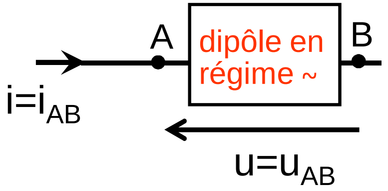
2. Dipôles en regime permanent sinusoïdal

2.1 Loi d'Ohm – impédance - admittance



Le dipôle est soumis à la ddp : $u(t) = U\sqrt{2}\cos\omega t \quad \Rightarrow \quad \underline{U} = Ue^{j0} = U$

il est alors parcouru par un courant i en retard de ϕ (algébrique) sur u :

$$i(t) = I\sqrt{2}\cos(\omega t - \phi) \quad \Rightarrow \quad \underline{I} = I\,e^{-j\phi}$$

Par analogie avec la loi d'Ohm, on définit l'impédance complexe $\underline{Z}$, d'un dipôle comme étant le quotient de $\underline{U}$ par $\underline{I}$.

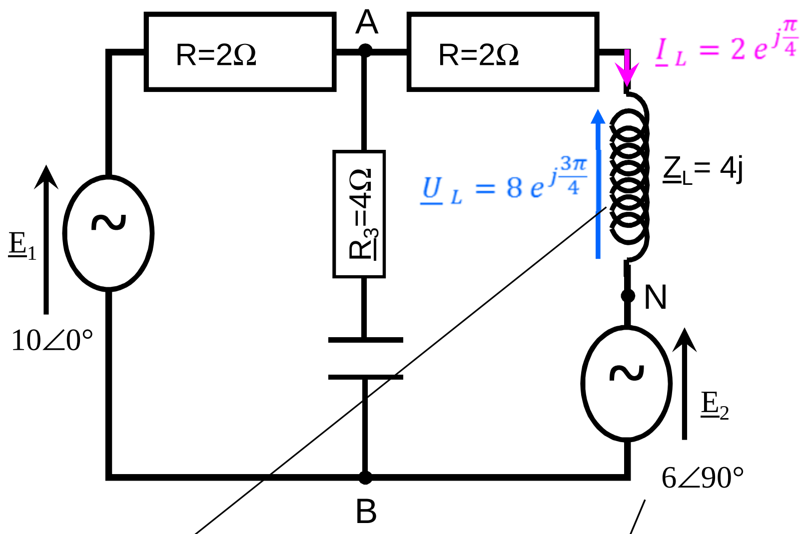
L' impédance complexe $\underline{Z}$ du dipôle : $\underline{Z} = \dfrac{\underline{U}}{\underline{I}} = \dfrac{U}{Ie^{-j\phi}} = \dfrac{U}{I}e^{j\phi} = Z\ e^{j\phi}$

$$\text{module: } \left|\underline{Z}\right| = \left|\frac{\underline{U}}{\underline{I}}\right| = \frac{\left|\underline{U}\right|}{\left|\underline{I}\right|} = \frac{U}{I} = Z \quad (\Omega) \qquad \text{et} \qquad \arg\underline{Z} = \arg\underline{U} - \arg\underline{I} = +\phi$$

L' admittance complexe $\underline{Y}$ du dipôle : $\underline{Y} = \dfrac{1}{\underline{Z}} = \dfrac{\underline{I}}{\underline{U}} = \dfrac{Ie^{-j\phi}}{U} = \dfrac{I}{U}e^{-j\phi} = Y\ e^{-j\phi}$

$$\text{module: } \left|\underline{Y}\right| = \left|\frac{\underline{I}}{\underline{U}}\right| = \frac{\left|\underline{I}\right|}{\left|\underline{U}\right|} = \frac{I}{U} = Y \quad (S) \qquad \text{et} \qquad \arg\underline{Y} = \arg\underline{I} - \arg\underline{U} = -\phi$$

Exemple de circuit



$$e_1(t) = 10\sqrt{2}\sin\left(5t + \frac{\pi}{2}\right)\ (V)$$

valeur efficace = *10 V*

phase = *0°*

D'abord… $e_1(t) = 10\sqrt{2}\cos(5t)$

$$\underline{e}_1(t) = 10\sqrt{2}\ e^{j5t}$$

Amplitude efficace complexe

$$\underline{E}_1 = 10\,e^{j0}$$

Amplitude efficace complexe

$$\underline{\boldsymbol{E}}_{\mathbf{2}} = 6\,e^{j\frac{\pi}{2}}$$

$$\boldsymbol{e_2(t)} = 6\sqrt{2}\cos\left(5t + \frac{\pi}{2}\right)\ (V)$$

$$\underline{Z}_L = \frac{\underline{u}_L(t)}{\underline{i}_L(t)} = \frac{\underline{U}_L}{\underline{I}_L} = \frac{8\,e^{j\frac{3\pi}{4}}}{2\,e^{j\frac{\pi}{4}}} = 4\,e^{j\frac{\pi}{2}} = 4j$$

$$\underline{Z} = Z\ e^{j\phi} = Z\cos\phi + j\ Z\sin\phi\ \ =\ \ R\ +\ j\ X$$

module: $\left|\underline{Z}\right| = Z = \sqrt{R^2 + X^2}$ (Ω) et $\arg\underline{Z} = arctg\left(\dfrac{X}{R}\right)$

Impédance complexe

résistance

réactance

$$\underline{Y} = Y\ e^{-j\phi} = Y\cos\phi - j\ Z\sin\phi\ \ =\ \ G\ +\ j\ B$$

module: $\left|\underline{Y}\right| = Y = \sqrt{G^2 + B^2}$ (S) et $\arg\underline{Y} = arctg\left(\dfrac{B}{G}\right)$

admittance complexe

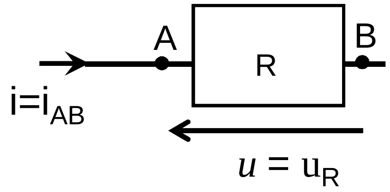
conductance

susceptance

2.2 Impédance élémentaire

2.2.1 Résistance linéaire R

La résistance R est parcourue par un courant alternatif d'expression :



$$i(t) = I\sqrt{2}\cos(\omega t) \quad \Rightarrow \quad u_R(t) = R\ i(t)$$

$$\Rightarrow u(t) = R\,I\sqrt{2}\cos(\omega t) = U_R\sqrt{2}\cos(\omega t) \quad \text{avec} \quad U_R = R\,I$$

$$\Rightarrow \quad \underline{u(t) \ \ et \ \ i(t) \ \ sont \ \ en \ \ phase}$$

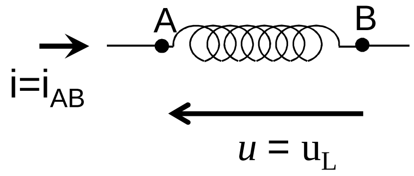
- Diagramme de Fresnel

- impédance complexe

$$\underline{Z}_R = R = R\angle 0° \quad \text{module} \ : \left|\underline{Z}_R\right| = R \quad \text{et} \quad \arg = \phi = 0$$

2.2.2 Bobine idéale (r=0) L

La bobine (self) L est parcourue par un courant alternatif d'expression :

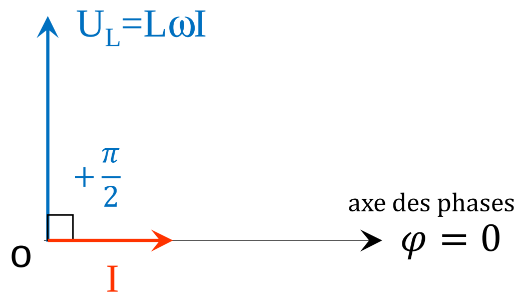


$$i(t) = I\sqrt{2}\cos(\omega t) \qquad \textit{par définition} \qquad u_L(t) = L\,\frac{di(t)}{dt}$$

$$\Rightarrow u(t) = -L\omega I\sqrt{2}\sin(\omega t) = U_L\sqrt{2}\cos(\omega t + \frac{\pi}{2}) \qquad \text{avec} \qquad U_L = L\omega I$$

$$\Rightarrow \quad u(t) \ \underline{\textit{est}\ \ \textbf{en quadrature}\ \ \textbf{avance}} \ \ \textbf{sur} \ \ i(t)$$

- Diagramme de Fresnel

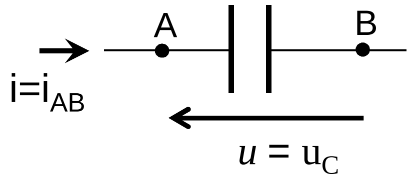


- impédance complexe

$$\underline{Z}_L = L\omega e^{j\frac{\pi}{2}} = jL\omega = L\omega\angle + \frac{\pi}{2} \quad \text{module} \quad : \left|\underline{Z}_L\right| = L\omega \quad \text{et} \quad \arg = \phi = +\frac{\pi}{2}$$

2.2.3 condensateur idéal C

Le condensateur C est parcourue par un courant alternatif d'expression :



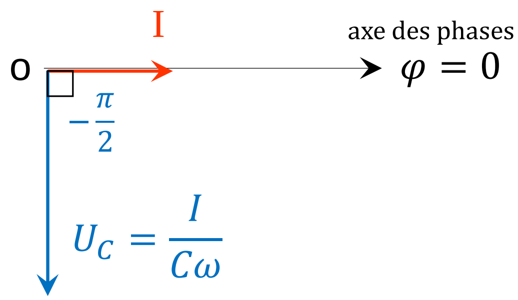
$$i(t) = I\sqrt{2}\cos(\omega t) \quad \textit{par definition} \quad i(t) = C\frac{du_c}{dt}$$

$$\Rightarrow u(t) = \frac{1}{C}\int i(t)dt = \frac{1}{C}\int I\sqrt{2}\cos(\omega t)dt = \frac{I\sqrt{2}}{C\omega}\sin(\omega t)$$

$$\Rightarrow u(t) = \frac{I\sqrt{2}}{C\omega}\cos(\omega t - \frac{\pi}{2}) \quad \text{avec} \quad U_C = \frac{I}{C\omega}$$

$$\Rightarrow \quad \underline{u(t)\ \textit{est}\ \text{en quadrature}\ \ \text{retard}}\ \ \text{sur}\ \ i(t)$$

- Diagramme de Fresnel



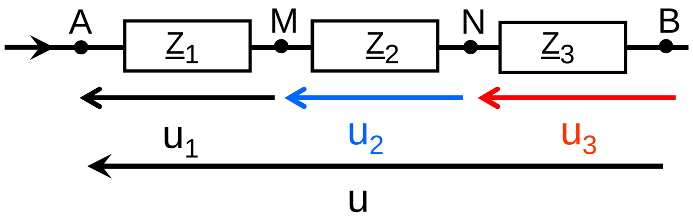
- impédance complexe

$$\underline{Z}_C = \frac{1}{C\omega}e^{-j\frac{\pi}{2}} = -\frac{j}{C\omega} = \frac{1}{C\omega}\angle -\frac{\pi}{2} \quad \text{module} \quad : \left|\underline{Z}_C\right| = \frac{1}{C\omega} \quad \text{et} \quad \arg = \phi = -\frac{\pi}{2}$$

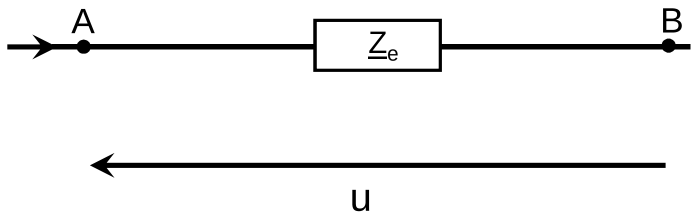
2.3 Association d'impédances complexes

2.3.1 Association d'impédances en série

2.3.1.1 impédance complexe équivalente

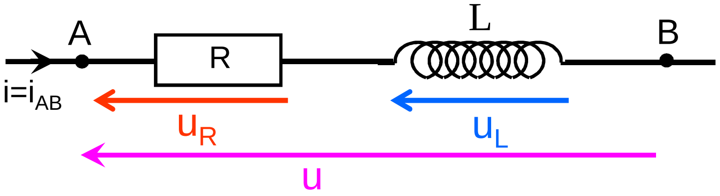


$\Leftrightarrow$



$$u(t) = u_1(t) + u_2(t) + u_3(t) \quad \Rightarrow \quad \boxed{\underline{Z}_e = \underline{Z}_1 + \underline{Z}_2 + \underline{Z}_3}$$

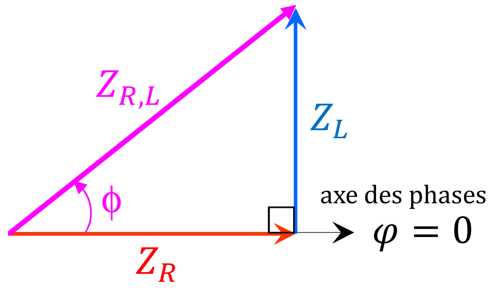
Exemple 1: circuit R,L (bobine réelle)

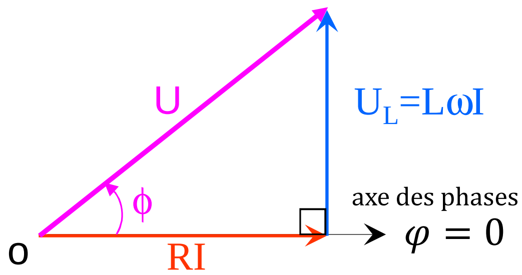


$$u(t) = u_R(t) + u_L(t)$$

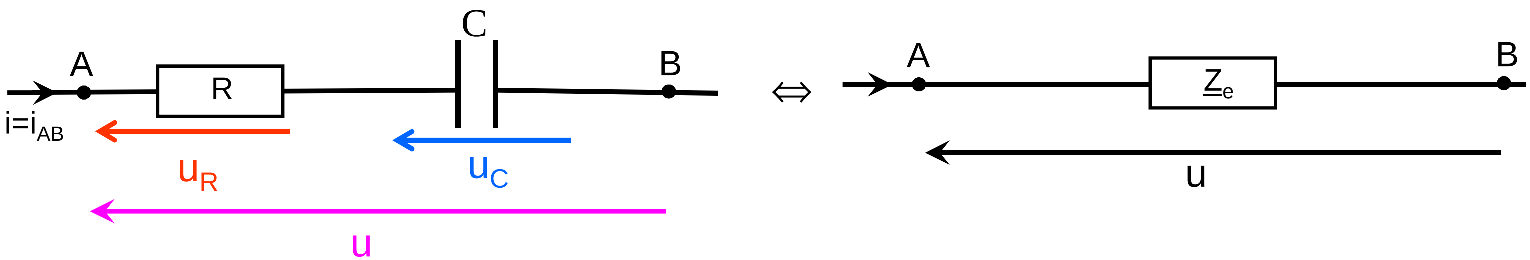
$$\Rightarrow \quad \underline{Z}_e = \underline{Z}_R + \underline{Z}_L = R + jL\omega$$

module $\left|\underline{Z}_e\right| = \sqrt{R^2 + (L\omega)^2}$ et $\arg \underline{Z}_e = \phi = arctg\left(\frac{L\omega}{R}\right)$





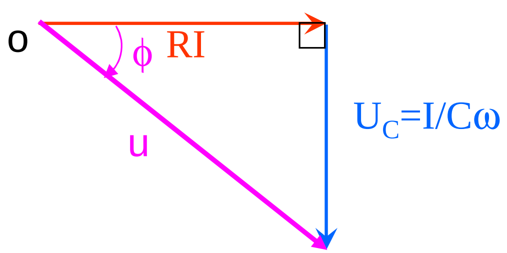
Exemple2: circuit R,C (bobine réelle)

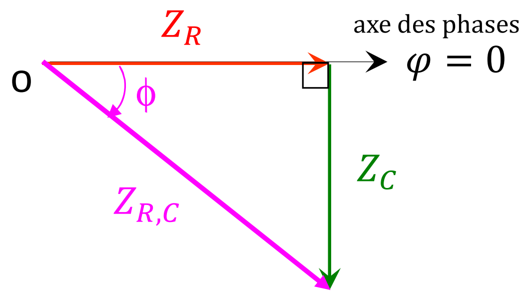


$$u(t) = u_R(t) + u_C(t)$$

$$\Rightarrow \quad \underline{Z}_e = \underline{Z}_R + \underline{Z}_C = R - j\frac{1}{C\omega}$$

$$\text{module } \left|\underline{Z}_e\right| = \sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2} \quad \text{et} \quad \arg \underline{Z}_e = \phi = arctg\left(-\frac{1}{RC\omega}\right)$$



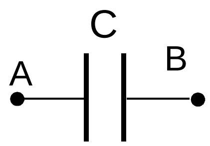


Exercices d'applications

$$f = 150\ Hz \qquad C = 530{,}5\ \mu F$$

$$\text{module} : |\underline{Z}_C| = Z_C = \frac{1}{C\omega} = \frac{1}{530{,}5 \cdot 10^{-6} \cdot 2 \cdot \pi \cdot 150} = 1{,}99999 \approx 2\ \Omega$$

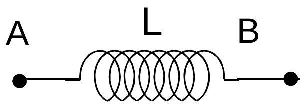
$$\underline{Z}_C = -\frac{j}{C\omega} = -2j$$



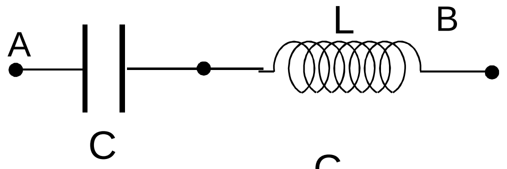
$$f = 150 Hz \qquad L = 23{,}34\, mH$$

$$\text{module} : |\underline{Z}_L| = Z_L = L\omega = 23{,}34 \cdot 10^{-3} \cdot 2 \cdot \pi \cdot 150 = 21{,}997 \approx 22\ \Omega$$

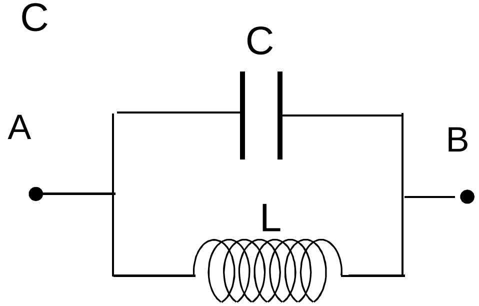
$$\underline{Z}_L = jL\omega = 22j$$



$$\underline{Z}_{AB} = \underline{Z}_C + \underline{Z}_L = -2j + 22j = 20j$$

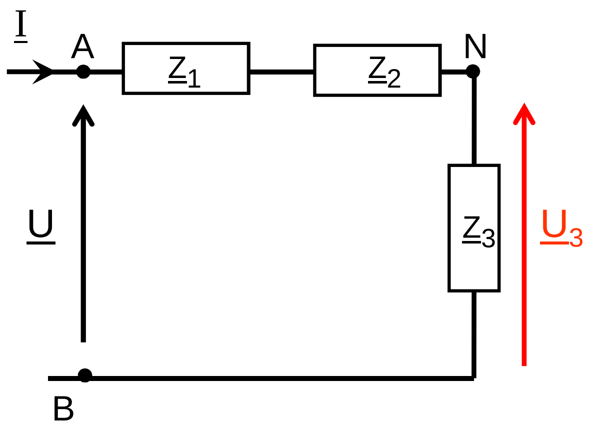


$$\underline{Z}_{AB} = \frac{\underline{Z}_C \cdot \underline{Z}_L}{\underline{Z}_C + \underline{Z}_L} = \frac{-2j \cdot 22j}{20j} = -2{,}2j$$



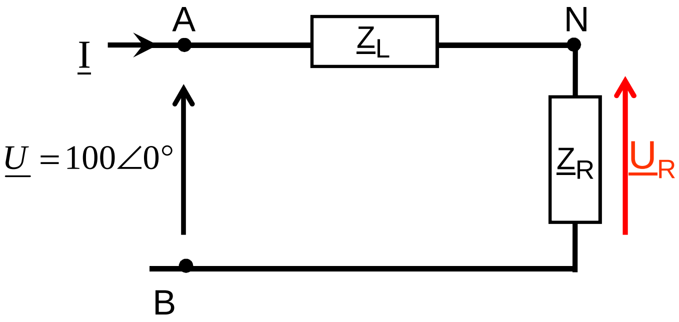
2.3.1.2 Diviseur de tension, utilisation des “phaseurs”

Cas de 3 impédances $\underline{Z}_1,\underline{Z}_2,\underline{Z}_3$ en série



$$\underline{U}_3 = \underline{Z}_3\,\underline{I} = \underline{U}\left(\frac{\underline{Z}_3}{\underline{Z}_1+\underline{Z}_2+\underline{Z}_3}\right)$$

Exemple Calculer $\underline{U}_R$ dans le cas du circuit (R,L)



$$\underline{U}_R = \underline{Z}_R\,\underline{I} = \underline{U}\left(\frac{\underline{Z}_R}{\underline{Z}_R+\underline{Z}_L}\right) = \underline{U}\left(\frac{R}{R+jL\omega}\right)$$

$$\underline{U}_R = \underline{U}\left(\frac{R(R-jL\omega)}{R^2+L^2\omega^2}\right) = \underline{U}\left(\frac{R^2-jL\omega R}{R^2+L^2\omega^2}\right)$$

$$\left|\underline{U}_R\right| = \left|\underline{U}\right|\frac{R\sqrt{R^2+L^2\omega^2}}{R^2+L^2\omega^2} = \left|\underline{U}\right|\frac{R}{\sqrt{R^2+L^2\omega^2}} \quad et \quad \arg = \arctan\left(-\frac{L\omega}{R}\right)$$

$$R = 30\Omega, \qquad L\omega = 40\Omega$$

$$\left|\underline{U}_R\right| = 100\frac{30}{\sqrt{2500}} = 60 \quad et \quad \arg = \arctan\left(-\frac{40}{30}\right) \Rightarrow \phi = -59°$$

$$\underline{U}_R = 60V\angle -59°$$