2}{3}}\,\mathrm{F}_{5/3}$$
$$+\frac{1}{8}t_0\rho\Big[2\,(2+x_0)-(1+2x_0)\,\mathrm{F}_2\Big]+\frac{1}{48}t_3\rho^{\alpha+1}\Big[2\,(2+x_3)-(1+2x_3)\,\mathrm{F}_2\Big]$$
$$+\frac{3}{40}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}}\rho^{\frac{5}{3}}\Big[\Theta_\mathrm{v}\mathrm{F}_{5/3}+\frac{1}{2}\left(\Theta_\mathrm{s}-2\Theta_\mathrm{v}\right)\mathrm{F}_{8/3}\Big]$$

$$\mathrm{F}_m(Y_\mathrm{p}) = 2^{m-1}\Big[Y_\mathrm{p}^m + \left(1-Y_\mathrm{p}\right)^m\Big], \qquad \mathrm{F}_m(I) = \frac{1}{2}\Big[(1+I)^m + (1-I)^m\Big].$$

Isospin effects. Importance of symmetry energy.

- **Empirical value (from masses using models).**

  Accepted value

$$\boxed{28\ \mathrm{MeV} \leq a_{\mathrm{I}} \leq 32\ \mathrm{MeV}.}$$

- **From the equation of state:**

$$a_{\mathrm{I}} = \frac{1}{2} \frac{\mathrm{d}^2}{\mathrm{d}I^2} \frac{E}{A}(I, \rho)\bigg|_{I=0}$$

From Skyrme EoS

$$a_{\mathrm{I}} = \frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}I^2}\frac{E}{A}(I,\rho)\bigg|_{I=0}$$
$$= \frac{1}{6}\frac{\hbar^2}{m}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}}\rho^{\frac{2}{3}} - \frac{1}{8}t_0\,(2x_0+1)\,\rho - \frac{1}{24}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}}(3\Theta_{\mathrm{v}} - 2\Theta_{\mathrm{s}})\,\rho^{\frac{5}{3}}$$
$$- \frac{1}{48}t_3\,(2x_3+1)\,\rho^{\alpha+1}$$

$$3\Theta_{\mathrm{v}} - 2\Theta_{\mathrm{s}} = 3t_1x_1 - t_2\,(4+5x_2).$$

About the symmetry energy. Dependence on ρ

- **We have mentioned the correlations between some quantities characterizing the EoS. A study of the correlations $\{K,\rho_0, a_I\}$ would allow a better control on some terms of the interaction (or the energy density functional). Density-dependent term?**

- **Heavy ion collisions, fragmentation: one of the objectives is to get a better knowledge of the density dependence of the symmetry energy. This scenario is richer if exotic ions are considered (dependence on neutron/proton asymmetry). Marie-France Rivet Lecture**

- **Collective modes in asymmetric nuclei?**

P.-G. Reinhard, Nucl. Phys. A **649**, 305c (1999).

K. Morawetz, U. Fuhrmann, R. Walke, *Isospin Physics in Heavy-Ion Collisions at Intermediate Energies*, édité par Bao-An Li, W.U. Schroeder (Nova Science Pub. Inc., New-York, 2001), nucl-th/0001032.

- **Neutron star masses and radii. URCA process and cooling (role of the symmetry energy)**

J.M. Lattimer, M. Prakash, Phys. Rep. **333**, 121 (2000).

M. Prakash, T.L. Ainsworth, J.M. Lattimer, Phys. Rev. Lett. **61**, 2518 (1988)

8. Spin instabilities of nuclear matter and the Skyrme interaction

Landau-Migdal parameters

In Landau theory of Fermi liquids nuclear matter properties can be written in terms of matrix elements of the interaction at Fermi surface

$$\langle \mathbf{k}_1 \mathbf{k}_2 | V | \mathbf{k}_1 \mathbf{k}_2 \rangle = N_0^{-1} \left[F(\theta) + F'(\theta)\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + G(\theta)\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + G'(\theta)\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \right]$$

The coefficients F, F', G, G' are functions of the angle between k_1 and k_2. A multipole expansion provides the Landau-Migdal parameters F_l, F'_l, G_l, G'_l (only l=0 and 1 for a zero-range interaction like Skyrme).

The stability of HF solutions constrains the values of these parameters (sum rules)

A detailed analysis in:

PHYSICAL REVIEW C **66**, 014303 (2002)

Instabilities of infinite matter with effective Skyrme-type interactions

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(Received 10 April 2002; published 3 July 2002)

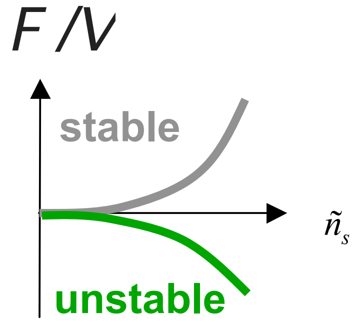
The stability of the equation of state predicted by Skyrme-type interactions is examined. We consider simultaneously symmetric nuclear matter and pure neutron matter. The stability is defined by the inequalities that the Landau parameters must satisfy simultaneously. A systematic study is carried out to define interaction parameter domains where the inequalities are fulfilled. It is found that there is always a critical density ρ_{cr} beyond which the system becomes unstable. The results indicate in which parameter regions one can find effective forces to describe correctly finite nuclei and give at the same time a stable equation of state up to densities of 3–4 times the saturation density of symmetric nuclear matter.

Ferromagnetic phase diagram

Symmetric nuclear matter :

Spin asymmetry density : $\tilde{n}_s = \tilde{n}_\uparrow - \tilde{n}_\downarrow$

Susceptibility : $$\frac{1}{\chi} = \left.\frac{\partial^2 F/V}{\partial \rho_s^2}\right|_{\tilde{n}_s = 0}$$



From Sagawa, Nuclear Bulk Properties, MSU 2008

Microscopic calculations of the ferromagnetic instabilty

S. Fantoni, et al., PRL 87, 018110 (2001)
I. Vidana et al., PRC 65, 035804 (2002), 66, 045801 (2002)
I. Bombaci et al., PLB 632, 638 (2006), ...

Magnetic susceptibility

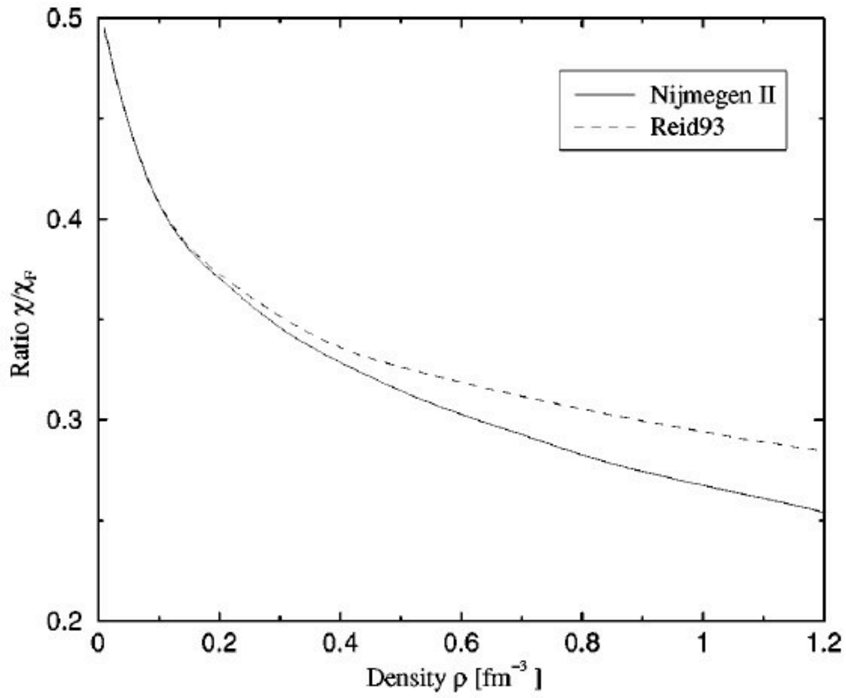


FIG. 4. Ratio χ/χ_F as a function of the density. The solid line shows the result for the Nijmegen II interaction, while the dashed line corresponds to the one obtained with Reid93.

Results:
No ferromagnetic instability

On the other hand, Skyrme functional is unstable at high density or large asymmetries

J. Meyer 2003

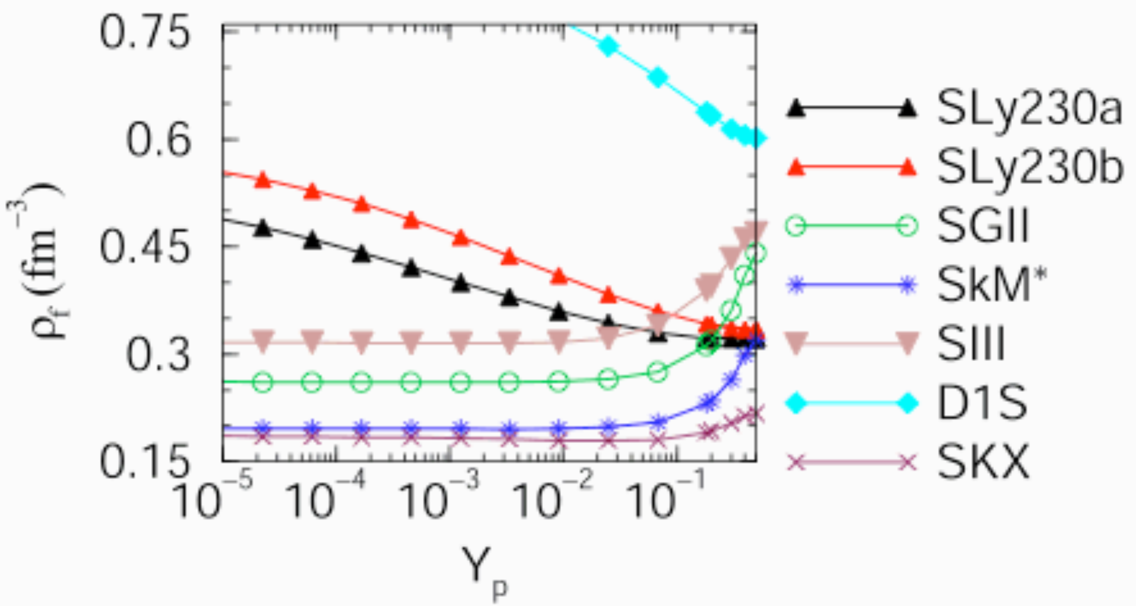


Figure 15. Densités ρ_f (en fm^{-3}) d'apparition d'une instabilité ferromagnétique en fonction de l'asymétrie neutron-proton Y_p pour différentes forces effectives [243].

[Densities ρ_f (fm^{-3}) where a ferromagnetic instability occurs as a function of the Y_p neutron-proton asymetry for some effective forces [243].]

Margueron 2002

How to cure? An extended Skyrme interaction

Proposed in **Margueron and Sagawa, to be published in J. Phys. G** (work on nuclear matter). New terms:

$$V^{s,st}(\mathbf{r}_1,\mathbf{r}_2) = \frac{1}{6}t_3^s(1+x_3^s P_\sigma)[\rho_s(\mathbf{R})]^{\gamma_s}\delta(\mathbf{r}) + \frac{1}{6}t_3^{st}(1+x_3^{st} P_\sigma)[\rho_{st}(\mathbf{R})]^{\gamma_{st}}\delta(\mathbf{r})$$

where the spin and spin-isospin densities are:

$$\rho_s \equiv \rho_\uparrow - \rho_\downarrow \qquad \rho_{st} \equiv \rho_{n\uparrow} - \rho_{n\downarrow} - \rho_{p\uparrow} + \rho_{p\downarrow}$$

Applied to ground state of nuclei in:

Extended Skyrme interaction (II): ground state of nuclei and of nuclear matter

J. Margueron[1], S. Goriely[2], M. Grasso[1], G. Colò[3] and H. Sagawa[4]

To be published J. Phys. G

Conclusions: necessity of improving predicting power (exotic nuclei)

- Fitting criteria ? Changing observables ?
- Functional and/or interaction ?
- Models ?