Nucleons and Nuclei: Interactions, Geometry, Symmetries

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Part I

Nuclear Relativistic Mean Field Theory: Underlying Symmetries

A Few Remarks about the Mean-Field Concept

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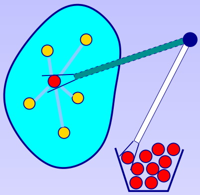
- **A mean-field interaction can be seen as an algorithm probing the two-body interactions through the generalized weighted average $\widehat{\mathbf{V}}$**

$$\widehat{V}(\hat{x}) = \frac{1}{N-1} \sum_{j=1}^{(N-1)} \int dx_j \psi^*(x_j)\, \widehat{V}(\hat{x}, \hat{x}_j)\, \psi(x_j)$$

An N–Body System



Schematic: Probing 2-body interactions with an 'external' test-particle

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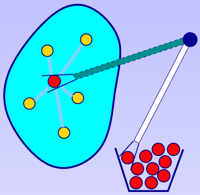

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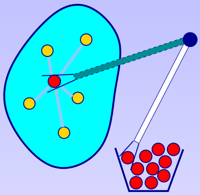

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- **Notice also that the mean-potential $\widehat{V} = \widehat{V}(\hat{x})$ is a one-body operator only**
- **Relativistic theory illustrated in the following provides a similar concept but using a quantum field theory basis**

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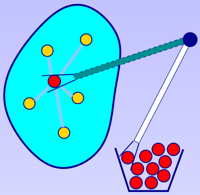

Schematic: Probing 2-body interactions with an 'external' test-particle

Quark confinement allows to use the independent nucleon approximation

Confinement & Low Energy Sub-Atomic Phenomena

- **In analogy to quantum electrodynamics whose Lagrangian-density***

$$\mathcal{L}_{\text{QED}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Maxwell}} + \mathcal{L}_{\text{int}}^{\text{EM}}$$

or more explicitly

$$\mathcal{L}_{\text{QED}} = \bar{\psi}(i\gamma^{\mu}p_{\mu} - m)\psi - \tfrac{1}{4}[F_{\mu\nu}]^2 + e(\bar{\psi}\gamma^{\mu}\psi)A_{\mu}$$

- **... we may introduce the so-called Yukawa interaction density:**

$$\mathcal{L}_{\text{Yukawa}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Klein-Gordon}} + \mathcal{L}_{\text{int}}^{\text{strong}}$$

- **In subatomic physics this theory leads to coupled systems of the relativistic equations ignoring the existence of quarks. Their form:**

$$\left.\begin{array}{lcl} [\text{Dirac Equations for Nucleons}] & = & [\text{Nucleons Coupled with Mesons}] \\ [\text{Klein}-\text{Gordon Eqs for Mesons}] & = & [\text{Mesons Coupled with Nucleons}] \end{array}\right\}$$

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- **In such theories we obtain Dirac-type relativistic wave-equations for the nuclens moving in the average fields of all other particles ...**

- **We obtain relativistic Klein-Gordon-type wave equations for mesons moving in average fields of all other particles; both sets are coupled**

- **Those coupled equations are iterated to obtain a self-consistent final solution for the wave-functions: ψ (nucleons) and ϕ (mezons)**

- **They turn out to be very successful in calculations which can be compared with numerous types of experimental data - e.g. masses**

- **Observe that neither quarks nor gluons will ever appear explicitly**

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Free Dirac Equation - A Short Reminder

- **The so-called covariant form of the free Dirac equation reads***

$$(\gamma^\mu \hat{\mathbf{p}}_\mu - \mathbf{m}\,\mathbf{c})\,\psi = \mathbf{0}; \quad \{\hat{\mathbf{p}}_\mu\} \equiv \left\{\mathbf{i}\left(\frac{\hbar}{\mathbf{c}}\right)\frac{\partial}{\partial \mathbf{t}},\, \mathbf{i}\hbar\hat{\boldsymbol{\nabla}}\right\}$$

***We use occasionally Einstein's summation convention: Repeated indices as e.g** $\gamma^\mu \hat{\mathbf{p}}_\mu \Leftrightarrow \sum_{\mu=0}^{4} \gamma^\mu \hat{\mathbf{p}}_\mu$

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- **Schrödinger-like form of the free Dirac equation - (just insert $\hat{p}_\mu$)**

$$i\hbar\,\frac{\partial\psi}{\partial t} = -i\hbar c\left(\hat{\alpha}\cdot\hat{\nabla}\right)\psi + \beta\left(mc^2\right)\psi; \quad \psi \sim \varphi\, e^{\pm i\,\frac{\mathcal{E}\,t}{\hbar}}$$

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- **An equivalent, stationary form of the free Dirac equation is now:**

$$\left[-i\hbar c\left(\hat{\alpha}\cdot\hat{\nabla}\right) + \beta\,(mc^2)\right]\varphi = \mathcal{E}\,\varphi,$$

where $\hat{\alpha} \equiv \{\alpha_1, \alpha_2, \alpha_3\}$ and β are the standard 4×4 Dirac matrices

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Mesons Mediating Nucleon-Nucleon Interactions

- **In principle the nucleons interact through exchange of q-q̄ pairs:**

π^+, π^0, π^-	—	**isovector, pseudoscalar;**
η	—	**isoscalar, pseudoscalar;**
ρ^+, ρ^0, ρ^-	—	**isovector, vector;**
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• **Using relativistic quantum field theory we may derive the Dirac equation for the nucleons in the presence of the exchange of mesons**

$$\boxed{\{c\,\vec{\alpha}\cdot\hat{p} + \hat{V}(\vec{r})\;\mathbb{I}_4 + \beta\,[m_0c^2 + \hat{S}(\vec{r})]\}\psi_n = \mathcal{E}_n\psi_n,}$$

Above: $\hat{V}$ and $\hat{S}$ are known functions originating from vector and scalar meson exchange, respectively (pseudo-scalars treated approx.)

Dirac Equation for Nucleons (with Interactions)

1. The bound nucleons satisfy the "Dirac equation with interaction"

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2. Vector- and scalar-meson potentials $\hat{V}(\vec{r})$ and $\hat{S}(\vec{r})$, respectively

$$\hat{S}(\vec{r}\,) = g_\sigma\,\sigma(\vec{r}\,) + g_3\,\sigma^3(\vec{r}\,)$$

and

$$\hat{V}(\vec{r}\,) = g_\omega\omega_0(\vec{r}\,) + g_\rho\hat{\tau}_3\rho(\vec{r}\,) + \tfrac{1}{2}(\mathbb{I} + \hat{\tau}_3)g_eA_0(\vec{r}\,)$$

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A Mathematical Simplification: Pauli-Schrödinger Formalism

Standard Pauli-Schrödinger Reduction

- **Representing nucleon's ψ in terms of 'big' and 'small' components:**

$$\psi \equiv \begin{pmatrix} \xi \\ \eta \end{pmatrix}; \quad \xi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}; \quad \eta \equiv \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix}; \quad \vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$$

we may write two Schrödinger-like equations for spinors ξ and η

$$\hat{\mathrm{H}}_\xi \, \xi_\mathrm{n} = \mathcal{E}_\mathrm{n} \, \xi_\mathrm{n} \qquad \text{and} \qquad \hat{\mathrm{H}}_\eta \, \eta_\mathrm{n} = \mathcal{E}_\mathrm{n} \, \eta_\mathrm{n}$$

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- **These Schrödinger-type Hamiltonians are non-linear in energy:**

$$\hat{H}_\xi \equiv (c\,\vec{\sigma}\cdot\hat{p}) \, \frac{1}{[\mathcal{E} + m_0c^2 - (\hat{V} - \hat{S})]} \, (c\,\vec{\sigma}\cdot\hat{p}) + \left[m_0c^2 + (\hat{V} + \hat{S})\right]$$

$$\hat{H}_\eta \equiv (c\,\vec{\sigma}\cdot\hat{p}) \, \frac{1}{[\mathcal{E} - m_0c^2 - (\hat{V} + \hat{S})]} \, (c\,\vec{\sigma}\cdot\hat{p}) - \left[m_0c^2 - (\hat{V} - \hat{S})\right]$$

Standard Pauli-Schrödinger Reduction - Properties

• **Eigen-energies $\mathcal{E}_n$ are common for both equations; they can be obtained by solving only one of them, usually for big component ξ_n**

$$\boxed{\hat{H}_\xi\, \xi_n = \mathcal{E}_n\, \xi_n \quad \text{or} \quad \hat{H}_\eta\, \eta_n = \mathcal{E}_n\, \eta_n}$$

• **The two Schrödinger-type equations are strictly equivalent to the original Dirac equation - <u>there are no approximations here</u>**

• **The potentials depend only (!) on $\vec{r}$: $\hat{V} = \hat{V}(\vec{r})$ and $\hat{S} = \hat{S}(\vec{r})$**

• **The eigen-energies appear non-linearly $\longrightarrow$ Bad News!**

• **Equations depend only on the sum and on the difference of the two original potentials - not on the individual ones $\longrightarrow$ Interesting!**
<u>Very Interesting!</u>

• **Calculations show that inside the nucleus**
$\langle \hat{S} \rangle \approx -400$ **MeV and** $\langle \hat{V} \rangle \approx +350$ **MeV**

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Position-Dependent Effective Mass: Definition

- **Let us recall the definition of the Pauli-Schrödinger Hamiltonian:**

$$\hat{H}_\xi \equiv (c\,\vec{\sigma}\cdot\hat{p})\,\frac{1}{\{\mathcal{E}+m_0c^2-[\,\hat{V}-\hat{S}\,]\}}\,(c\,\vec{\sigma}\cdot\hat{p}) + \big[m_0c^2+[\,\hat{V}+\hat{S}\,]\big]$$

- By replacing $\mathcal{E}$ with $m_0c^2+\epsilon$, we may introduce the position-dependent effective mass $m^*(\vec{r})$

$$m^*(\vec{r}) \equiv \left\{m_0c^2 - \tfrac{1}{2}\,[\hat{V}(\vec{r}) - \hat{S}(\vec{r})\,]\right\}$$

and rewrite the denominator in the form:

$$\epsilon + 2m_0c^2 - [\hat{V}(\vec{r}) - \hat{S}(\vec{r})] \equiv \epsilon + 2m^*(\vec{r})$$

- Since $m_0c^2 \approx 1000$ MeV and since inside the nucleus we have $\langle\tfrac{1}{2}[\hat{V}(\vec{r}) - \hat{S}(\vec{r})]\}\rangle \approx 375$ MeV we find that $\langle 2m^*(\vec{r})\rangle \approx 750$ MeV

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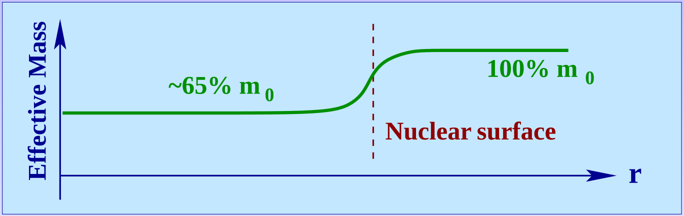
Position-Dependent Effective Mass: Estimates

- **Using the estimates $\langle\hat{S}\rangle \approx -400$ MeV and $\langle\hat{V}\rangle \approx +350$ we find**

$$\frac{1}{2m_0c^2 + \epsilon - (\hat{V} - \hat{S})} = \frac{1}{\epsilon + 2m^*} \simeq \frac{1}{2m^*}\left(1 - \frac{\epsilon}{2m^*}\right) \simeq \frac{1}{2m^*}$$

- **In the above relations $2m^* \approx 1300$ MeV. For the levels close to the Fermi energy we have $|\epsilon| \sim$ (0 to 10) MeV $\longrightarrow \epsilon/2m^* \sim 0.01$ Thus Hamiltonians discussed are energy independent to 1% error**



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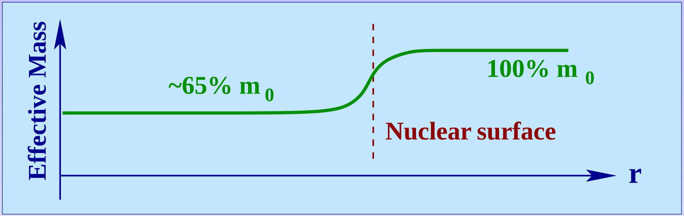

Linearized Pauli-Schrödinger Equation

- **The approximately linearised Pauli-Schrödinger equation then is:**

$$\Big\{(\vec{\sigma}\cdot\hat{\mathsf{p}})\frac{1}{2\mathsf{m}^*(\vec{\mathsf{r}})}(\vec{\sigma}\cdot\hat{\mathsf{p}}) + \underbrace{\big[\,\hat{\mathsf{S}}(\vec{\mathsf{r}}) + \hat{\mathsf{V}}(\vec{\mathsf{r}})\,\big]}_{\sim\,-60\ \mathrm{MeV}}\Big\}\,\xi_{\mathsf{n}} = \epsilon_{\mathsf{n}}\,\xi_{\mathsf{n}}$$

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$$m^*(\vec{r}) = \Big\{m_0c^2 - \tfrac{1}{2}\,\underbrace{\big[\,\hat{V}(\vec{r}) - \hat{S}(\vec{r})\,\big]}_{\sim\,+750\ \mathrm{MeV}}\Big\}$$

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- **The potential that binds the nucleons in the nucleus is the sum of the scalar- and vector-meson exchange contributions:**

$$\boxed{W(\vec{r}) \stackrel{\text{df}}{=} \hat{S}(\vec{r}) + \hat{V}(\vec{r}) \approx -60\ \mathrm{MeV}}$$

Form of the Generalized Kinetic Energy Operator

- **The operator quadratic in linear momenta can be transformed:**

$$(\vec{\sigma}\cdot\hat{p})\frac{1}{2m^*(\vec{r})}(\vec{\sigma}\cdot\hat{p}) = \frac{1}{2m^*(\vec{r})}\,\hat{p}^{\,2} + \hat{V}_{\vec{p}}(\vec{r},\hat{p}) + \hat{V}_{so}(\vec{r},\hat{p},\hat{s})$$

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- **We recognise two new operators called 'potentials' despite the fact that they originate from the kinetic energy operator:**

$$\hat{V}_{so}(\vec{r},\hat{p},\hat{s}) \equiv \frac{2}{[2m^*(\vec{r})]^2}\{[\vec{\nabla}\underbrace{(\hat{V}(\vec{r})-\hat{S}(\vec{r}))}_{\sim 750\ \mathrm{MeV}}]\wedge\hat{p}\}\cdot\hat{s}$$

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- **In the following we find the interpretation of the above operators**

Part II

Physical Interpretation

Prediction of the Spin-Orbit Splitting Mechanism

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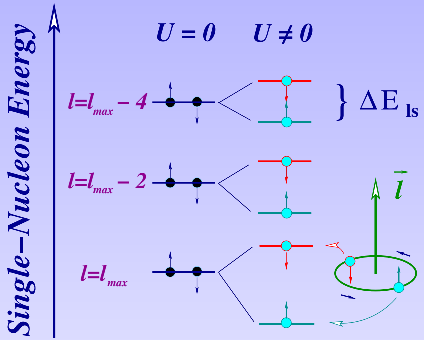
- **The Simplest Case: Spherical Symmetry**

$$\mathbf{U}(\vec{r}) \equiv \mathbf{U}(r) \equiv \hat{V} - \hat{S} \;\rightarrow\; [\nabla U \wedge \hat{p}]\cdot\hat{s} = \frac{1}{r}\frac{dU}{dr}\overbrace{(\vec{r}\wedge\hat{p})}^{\hat{\ell}}\cdot\hat{s} = \frac{1}{r}\frac{dU}{dr}\hat{\ell}\cdot\hat{s}$$



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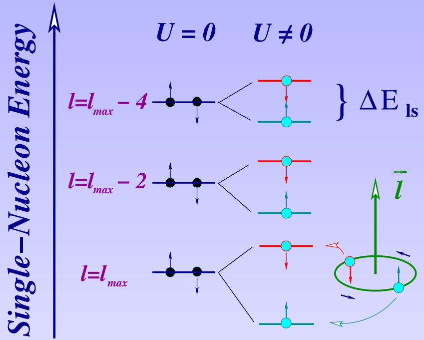

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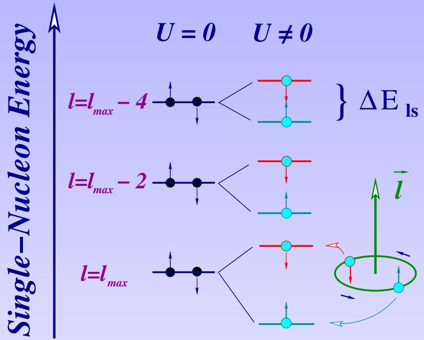
Notice the correct sign of $\Delta E_{\ell s}$

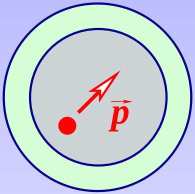
$$U = V - S > 0 \;\rightarrow\; \frac{dU}{dr} < 0$$

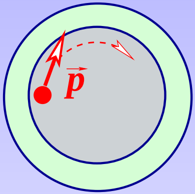


Potentials V_{so} and V_p - An Illustration

- **Potential $\hat{V}_p$ is responsible for 'de-acceleration' proportional to $\hat{p}$**
- **Both potentials stop acting at the limit $\vec{v} \sim \vec{p}/m_0 \to 0$ ('kinetic')**

$$V_p \sim \frac{dU}{dr}\left(\frac{\vec{r}}{r}\right)\cdot\hat{p} \qquad\qquad V_{so} \sim \frac{1}{r}\frac{dU}{dr}\,\hat{\ell}\cdot\hat{s}$$





Potential V_p: It is transparent to the circular motion, and it is independent of spin

Potential V_{so}: It is indifferent to the radial motion while its action depends on spin

Orders of Magnitude: Realistic $\hat{V}$ and $\hat{S}$ Potentials

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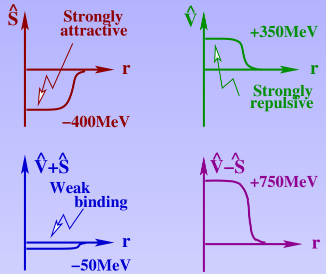

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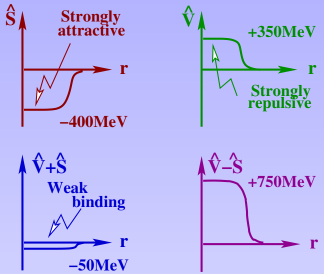

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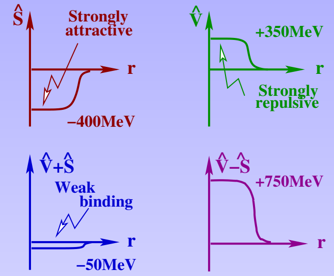

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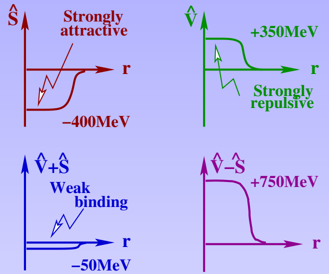

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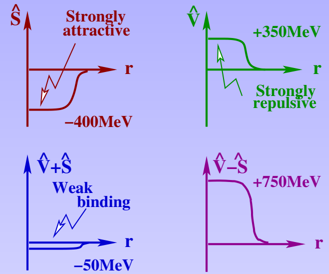

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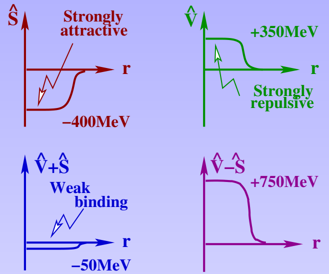

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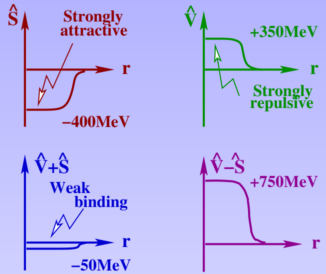

⋄ **Do you know WHY is the V+S potential so shallow? No? - Then please listen ...**

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- **The nuclear interactions originate from the exchange of mesons**
- **The scalar mesons contribute to a strong attraction (∼400 MeV)**
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- **Tangential orbits couple with the spin: bouncing-ball ones do not!**
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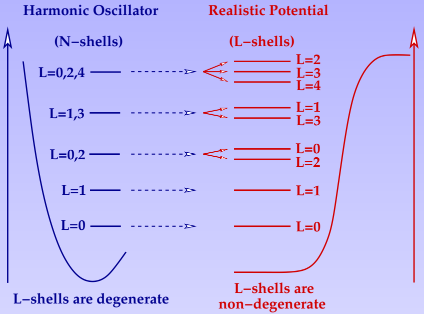
Part III

Mean-Field Theory: Link with Experiment

Quantum Mechanics: Memory Refreshing Facts

• **In the harmonic-oscillator case there exists a special symmetry that makes L-shells degenerate; for realistic nuclear potentials this symmetry does not hold anymore. Observe: N-shells and L-shells:**



• **Levels E_{LM} are M-degenerate, $E_{LM} = E_{LM'}$ $(-L \leq M, M' \leq +L)$. This 'magnetic' degeneracy results from the spherical symmetry**

Quantum Mechanics: Memory Refreshing Facts (II)

- **It is well known from elementary quantum mechanics that for hamiltonians with spherical symmetry:**

$$[\hat{H},\hat{j}^2]=0,\quad [\hat{H},\hat{j}_z]=0,\quad [\hat{H},\hat{\ell}^2]=0,\quad [\hat{H},\hat{s}]=0,\quad \hat{j}\equiv\hat{l}+\hat{s}$$

- **The solutions are simultaneous eigenstates of $\hat{H}$, $\hat{j}^2$, $\hat{j}_z$ and $\hat{\ell}^2$**

$$\hat{H}\psi_{n;j\ell m}=E_{n;j\ell m}\psi_{n;j\ell m}$$

- **This allows to introduce the spectroscopic notation based on:**

$$\left.\begin{array}{cccccccccc} \ell & = & 0 & 1 & 2 & 3 & 4 & 5 & 6 & \ldots \\ & & s & p & d & f & g & h & i & \ldots \end{array} :\right\} \quad n_r\ell_j$$

for instance $1s_{1/2}$, $2d_{5/2}$, $3p_{1/2}$, $1i_{13/2}$ etc.

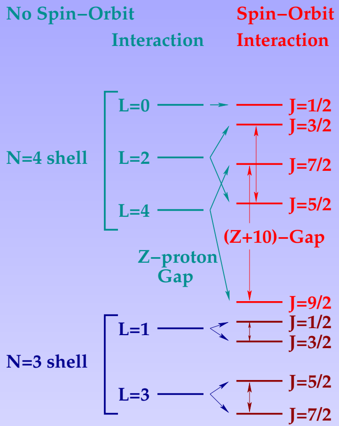
Spin-Orbit Splitting and Nobel Prize

- **Left: results with no-spin-orbit potential; Right: with the spin-orbit potential**

- **Vertical arrows denote the so-called spin-orbit splitting**

- **In atomic nuclei this splitting is very large, ejecting the lowest energy, the highest-J orbital, to the (N-1st)-shell below**

- **The ejected orbitals are called 'intruders'; for their discovery M. Göppert-Mayer and J. Jensen received the Nobel Prize in 1963**



Spin–Orbit Splitting Mechanism

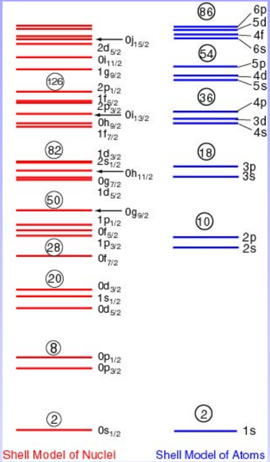
Spin-Orbit Splitting and Nobel Prize

- **At the discovery time, the mechanism of spin-orbit splitting was not trivial at all: observe the differences between nuclear and atomic cases**

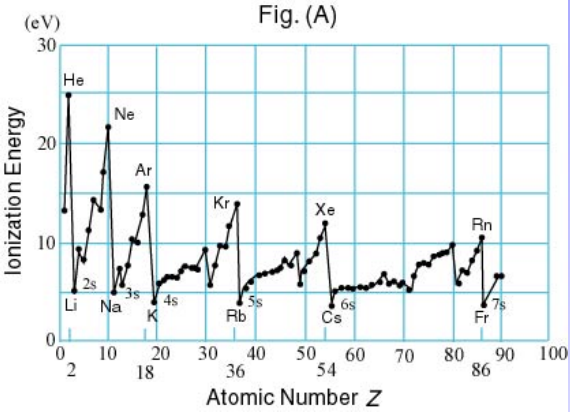
- **The 1963 Nobel Prize for explanation of the nuclear Göppert-Mayer and Johannes Jensen [together with Eugene Wigner]**

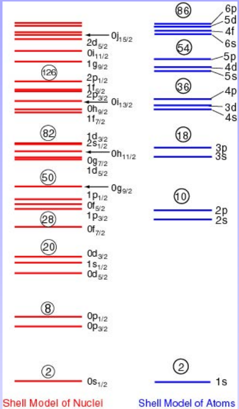
- **Today we know that the spin-orbit potential describing the magic numbers is in fact spin-orbit kinetic energy**

- **Gaps in the spectra are measurable quantities; measurements fully confirm the discussed mechanism**



Energy Gaps and Experimental Confirmation





- **Correlation: Maxima in ionization energy and the big gaps**

Part IV

Characteristic Functional Dependencies - or: Who Is Who?

- **We will follow literally the least action procedure now**

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- **Instead of solving the RMF equations self-consistently, we will parametrize them using realistic Woods-Saxon form-factors for $S(\vec{r})$ and $V(\vec{r})$ - with great advantages:**

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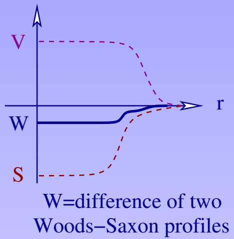
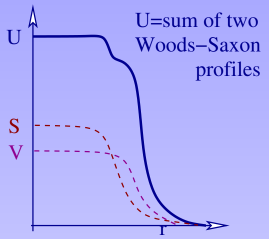
- **Instead of solving the RMF equations self-consistently, we will parametrize them using realistic Woods-Saxon form-factors for $S(\vec{r})$ and $V(\vec{r})$ - with great advantages:**

- **Mathematical simplicity when examining qualitatively the parametric dependencies - and the symmetry issues**

Remarks about Some Functional Dependencies





$$\hat{\mathbf{H}}_{\mathbf{int}} = \underbrace{\hat{\mathbf{S}} + \hat{\mathbf{V}}}_{\hat{\mathbf{W}} \sim -50\ \mathbf{MeV}} + \underbrace{\hat{\mathbf{V}}_{\mathbf{so}} + \hat{\mathbf{V}}_{\mathbf{p}}}_{\mathbf{surface\ peaked}}$$

Conclusion: In the simplest picture the gradient contributions from the V_{so} and V_p potentials have not a single Woods-Saxon but a double Woods-Saxon profile → Importance of knowing who-is-who.

Geometry of the Deformed Woods-Saxon Potential

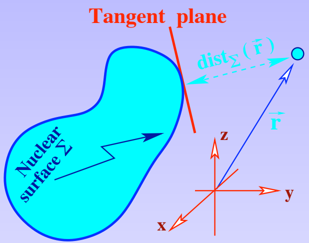
- **Nuclear surface Σ is parametrized in terms of spherical harmonics:**

$$R(\vartheta,\varphi) = c(\{\alpha_{\lambda\mu}\})\,[r_o * A^{1/3}]\,\{1 + \sum\sum \alpha_{\lambda\mu}\, Y_{\lambda,\mu}(\vartheta,\varphi)\}$$

- **Geometrical interpretation of the distance function and related deformed Woods-Saxon potential:**

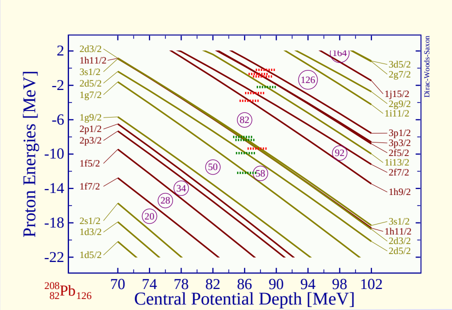
$$V_{WS}(\vec{r};\, r_o, a, V_o) =$$

$$= \frac{V_o}{1 + \exp[\,\mathrm{dist}_{\Sigma(r_o)}(\vec{r})/a\,]}$$



Central-Potential Depth-Parameter

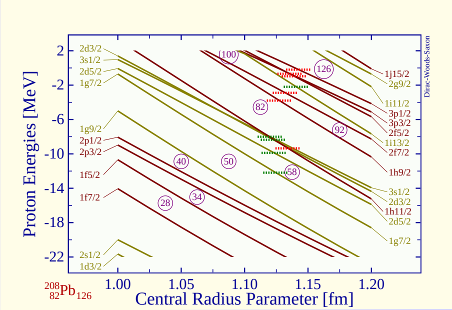
- Nuclear Dirac Woods-Saxon potentials have a very important geometrical feature - *each parameter dominates a certain mechanism*



Mechanism No. 1: The potential depth parameter is primarily responsible for the nucleonic binding energies. Observe nearly ideal description of the experimental levels: here in ^{208}Pb - as well as nearly linear dependence of the energies on V_o.

Central-Potential Radius-Parameter

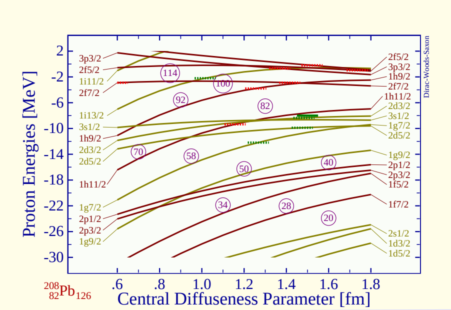
- The nucleonic binding energies vary nearly linearly in function of the central radius (although some levels may cross



Mechanism No. 2: The central-radius parameter is primarily responsible for the nucleonic binding energies but also for the calculated values of the r.m.s. radii. Here: ^{208}Pb.

Central-Potential Diffuseness-Parameter

- The central diffuseness parameter is the only one that can clearly distinguish among the eigen-energies of various quantum numbers



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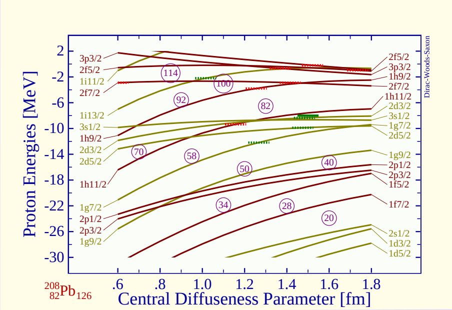


Mechanism No. 3: Observe the existence of families of nearly parallel lines which are characterized by common ℓ quantum number: These are spin-orbit partners.

Mean-Field Geometry: Spin-Orbit Potential

The spherically-symmetric W-S spin-orbit form-factor has the form:

$$\mathbf{V}^{so}_{ws}(r;\lambda,r_{so},a_{so}) \overset{df}{=} \frac{\lambda}{r}\frac{d}{dr}\cdot\left\{\frac{1}{1+\exp\left[(r-R_{so})/a_{so}\right]}\right\}$$

$$= \frac{\lambda}{2a_{so}}\frac{1}{r}\left\{\frac{1}{1+\cosh[(r-R_{so})/a_{so}]}\right\}$$

- λ - spin-orbit strength parameter
- $\mathbf{r}_{so}$ - spin-orbit radius parameter
- $\mathbf{a}_{so}$ - spin-orbit diffuseness parameter

Geometry: Radial Structure - Spin Orbit Potential

• The central diffuseness parameter is the only one that can clearly distinguish among the eigen-energies of various quantum numbers

• The matrix elements of the spin-orbit potential are calculated through the integration of the functions of general structure

$$\left(\frac{\vec{\mathbf{r}}}{\mathbf{r}}\right)\frac{\mathbf{df(r)}}{\mathbf{dr}}\,\vec{\ell}\cdot\vec{\mathbf{s}}\;\times\;\mathbf{r}^2\quad\text{where}\quad\mathbf{f(r)}=\frac{\mathbf{1}}{\mathbf{1}+\mathbf{exp}[\underbrace{(\mathbf{r}-\mathbf{r}_{\ell s})/\mathbf{a}_{\ell s}}_{\mathbf{x}}]}$$

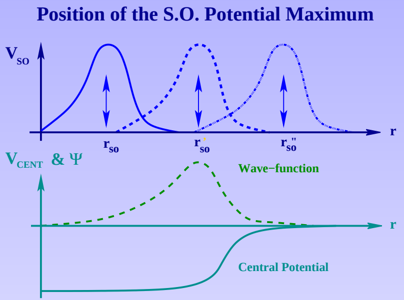
and the derivatives

$$\boxed{\left|\frac{\mathbf{df(x)}}{\mathbf{dx}}\right|=\frac{\mathbf{1}}{\mathbf{4}}\;\left[\frac{\mathbf{e}^{-\mathbf{x}/2}}{\mathbf{cosh}(\frac{\mathbf{x}}{\mathbf{2}})}\right]\times\left[\mathbf{1}+\mathbf{tanh}\left(\frac{\mathbf{x}}{\mathbf{2}}\right)\right]}$$

• **Conclusion:** The function of interest (spin-orbit potential) has always one extremum close to $x \sim 0$ or, in other words, when $r \sim r_{\ell s}$

Consequences of Single Maximum Mechanism

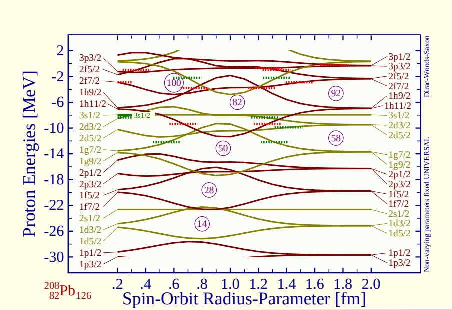
- Consider fixed central potential and let vary only one spin-orbit parameter *viz.* $r_{\ell s}$ so that $r_{\ell s} < r'_{\ell s}$, next $r'_{\ell s} < r''_{\ell s}$, etc. We have:



The radial wave function of a state is bound by the central potential whose geometry is considered fixed. Shifting the position of the maximum of the spin-orbit potential will first cause increasing of the integral (and thus the matrix elements), then a decrease.

Geometrical Consequences: Two Physical Solutions

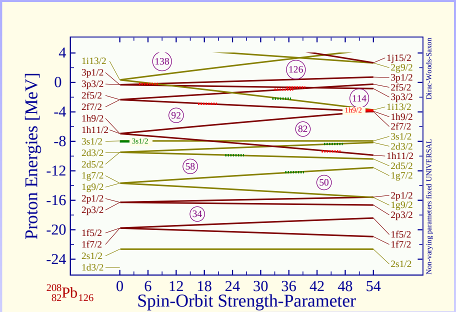
- Observe an increase of the spin-orbit splitting first, then a decrease and a characteristic 'bubble' structures in all the $\ell \neq 0$ solutions



Mechanism No. 4: A structure with two solutions: The 'standard' one (with the s.o. radius parameter $r^o_{s.o.} \sim 1.25$ *Fm) - and the 'compact' one* $r^o_{s.o.} \sim 0.75$ *Fm*

Nuclear Mean-Field Geometry: Spin-Orbit Strength

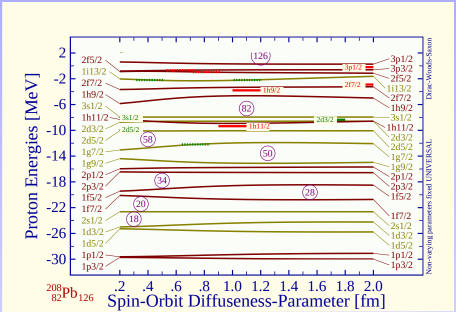
Single nucleon levels in function of *s-o strength parameter*



Mechanism No. 5: Observe a clear straight-line pattern (linear λ-dependence) of energies and the opening-angles increasing with ℓ of the corresponding orbitals

Nuclear Mean-Field Geometry: Spin-Orbit Diffuseness

Single nucleon levels in function of *s-o diffuseness parameter*



Mechanism No. 6: Observe a regular increase of the spin-orbit splitting with a_{so}

Part V

Nuclear Relativistic Mean Field Theory: Role of the SU(2) Symmetries

Spin, Pseudo-Spin and Dirac Hamiltonian

- Let us introduce the helicity $\hat{h}$ as the spin-projection on the $\vec{p}$-axis

$$\hat{h} \stackrel{df}{=} \vec{\sigma} \cdot \hat{p}; \quad \hat{p} \equiv \vec{p}/||\vec{p}||$$

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$$\boxed{\tilde{s}_i \equiv (\vec{\sigma} \cdot \hat{p}) \, s_i \, (\vec{\sigma} \cdot \hat{p})} \quad \text{and} \quad \hat{\mathcal{S}}_i \equiv \begin{pmatrix} \tilde{s}_i & 0 \\ 0 & s_i \end{pmatrix}$$

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- Recall Dirac equation for nucleons with meson-transmitted $\hat{V}$ and $\hat{S}$

$$\hat{\mathcal{H}}_{\mathbf{D}} = \mathbf{c}\vec{\alpha} \cdot \vec{\mathbf{p}} + \hat{\mathbf{V}}(\vec{\mathbf{r}}\,)\, \mathbb{I}_4 + \beta\, [\mathbf{m_0 c^2} + \hat{\mathbf{S}}(\vec{\mathbf{r}}\,)]$$

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$$\hat{\mathcal{H}}_D = c\vec{\alpha} \cdot \vec{p} + \hat{V}(\vec{r})\, \mathbb{I}_4 + \beta\,[m_0c^2 + \hat{S}(\vec{r})]$$

Explicitly:

$$\hat{\mathcal{H}}_D = \begin{pmatrix} +[m_0c^2 + (\hat{S} + \hat{V})]\mathbb{I}_2 & , & c\,(\vec{\sigma} \cdot \vec{p}) \\ c\,(\vec{\sigma} \cdot \vec{p}) & , & -[m_0c^2 + (\hat{S} - \hat{V})]\mathbb{I}_2 \end{pmatrix}$$

A New Nuclear Symmetry: Pseudo-Spin Symmetry

- Calculating the commutator shows that it 'almost' vanishes:

$$[\hat{\mathcal{H}}_D, \hat{\mathcal{S}}_i] = \left[\begin{pmatrix} \hat{\mathcal{H}}_D^{11} & , \hat{\mathcal{H}}_D^{12} \\ \hat{\mathcal{H}}_D^{21} & , \hat{\mathcal{H}}_D^{22} \end{pmatrix}, \begin{pmatrix} \hat{\mathcal{S}}_i^{11} & , \hat{\mathcal{S}}_i^{12} \\ \hat{\mathcal{S}}_i^{21} & , \hat{\mathcal{S}}_i^{22} \end{pmatrix} \right] = \begin{pmatrix} \hat{X} \neq 0 & , 0 \\ 0 & , 0 \end{pmatrix}$$

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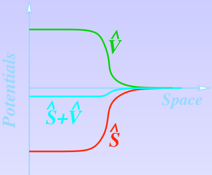
- Exact symmetry limit requires that

$$\hat{S} + \hat{V} = 0$$

but then our Universe disappears!



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• For unfortunate historical reasons we call it *Pseudo-Spin Symmetry* - there is nothing 'less valuable' in the 'p s e u d o - SU_2' symmetry

Nuclear Mean Field and $SU_2 \times SU_2$ Symmetry

- We defined pseudospin using spin projection $\tilde{s}_i \equiv (\vec{\sigma} \cdot \hat{p}\,)\, s_i\, (\vec{\sigma} \cdot \hat{p}\,)$

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- We have formally introduced helicity with its algebraic properties

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- From definition $\tilde{s}_j = \hat{h} \, s_j \, \hat{h}^{-1}$ and from unitarity of $\hat{h}$ it follows that

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[This happens if and only if there is no nuclear binding: $(S+V)\rightarrow 0$]

Dirac Equation - Exact Symmetry Limit: $\hat{S}+\hat{V}\rightarrow 0$

- The original Dirac equation is equivalent to two following ones:

$$\begin{cases} \hat{\mathcal{H}}^{\xi}\,\xi = \mathcal{E}\,\xi; & \textbf{Here}: \hat{\mathbf{V}} + \hat{\mathbf{S}} \rightarrow \mathbf{0} \\ \hat{\mathcal{H}}^{\xi} \equiv (\mathbf{c}\,\vec{\sigma}\cdot\vec{\mathbf{p}}\,)\,\dfrac{1}{[\mathcal{E} + \mathbf{m_0 c^2} + (\hat{\mathbf{S}} - \hat{\mathbf{V}})]}\,(\mathbf{c}\,\vec{\sigma}\cdot\vec{\mathbf{p}}\,) + [\mathbf{m_0 c^2} + (\hat{\mathbf{S}} + \hat{\mathbf{V}})] \end{cases}$$

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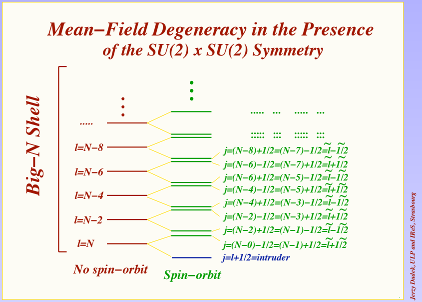
- Case ξ: One shows exactly that: $[\hat{\mathcal{H}}^{\xi},\,\tilde{s}_j]=0\rightarrow$ $\boxed{\xi=\xi_{n,\tilde{s},\tilde{s}_z}}$

Spin and Pseudospin - In Coexistence ???

• Nuclear Spin-Orbit splitting is huge; How is it possible that pseudo-spin and pseudo-orbit splitting can be negligible at the same time?



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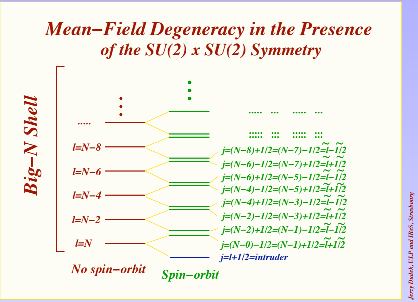
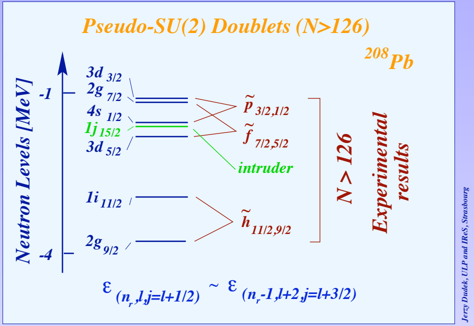


◇ We begin with the numerical exercise: we set spin orbit to zero (left). Then we increase the coupling constant until the experimental conditions are met (right). *The spin-orbit splitting increases dramatically - while the pseudo-spin pseudo-orbit splitting goes to zero! And YES: all that functions indeed in coexistence!!!*

Spin and Pseudospin *vs.* Experiment [1]

• The splitting of the orbitals (or: symmetry breaking) should be compared to numbers of the order of $\langle \hat{S} + \hat{V} \rangle \sim$ (-60 to -50) MeV



◇ In the exact $SU_2 \otimes SU_2$ symmetry limit the orbitals marked with symbol 'tilde' should be exactly degenerate. [Here: The lighter bound (N>126 particle) states]

Spin and Pseudospin *vs.* Experiment [2]

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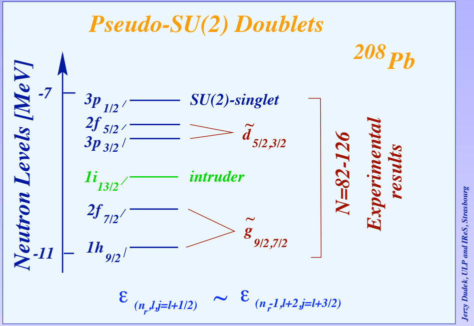

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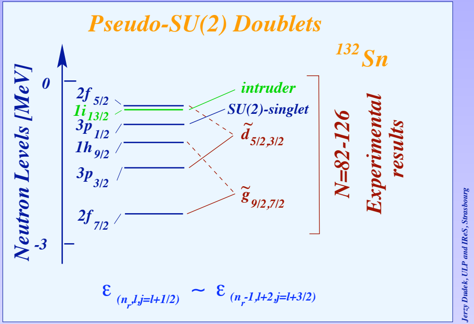
Spin and Pseudospin *vs.* Experiment [3]: ^{132}Sn Case

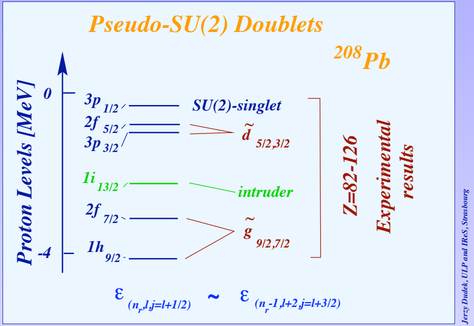
• The splitting of the orbitals (symmetry breaking) is stronger here compared to ^{208}Pb. Recall: symmetry gets exact if Surf./Vol. $\rightarrow 0$



◇ In the exact $SU_2 \otimes SU_2$ symmetry limit the orbitals marked with symbol 'tilde' should be exactly degenerate. [Here: 'ν-particles' in N=(82-125)-shell in ^{132}Sn]

Spin and Pseudospin *vs.* Experiment [4]

• The splitting of the orbitals (symmetry breaking) for protons in ^{208}Pb is comparable to that of the neutrons ('isospin-independence')



◇ In the exact $SU_2 \otimes SU_2$ symmetry limit the orbitals marked with symbol 'tilde' should be exactly degenerate. [Here: proton 'particle' states, Z=(82-126) shell]

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- Strong spin-orbit splitting (Goeppert-Mayer, Jenssen) receives a new partner: A Parallel - weak - pseudo-spin pseudo-orbit coupling!