(ne pas écrire dans cette marge)

Fonction 1

La fonction logarithme népérien, notée ln a pour ensemble de déf l'intervalle $]0, +\infty[$

Signe

$\ln x \leq 0$ pour $x \in]0, 1]$

$\ln x > 0$ pour $x \in]1, +\infty[$

Limite

$\lim_{x \to 0} \ln x = -\infty$ $\lim_{x \to +\infty} \ln x = +\infty$ $\begin{cases} \ln e = 1 \\ e = 2{,}718 \end{cases}$

Dérivation

$\ln'(x) = \frac{1}{x}$

on en déduit que la fonction $\ln x$ est et strictement croissante sur l'intervalle $]0, +\infty[$

propriétés algébriques

Pour tout x et $y \in]0, +\infty[$

$\ln xy = \ln x + \ln y$

$\ln \frac{1}{x} = -\ln x$

$\ln \sqrt{x} = \ln x^{1/2} = \frac{1}{2} \ln x$

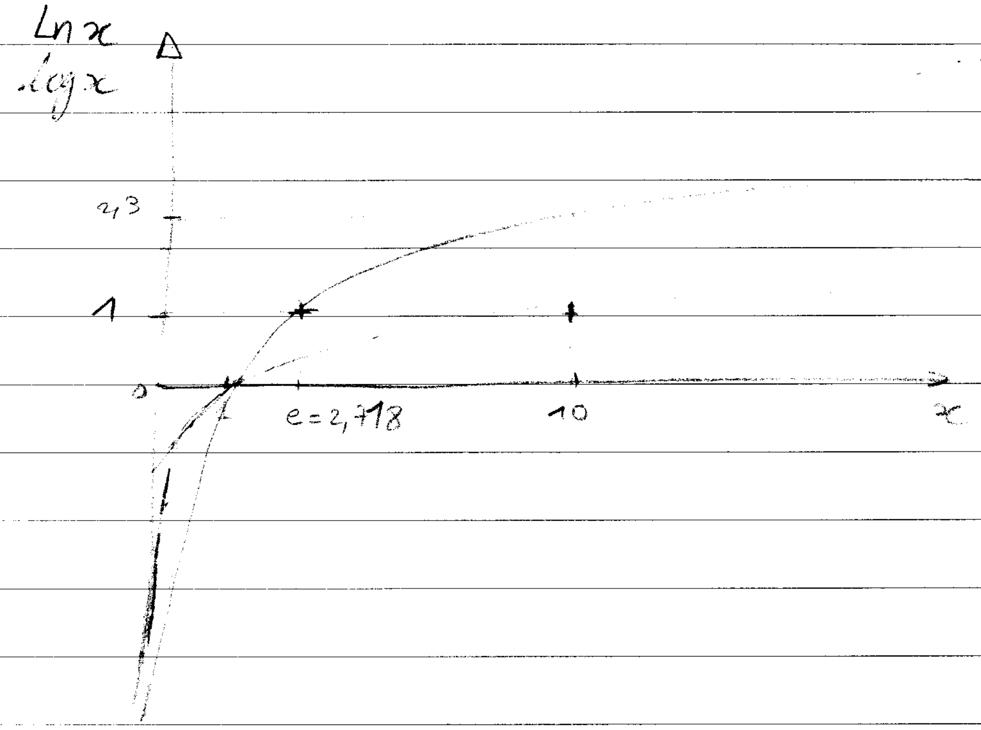
$\ln x^a = a \ln x$

On appelle fonction logarithme décimal, la fonction notée $\log_{10}$ ou $\log$ et définie sur l'intervalle $]0, +\infty[$ par :

$$\log_{10} x = \log_x = \frac{\ln x}{\ln 10}$$

$\ln 10 = 2{,}3$

(ne pas écrire dans cette marge)



2°/ Le logarithme népérien est un logarithme à base e (e = 2,718...)

$$Ln\,x = \frac{Ln\,x}{Ln\,e} = \frac{Ln\,x}{1}$$

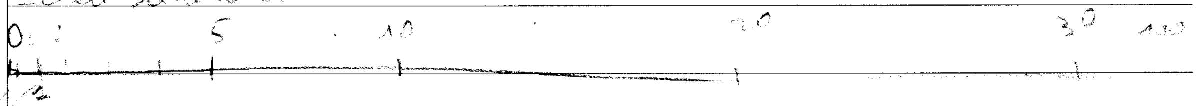
On peut définir un logarithme de base qlq.
Le plus répandu est celui de la base 10.

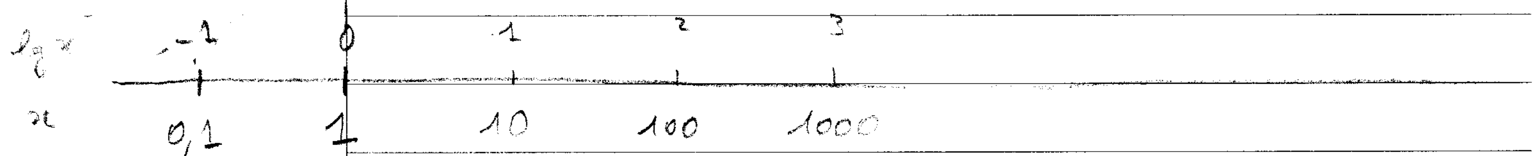
$$\log x = \frac{Ln\,x}{Ln\,10}$$

L'intérêt de l'échelle log x : contraction de l'échelle pour un grand domaine de variation de x.
La graduation de l'échelle est donnée par log 10 → 1 graduation (en cm)

Ex :

Echelle linéaire





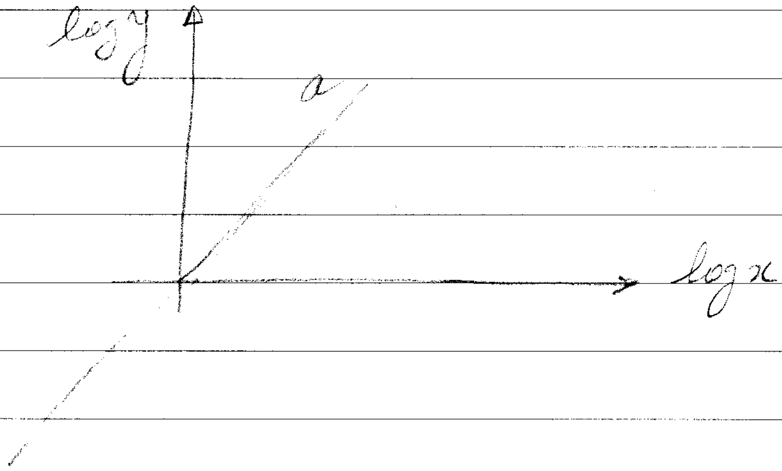
Cela a pour effet de dilater les faibles écarts et contracter les grands écarts.

(ne pas écrire dans cette marge)

(3) Coordonnées log-log.

ex : Soit la fonction $y = x^a$

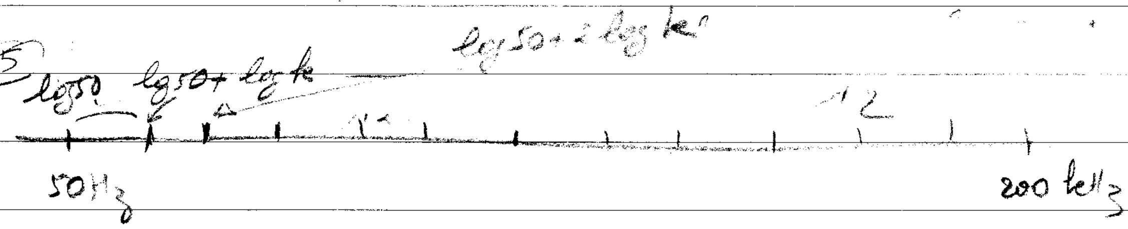
$\Rightarrow \log y = a \log x$



(4) Semi log

lorsque seul l'axe des abscisses est en coord. logarithmique.

(5)



1ᵉʳ gr. $\log 50 \longrightarrow 50\,Hz$.

2ᵉ $\log 50 + \log k \longrightarrow 50k\,(Hz)$

3ᵉ $\log 50 + 2\log k \longrightarrow 50k^2\,(Hz)$

4ᵉ $\log 50 + 3\log k \longrightarrow 50k^3\,(Hz)$

⋮

12 $\log 50 + 12\log k \rightarrow 50k^{12} = 200\,10^3\,Hz$.

$\Rightarrow k^{12} = \frac{200}{50}\,10^3 = 4000$

$\Rightarrow \log k^{12} = \log 4000 = 3 + \log 4$

$12 \log k = 3 + \log 4$

$\log k = \frac{3+\log 4}{12} = 3{,}602$

$\Rightarrow k = 1{,}996 \simeq 2$

les fréquence sont : $k=2$

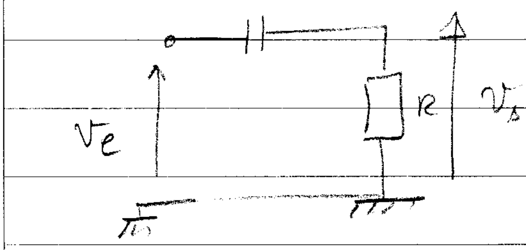
~~50 → 100 → 150 → 200 → 550~~

$\underset{0}{50} \longrightarrow \underset{1}{100} \longrightarrow \underset{2}{200} \longrightarrow 400 \longrightarrow 800 \longrightarrow 1600$

$\longrightarrow 3200 \longrightarrow 6400 \longrightarrow 12\,800 \longrightarrow 25\,600 \longrightarrow$

$51200 \longrightarrow 102400 \longrightarrow 204\,800$

Ex 2 :



$$\underline{V_s} = \frac{Z_R}{Z_R + Z_C} \underline{V_e}$$

$$\frac{V_s}{V_e} = \frac{R}{R + \frac{1}{jC\omega}} = \frac{RjC\omega}{1 + jRC\omega}$$

$$\underline{H}(j\omega) = \frac{jRC\omega}{1+jRC\omega} \qquad \text{on pose } \omega_0 = \frac{1}{RC}$$

$$\underline{H}(j\omega) = \frac{j\,\omega/\omega_0}{1 + j\frac{\omega}{\omega_0}} \qquad \text{on pose } x = \frac{\omega}{\omega_0}$$

$$\boxed{\underline{H}(jx) = \frac{jx}{1+jx}}$$

$$H(x) = \frac{x}{\sqrt{1+x^2}}$$

Diagramme de Bode.

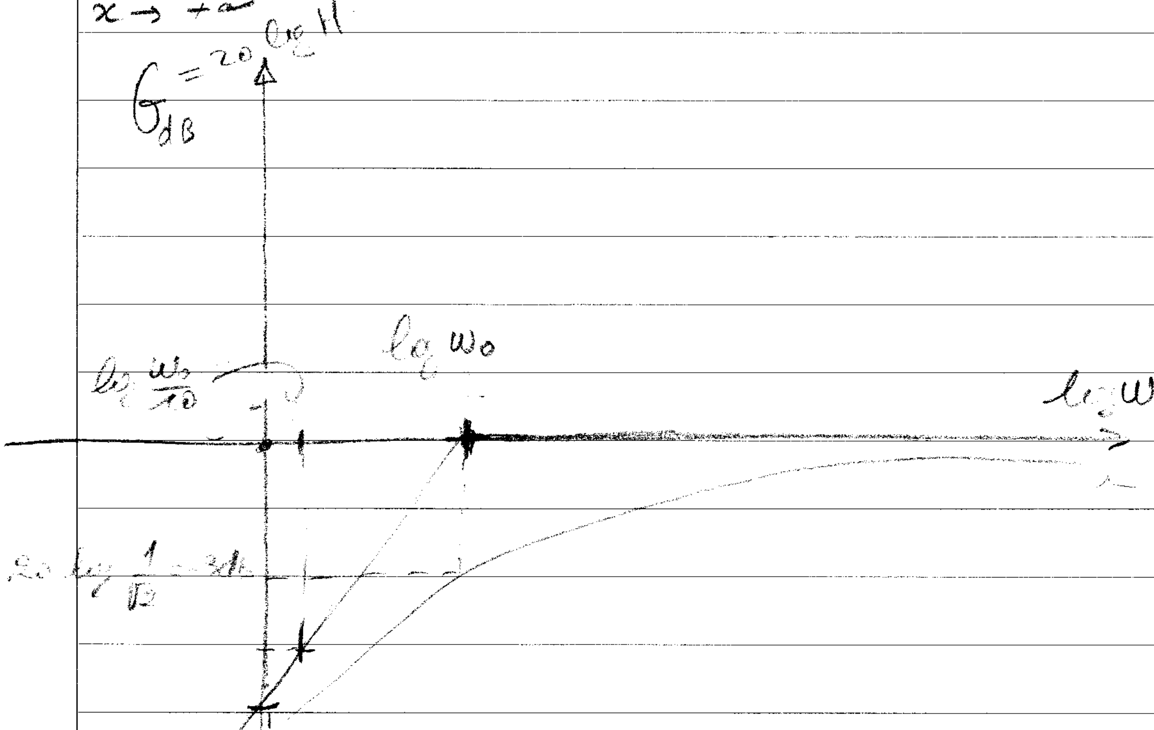
$$G_{dB}(x) = 20 \log H(x) = 20 \left[\log x - \frac{1}{2}\log(1+x^2)\right]$$

$$G_{dB} = 20 \log x - 10 \log(1+x^2)$$

$\lim\limits_{x \to 0} G_{dB} = \lim\limits_{x \to 0} 20 \lg x$ a pour limite asymptote la droite d'équ. $20 \lg x$

(ne pas écrire dans cette marge)

$$\lim_{x \to +\infty} G_{dB} = 20 \log x - 20 \log x = 0$$



valeur particulière pour $x = 1$ $(\omega = \omega_0)$

$H = \frac{1}{\sqrt{2}}$ ω_0 correspond donc à la fréquence de coupure

$$\frac{x}{\sqrt{1+x^2}} = \frac{1}{\sqrt{2}} \qquad \frac{x^2}{1+x^2} = \frac{1}{2} \Rightarrow 2x^2 = 1 + x^2$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = +1$$

asymptote $G_0 = 20 \log x = 20 \log \omega - 20 \log \omega_0$

pour $x = \frac{1}{10}$ $G_0 = 20 \log \frac{1}{10} = -20$

Déphasage :

$$\underline{H}(jx) = \frac{jx(1-jx)}{1+x^2} = \frac{x^2 + jx}{1+x^2}$$

$$\phi = \text{Arctan} \frac{1}{x} = \text{Arctan} \frac{\omega_0}{\omega}$$

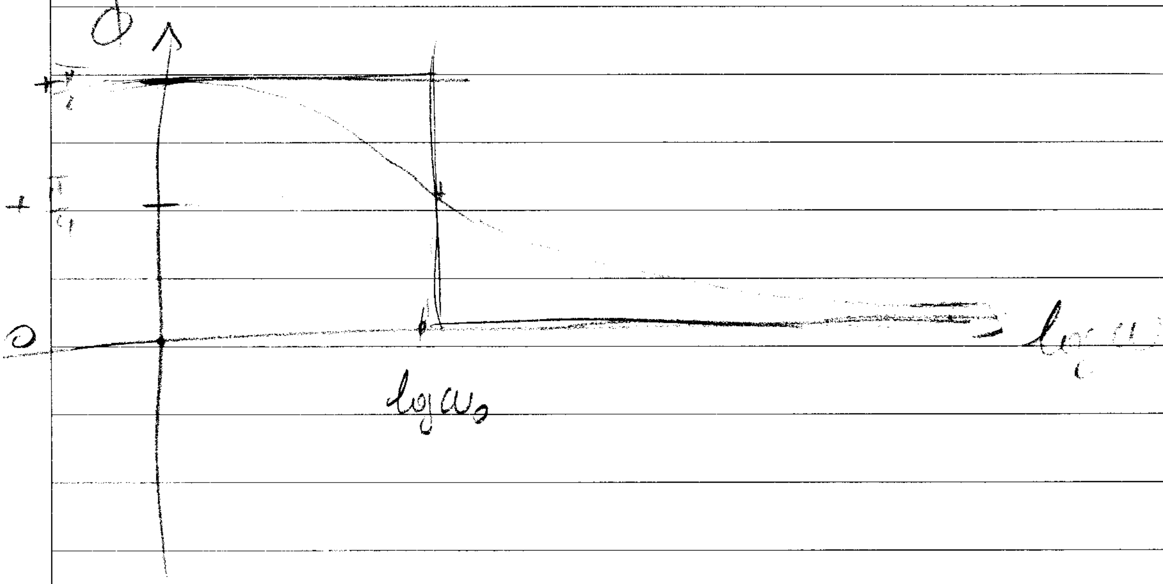
$$\tan \phi = \frac{\omega_0}{\omega}$$

$$\lim_{\omega \to 0} \tan \phi = +\infty \Rightarrow \phi \to +\frac{\pi}{2}$$

$$\lim_{\omega \to +\infty} \tan \phi = 0 \Rightarrow \phi \to 0$$

(ne pas écrire dans cette marge)

pour $\omega = \omega_0$ $\tan \phi = 1$ $\Rightarrow$ $\phi = +\frac{\pi}{4}$



[Exercice n° 3]

19

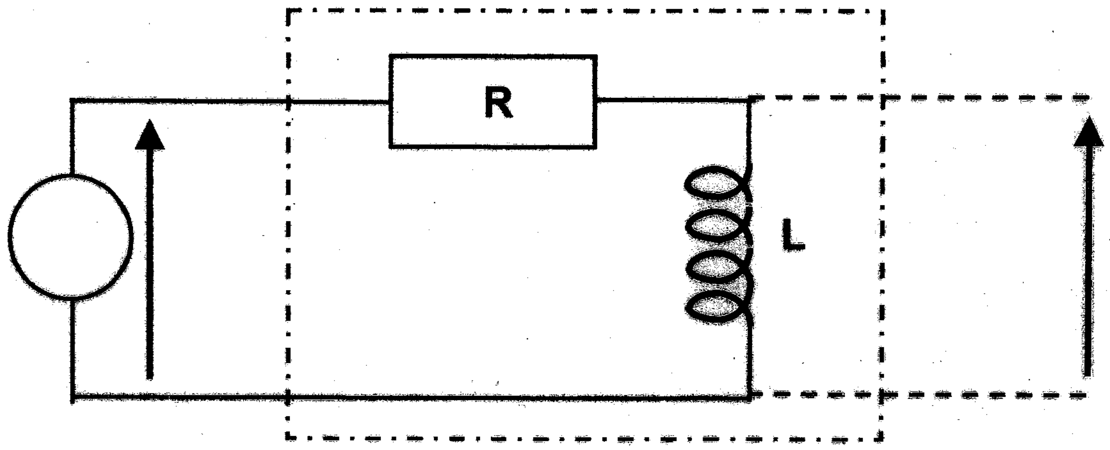
II-1

III)PRODUIT DE FONCTIONS DE TRANSFERT :

III-3 : Exemples de fonctions : $\underline{H} = \dfrac{j\omega/\omega_0}{1 + j\omega/\omega_0} = \underline{H_1}.\underline{H_2}$

III-3-1 : Exemple 1 : circuit R,C de II-2 ($\omega_0 = 1/RC$)

III-3-2 : Exemple 2 : circuit R,L

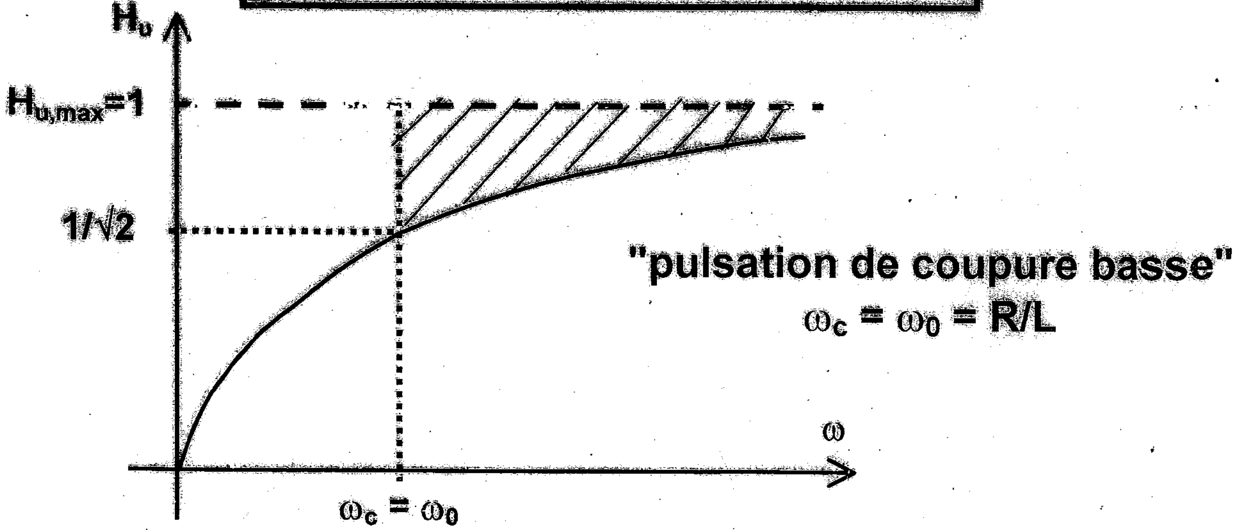


- fonction de transfert complexe en tension

$$\boxed{\underline{H_u} = \frac{\underline{U_s}}{\underline{U_e}} = \frac{jL\omega}{R + jL\omega} = \frac{j\omega/\omega_0}{1 + j\omega/\omega_0} \quad \text{avec} \quad \omega_0 = \frac{R}{L}}$$

- coefficient d'amplification en tension A_u

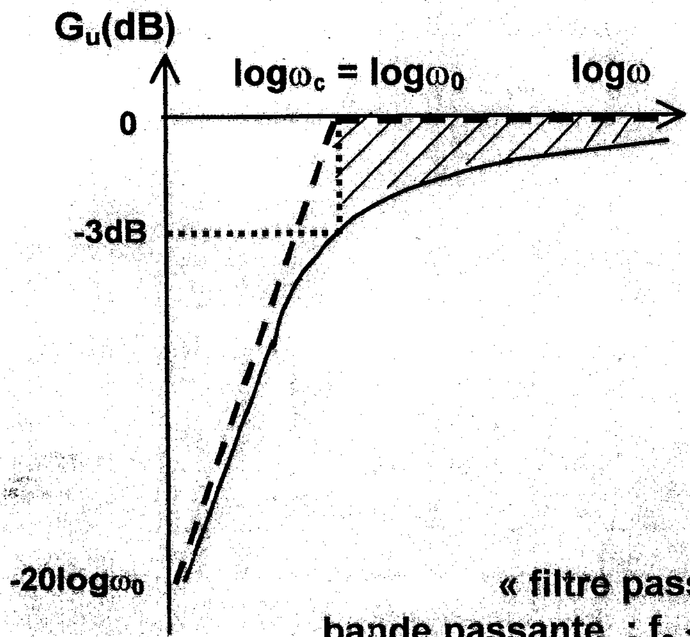
$$\boxed{H_u = \frac{L\omega}{\sqrt{R^2 + L^2\omega^2}} = \frac{\omega/\omega_0}{\sqrt{1 + \omega^2/\omega_0^2}}}$$

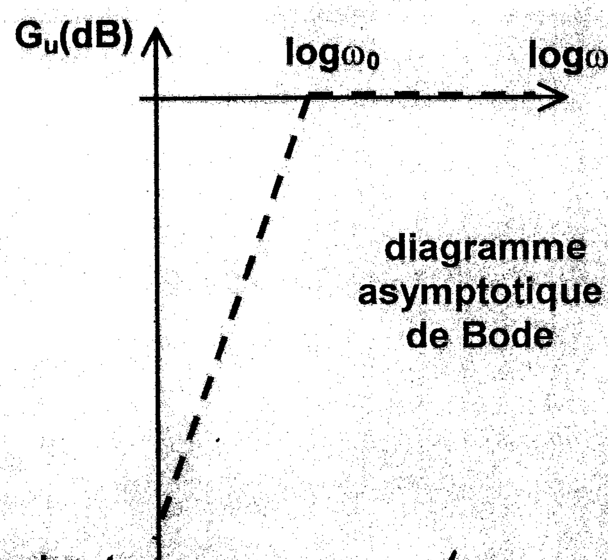


III-3-2 : Exemple 2 : circuit R,L (suite)

• gain G_u (dB)

$$\boxed{G_u(\mathrm{dB}) = 20\log\left[\frac{\omega/\omega_0}{(1+\omega^2/\omega_0^2)^{1/2}}\right]}$$



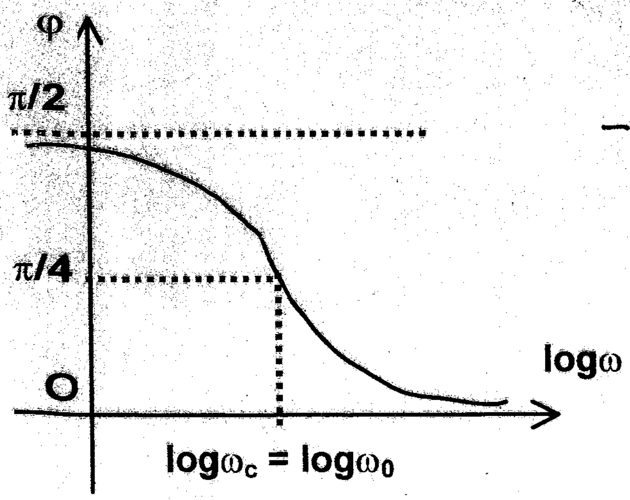


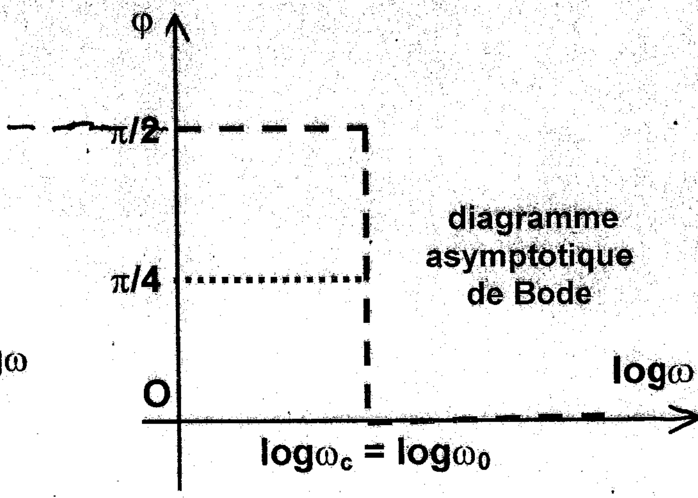
« filtre passe-haut »

bande passante : $f_c \leftrightarrow \infty$ avec $f_c=\omega_c/2\pi$ $= f_0$

• déphasage φ de u_s par rapport à u_e : $\underline{H} = \frac{jL\omega}{R+jL\omega} = \frac{1}{1-j\frac{R}{L\omega}}$

$$\boxed{\varphi = \arg \underline{H}\,\cancel{\underline{T}_u} = \mathrm{Arc}\tan\left(\frac{R}{L\omega}\right) = \mathrm{Arc}\tan\left(\frac{\omega_0}{\omega}\right)}$$



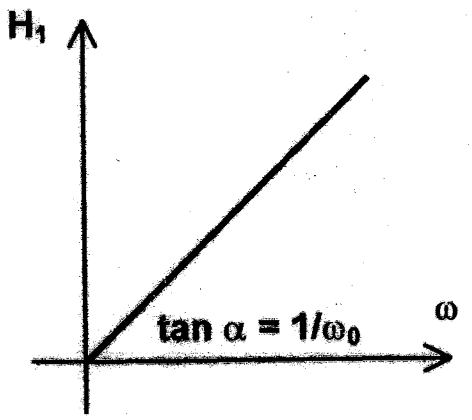


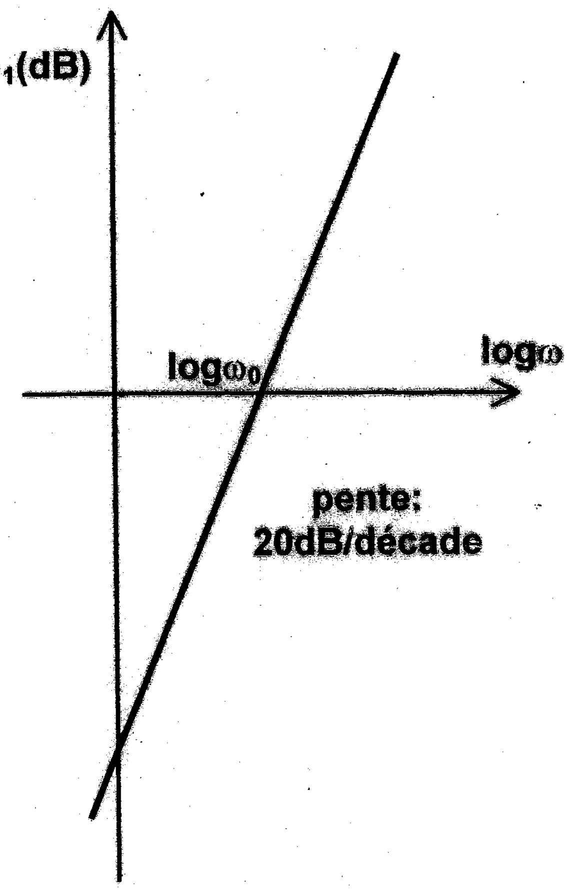
III)PRODUITS DE FONCTIONS DE TRANSFERT :

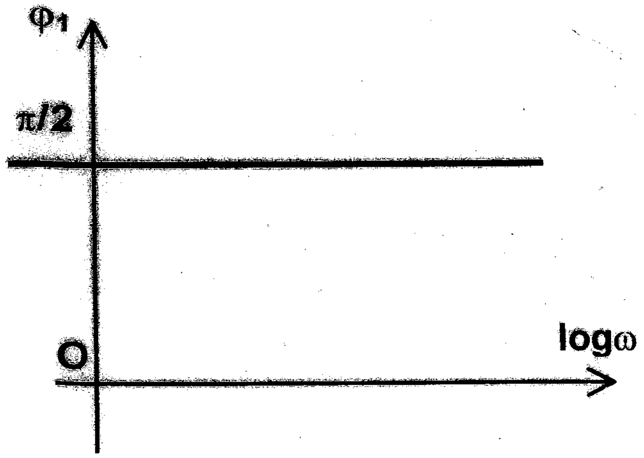
III-1 : Fonction : $\underline{H_1} = j\dfrac{\omega}{\omega_0}$

- coefficient d'amplification H ; gain G ; phase φ

$$\boxed{H_1 = \frac{\omega}{\omega_0} \quad ; \quad G_1(dB) = 20\log\omega - 20\log\omega_0 \quad ; \quad \varphi_1 = \pi/2}$$







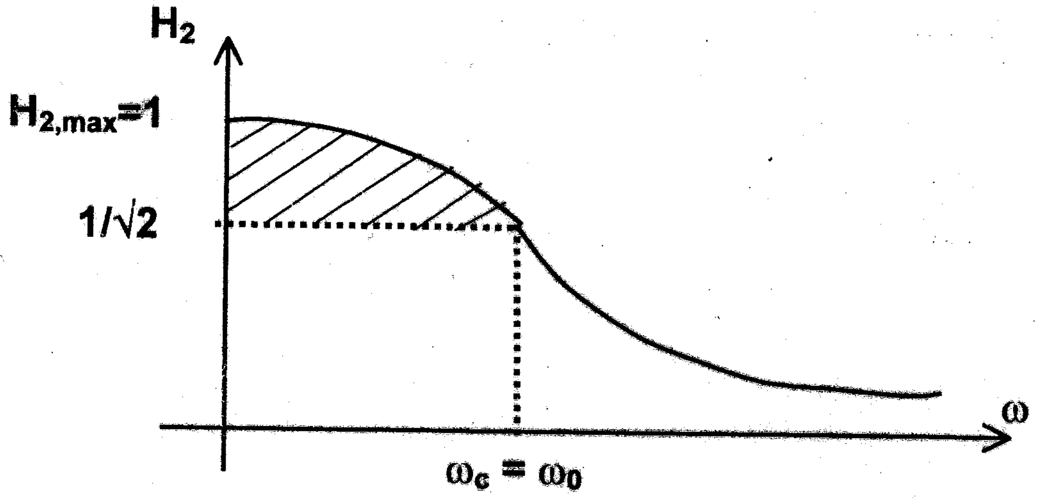
diagrammes

III)PRODUIT DE FONCTIONS DE TRANSFERT :

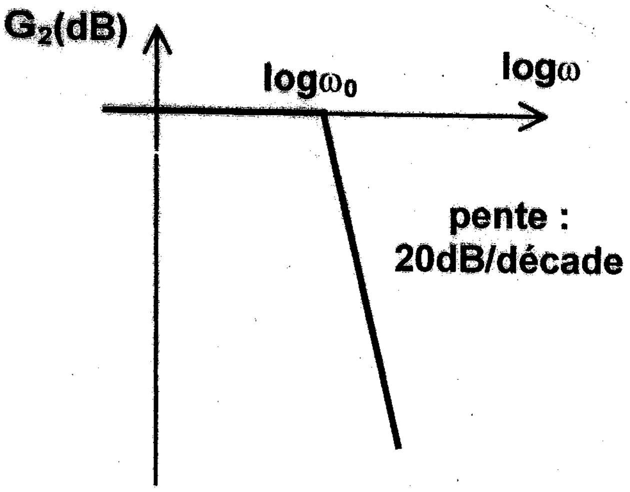
III-2 : Fonction : $\underline{H}_2 = \dfrac{1}{1 + j\dfrac{\omega}{\omega_0}}$

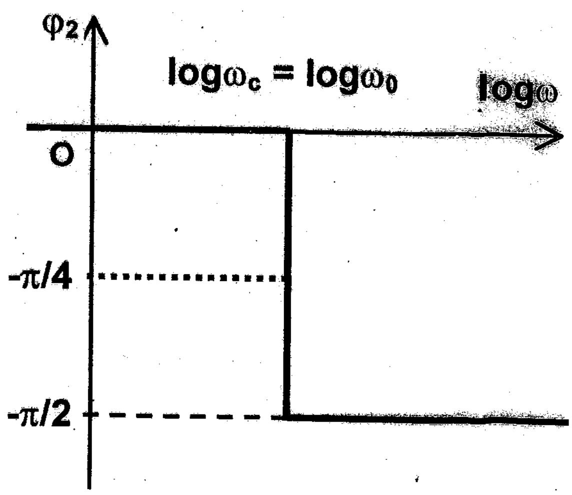
• coefficient d'amplification H ; gain G ; phase φ

$$H_2 = \frac{1}{\sqrt{1 + \frac{\omega^2}{\omega_0^2}}} \; ; \; G_2(dB) = -10\log\left(1 + \frac{\omega^2}{\omega_0^2}\right) \; ; \; \varphi_2 = -\arctan\left(\frac{\omega}{\omega_0}\right)$$



diagrammes asymptotiques
de Bode :

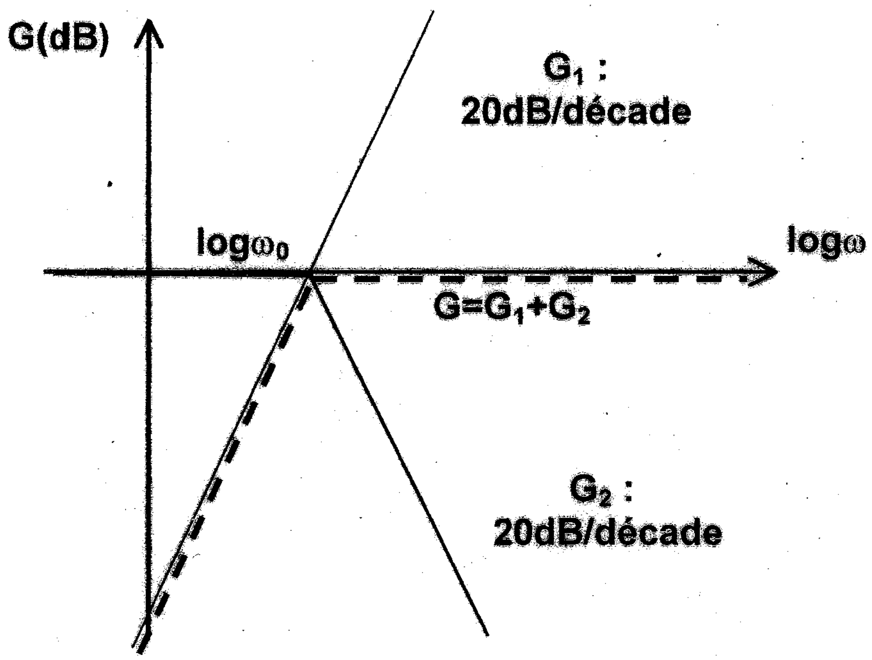


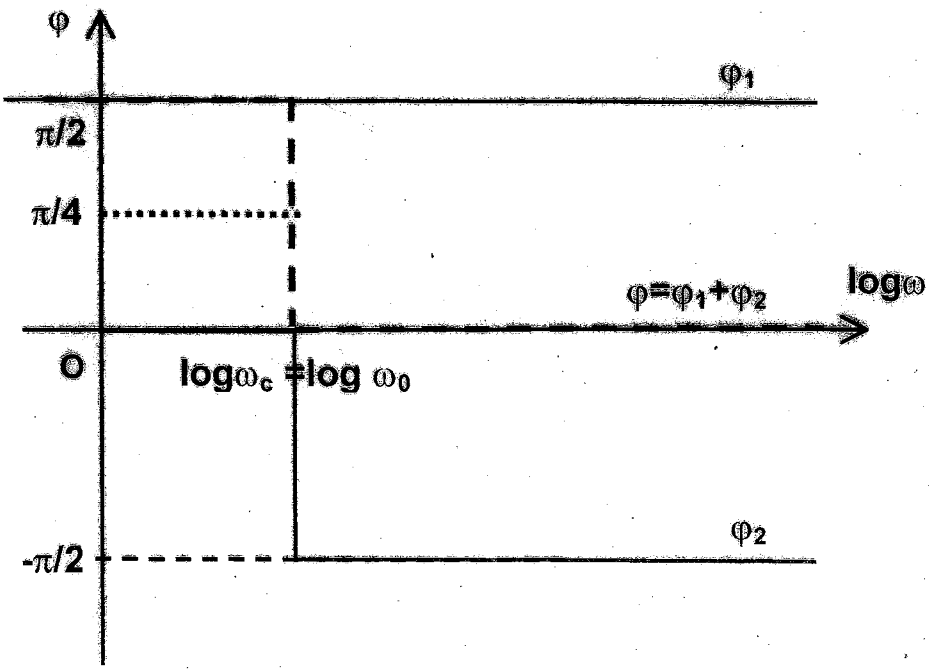


III-3 : Fonction : $\underline{H} = \dfrac{j\dfrac{\omega}{\omega_0}}{1+j\dfrac{\omega}{\omega_0}} = \underline{H_1}.\underline{H_2}$

- coefficient d'amplification H ; gain G ; phase φ

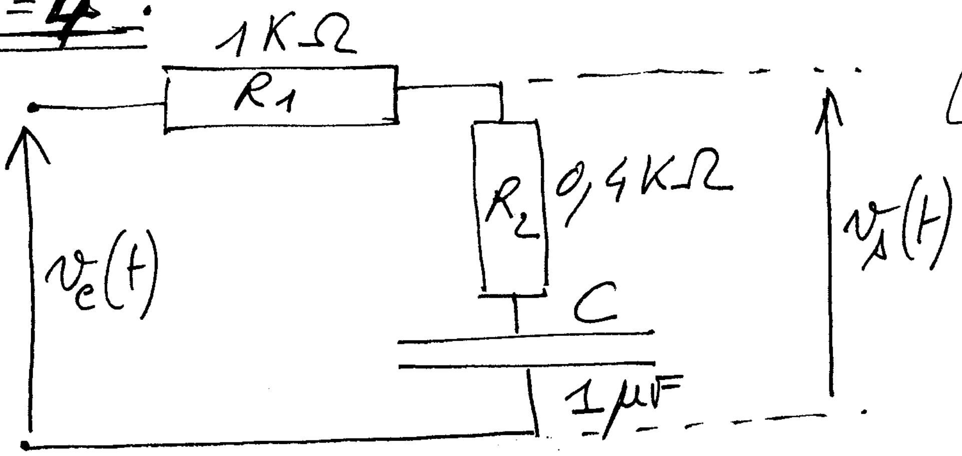
$$\boxed{H = H_1.H_2 \quad ; \quad G(dB) = G_1(dB) + G_2(dB) \quad ; \quad \varphi = \varphi_1 + \varphi_2}$$





Exercice n° 4 :

4-1



•) $\underline{\alpha} = \frac{R_1}{R_2} = \frac{1}{0,4} = \underline{2,5}$

$\underline{\tau} = R_2 C = 400\,10^{-6} = \underline{0,4\,ms}$

$$\underline{H} = \frac{V_s}{V_e} = \frac{R_2 - \frac{j}{C\omega}}{(R_1+R_2) - \frac{j}{C\omega}} = \frac{1 - \frac{j}{R_2 C\omega}}{\left(1+\frac{R_1}{R_2}\right) - \frac{j}{R_2 C\omega}}$$

$$\Rightarrow \underline{H} = \frac{1 - \frac{j}{\tau\omega}}{(1+\alpha) - \frac{j}{\tau\omega}} = \frac{\tau\omega - j}{(1+\alpha)\,\tau\omega - j}$$

$\underline{H_1} = \tau\omega - j = 4.10^{-4}\omega - j$

$\underline{H_2} = \underbrace{(1+\alpha)}_{3,5}\tau\omega - j = 14\,10^{-4}\omega - j$

$$\boxed{\underline{H} = \frac{\underline{H_1}}{\underline{H_2}} = \frac{4\,10^{-4}\omega - j}{14.10^{-4}\omega - j}}$$

$\underline{H_1} = 4.10^{-4}\omega - j$: $\begin{cases} A_1 = \sqrt{1+16.10^{-8}\omega^2} \;;\; G_1(dB) = 10\log(1+16\,10^{-8}\omega^2) \\ \varphi_1 = \text{Arctan}\left(\frac{-1}{4\,10^{-4}\omega}\right) = -\text{Arctan}\left(\frac{1}{4\,10^{-4}\omega}\right) \end{cases}$

$\underline{H_2} = 14.10^{-4}\omega - j$: $\begin{cases} A_2 = \sqrt{1+196\,10^{-8}\omega^2} \;;\; G_2(dB) = 10\log(1+196\,10^{-8}\omega^2) \\ \varphi_2 = \text{Arctan}\left(\frac{-1}{14\,10^{-4}\omega}\right) = -\text{Arctan}\left(\frac{1}{14\,10^{-4}\omega}\right) \end{cases}$

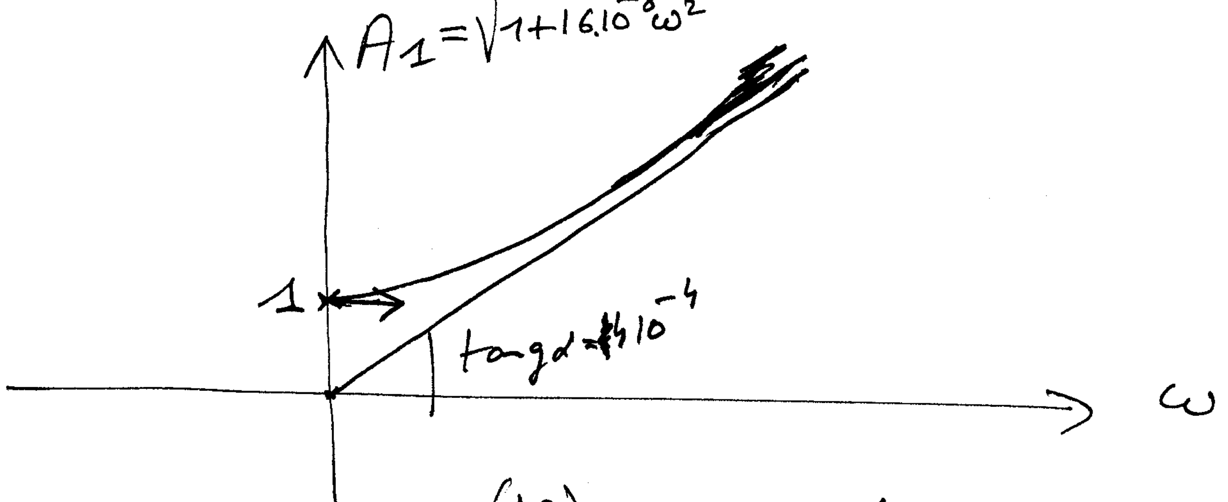
$\boxed{G = G_1 - G_2}$ et $\boxed{\varphi = \varphi_1 - \varphi_2}$

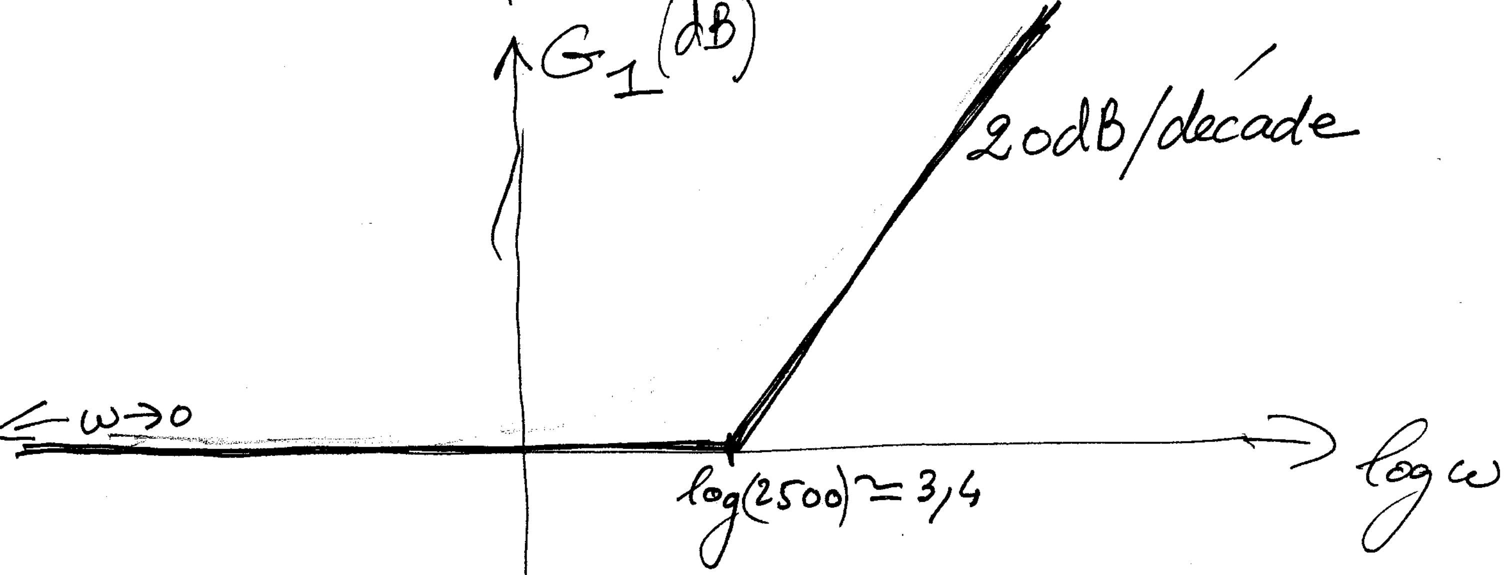
diagrammes asymptotiques :

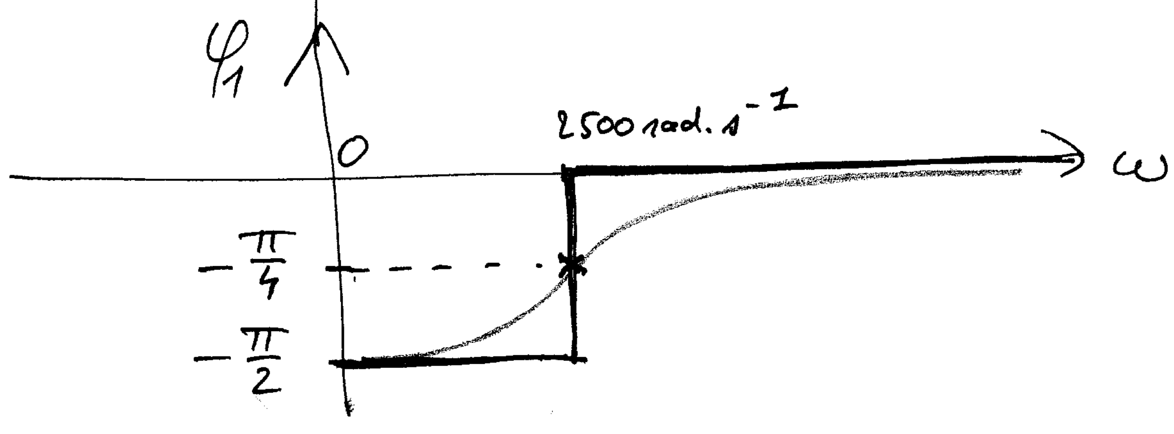
$$G_1 : \begin{cases} \omega = 0 \Rightarrow G_1 = 0 \\ \omega \to \infty \Rightarrow G_1 \to 20 \log(4.10^{-4}\omega) = 20 \log\left(\frac{\omega}{2500}\right) \end{cases}$$

$$\boxed{y_{1,\infty} = 20(\log\omega - 3{,}4)}$$

$$\varphi_1 : \begin{cases} \omega = 0 \Rightarrow \varphi_1 \to -\frac{\pi}{2} \\ \omega = 2500 \text{ rad.s}^{-1} \Rightarrow \varphi_1 = -\frac{\pi}{4} \\ \omega \to \infty \Rightarrow \varphi_1 \to 0 \end{cases}$$



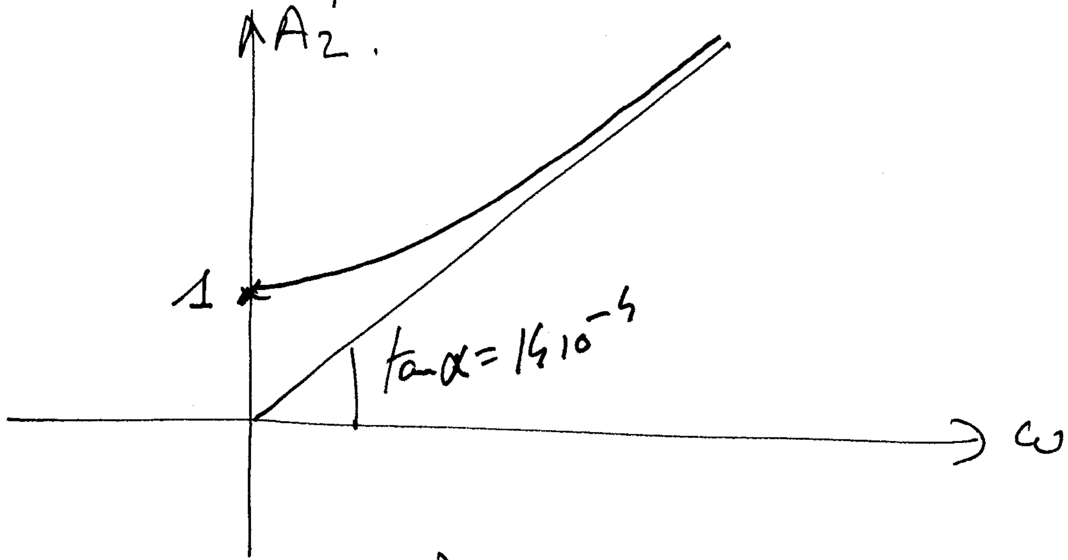


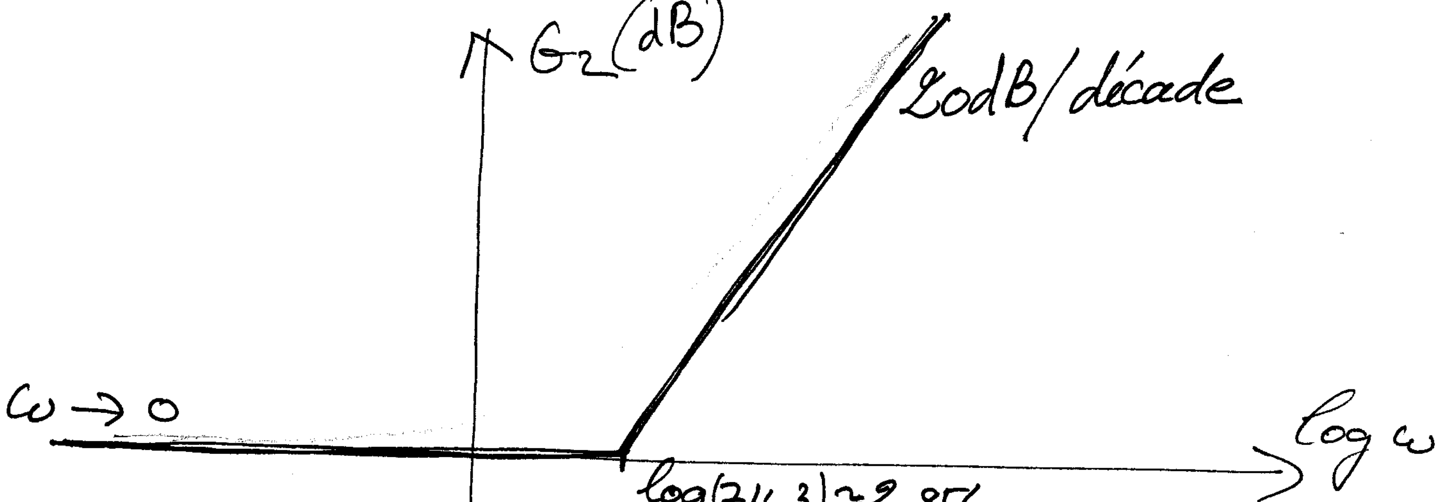


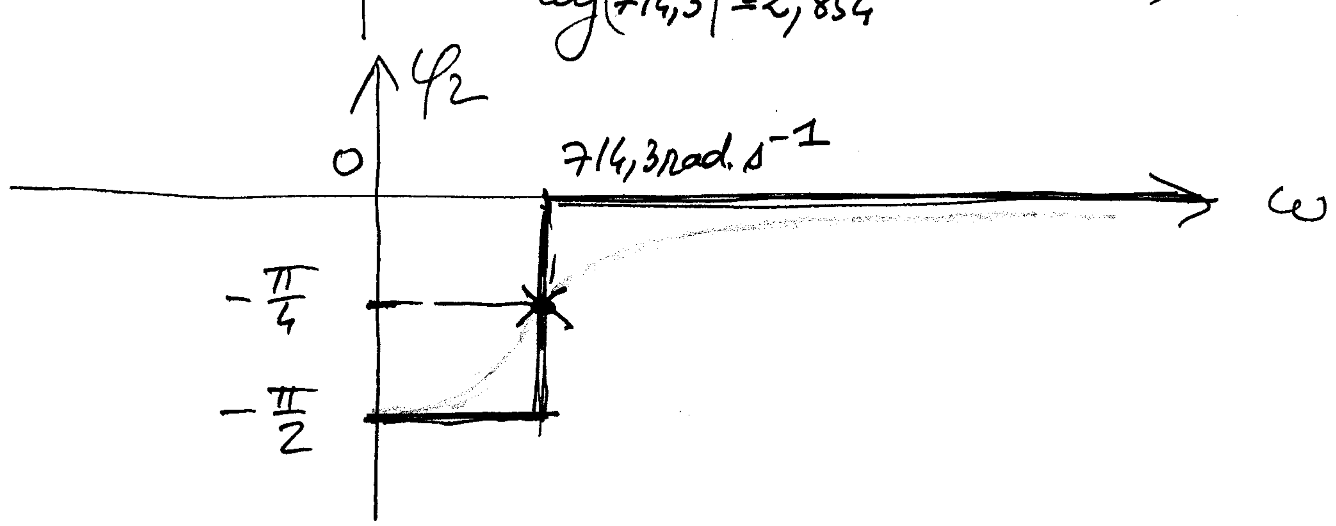
$$G_2 : \begin{cases} \omega = 0 \Rightarrow G_2 = 0 \\ \omega \to \infty \Rightarrow G_2 \to 20 \log(14\,10^{-4}\omega) = 20 \log\left(\dfrac{\omega}{714,3}\right) \end{cases}$$

$$y_{2,\infty} = 20(\log \omega - 2,854)$$

$$\varphi_2 : \begin{cases} \omega = 0 \Rightarrow \varphi_2 \to -\dfrac{\pi}{2} \\ \omega = 714,3 \Rightarrow \varphi_2 = -\dfrac{\pi}{4} \\ \omega \to \infty \Rightarrow \varphi_2 \to 0 \text{ (par val} < 0) \end{cases}$$



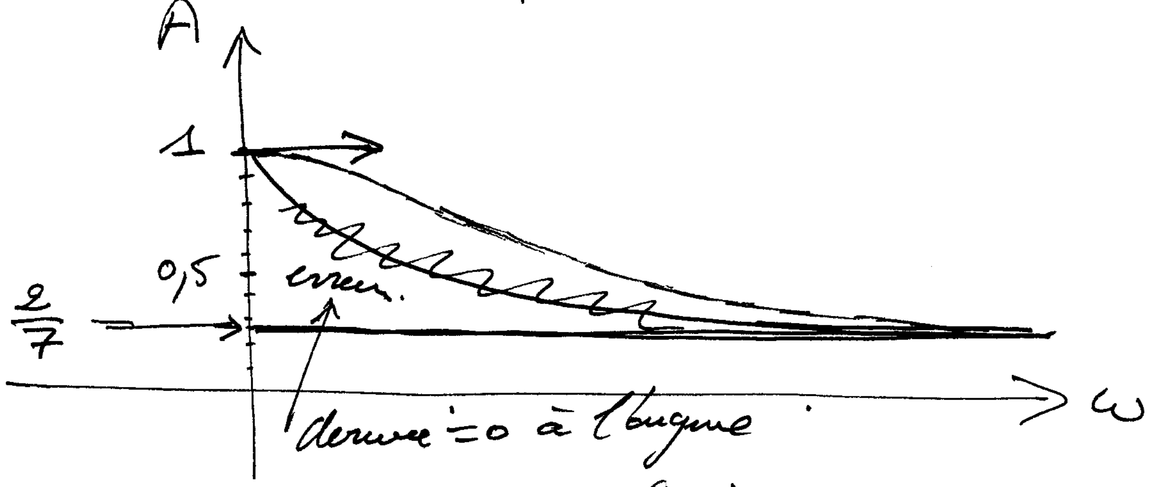


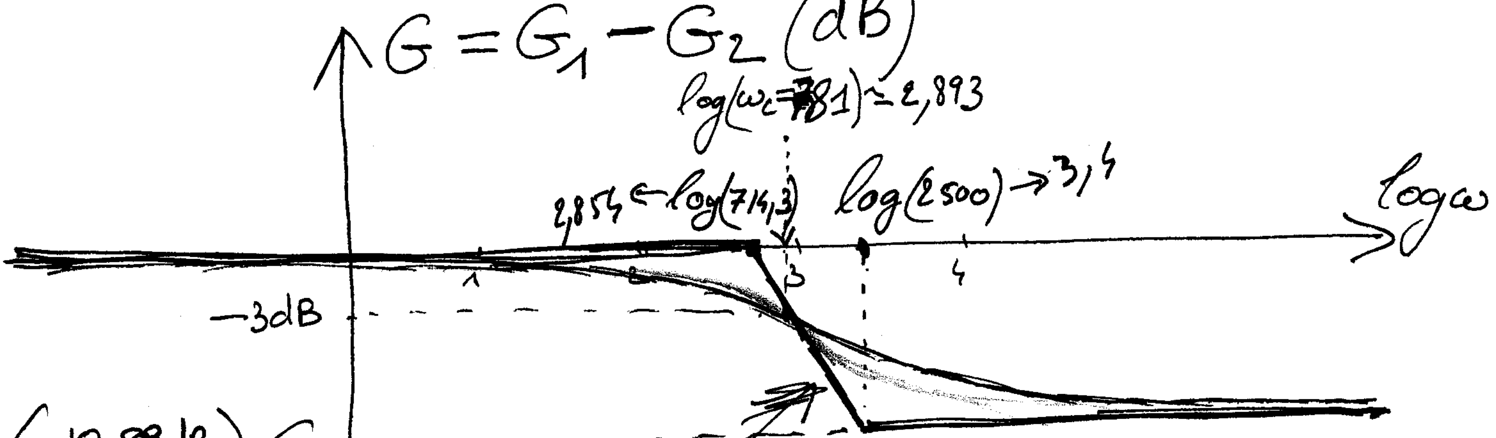


$$A = \frac{\sqrt{1+16.10^{-8}\,\omega^2}}{\sqrt{1+196.10^{-8}\,\omega^2}}$$

$\omega \to 0 \Rightarrow A \to 1$

$\omega \to \infty \Rightarrow A \to \frac{4\,10^{-4}\omega}{14\,10^{-4}\omega} = \frac{2}{7} \# 0,286$





(−10,88dB). $G_m = +20\log\left(\frac{2}{7}\right) = -10,88$ dB

"passe-bas" : 0 — 781 rad.s^{-1}

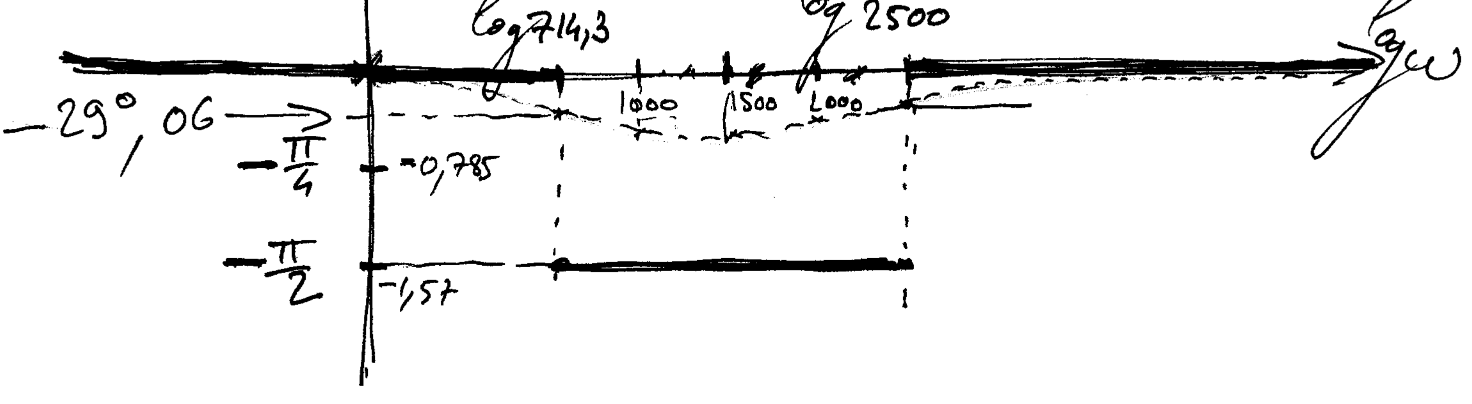
$G_m = -20\log\left(\frac{2500}{714,3}\right) = -20\log(3,5)$

$G_m = -10,88$ dB

$\boxed{\log\omega_0 = \frac{\log\omega_1 + \log\omega_2}{2}} \Rightarrow 2\log\omega_0 = \log\omega_1\omega_2.$

$\omega_0^2 = \omega_1\omega_2$

$\boxed{\omega_0 = \sqrt{\omega_1\omega_2}}$



4-5 (11)

$A_{max} = 1$

$\frac{A_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$ pour $\omega = \omega_c$

$$\frac{1 + 16.10^{-8}.\omega_c^2}{1 + 196.10^{-8}.\omega_c^2} = \frac{1}{2}$$

$$2\left(1 + 16.10^{-8}.\omega_c^2\right) = 1 + 196.10^{-8}\omega_c^2$$

$$164.10^{-8}\omega_c^2 = 1 \Rightarrow \omega_c^2 = \frac{10^8}{164}$$

$$\Rightarrow \underline{\omega_c} = \frac{10^4}{\sqrt{164}} \simeq \underline{781 \text{ rad.s}^{-1}}$$

$\omega = 1000$ rad.s^{-1} : $\underline{H} = \frac{0,4 - j}{1,4 - j}$ $\frac{\angle -1,190}{\angle -0,620} = -0,570$

$\omega = 1500$: $\underline{H} = \frac{0,6 - j}{2,1 - j}$ $\frac{\angle -1,0304}{\angle -0,4444} = -0,586$

$\omega = 2000$: $\underline{H} = \frac{0,8 - j}{2,8 - j}$ $\frac{\angle -0,89606}{\angle -0,34302} = -0,553$

$\omega = 500$: $\underline{H} = \frac{0,2 - j}{0,7 - j}$ $\frac{\angle -1,3734}{\angle -0,9601} = -0,4133$

$\omega = 2500$: $\underline{H} = \frac{1 - j}{3,5 - j}$ $\frac{\angle -0,7854}{\angle -0,2783} = -0,507$

Exercice n°5 :

4 (12)

$\underline{H} = \dfrac{V_s}{V_e}$ ~~$L//C$ ...~~

5-1

1)

$$\underline{H} = \frac{\underline{Z}_{L,C}}{R+\underline{Z}_{L,C}} = \frac{1}{1+R\,\underline{Y}_{L,C}} = \frac{1}{1+R\left(jC\omega - \frac{j}{L\omega}\right)}$$

$$\boxed{\underline{H} = \frac{1}{1+j\left(RC\omega - \frac{R}{L\omega}\right)} = \frac{H_0}{1+j\varphi\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}}$$

$\longleftarrow$ avec $H_0 = 1$

identification $\left.\begin{array}{l}\dfrac{\varphi}{\omega_0} = RC \\ \varphi\omega_0 = \dfrac{R}{L}\end{array}\right\}$ $\varphi^2 = R^2\dfrac{C}{L}$, $\varphi = R\sqrt{\dfrac{C}{L}}$

$\Rightarrow \omega_0 = \dfrac{\varphi}{RC} = \dfrac{1}{\sqrt{LC}}$

donc : $\boxed{\underline{H} = \dfrac{H_0}{1+j\varphi\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}}$ avec $\boxed{H_0 = 1}$ $\boxed{\omega_0 = \dfrac{1}{\sqrt{LC}}}$ $\boxed{\varphi = R\sqrt{\dfrac{C}{L}}}$

avec $\underline{\varphi = 4}$

$$\boxed{\underline{H} = \frac{1}{1+4j\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}} \quad \boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \quad \boxed{R\sqrt{\frac{C}{L}} = 4}$$

2) $$H = \frac{1}{\sqrt{1+16\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}}$$

V-2 (13)

$$\frac{dH}{d\omega} = \underbrace{-\frac{1}{2}}_{<0}\underbrace{\left[1+16\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2\right]^{-3/2}}_{>0} 32\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)\underbrace{\left(\frac{1}{\omega_0}+\frac{\omega_0}{\omega^2}\right)}_{>0}$$

$= 0$ si $\omega = \omega_0$
< 0 si $\omega > \omega_0$
> 0 si $\omega < \omega_0$

$\omega \to 0$

$\frac{dH}{d\omega} \to -\frac{1}{2}\frac{\left(-32\frac{\omega_0^2}{\omega^3}\right)}{\left(4\frac{\omega_0}{\omega}\right)^3}$

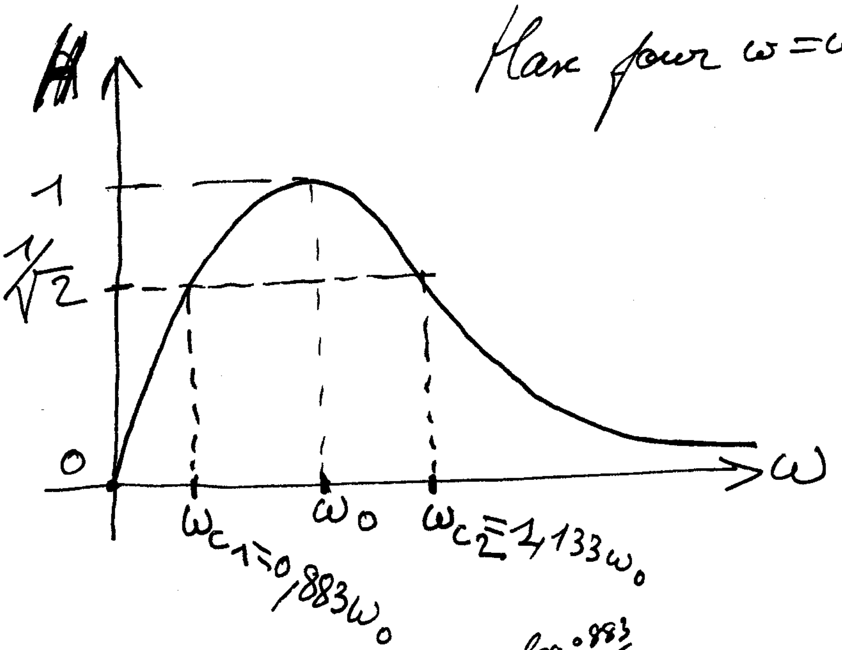
$\frac{1}{4\omega_0}$

Max. pour $\omega = \omega_0$

$\omega \to 0 \Rightarrow H \to 0$
$\omega \to \infty \Rightarrow H \to 0$

Max pour $\omega = \omega_0 \Rightarrow \boxed{H_{Max} = 1}$

$\hookrightarrow G \leq 0$



$\boxed{\omega \to 0}$:

$H \to \frac{\omega}{4\omega_0}$

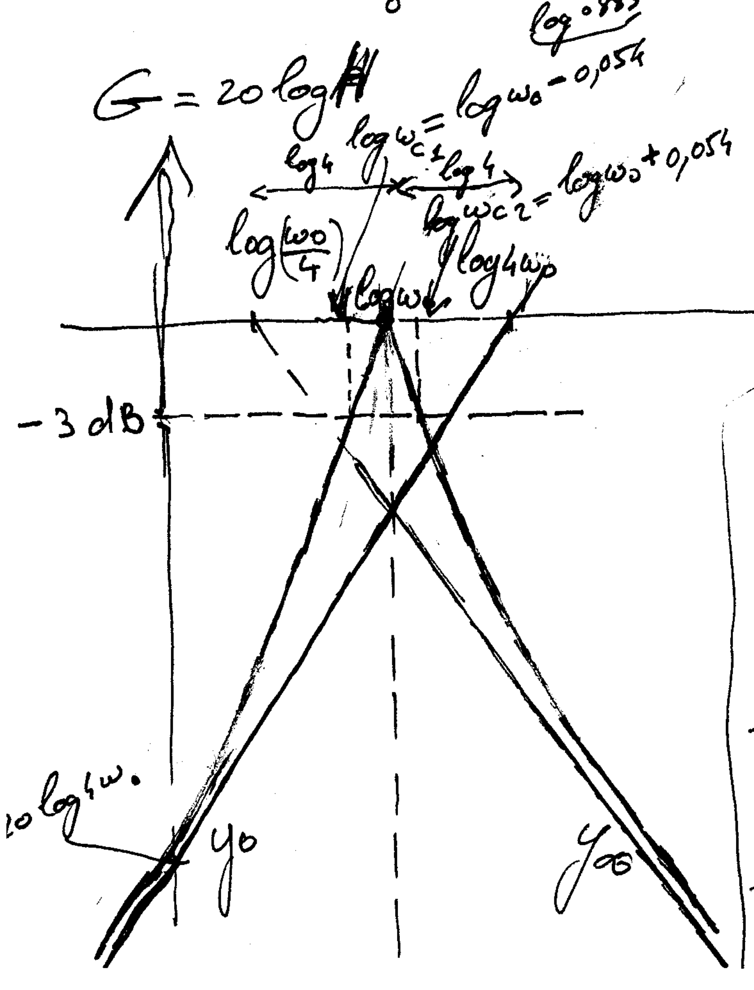
$\boxed{y_0 = 20(\log\omega) - 20\log(4\omega_0)}$

$\boxed{\omega \to \infty}$:

$H \to \frac{\omega_0}{4\omega}$

$\boxed{y_\infty = -20(\log\omega) + 20\log\left(\frac{\omega_0}{4}\right)}$

$G = 20\log H$



fréquences de coupure : ω_{c_1} et ω_{c_2} :

$$H = \frac{1}{\sqrt{2}} \Rightarrow 16\left(\frac{\omega_c}{\omega_0}-\frac{\omega_0}{\omega_c}\right)^2 = 1$$

$$\frac{\omega_c}{\omega_0}-\frac{\omega_0}{\omega_c} = \pm\frac{1}{4}$$

$\to \frac{\omega_c}{\omega_0}-\frac{\omega_0}{\omega_c} = +\frac{1}{4} \Rightarrow 4\omega_c^2 - \omega_c\omega_0 - 4\omega_0^2 = 0$

$\omega_c = \frac{\omega_0}{8}(1 \pm \sqrt{65}) \Rightarrow \boxed{\omega_{c_2} = 1,133\omega_0}$

$\to \frac{\omega_c}{\omega_0}-\frac{\omega_0}{\omega_c} = -\frac{1}{4} \Rightarrow 4\omega_c^2 + \omega_c\omega_0 - 4\omega_0^2 = 0$

$\omega_c = \frac{\omega_0}{8}(-1 \pm \sqrt{65}) \Rightarrow \boxed{\omega_{c_1} = 0,883\omega_0}$

5-3 (13')

$$\cancel{\varphi_2} \quad \underline{H} = \frac{1}{1+4j\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}$$

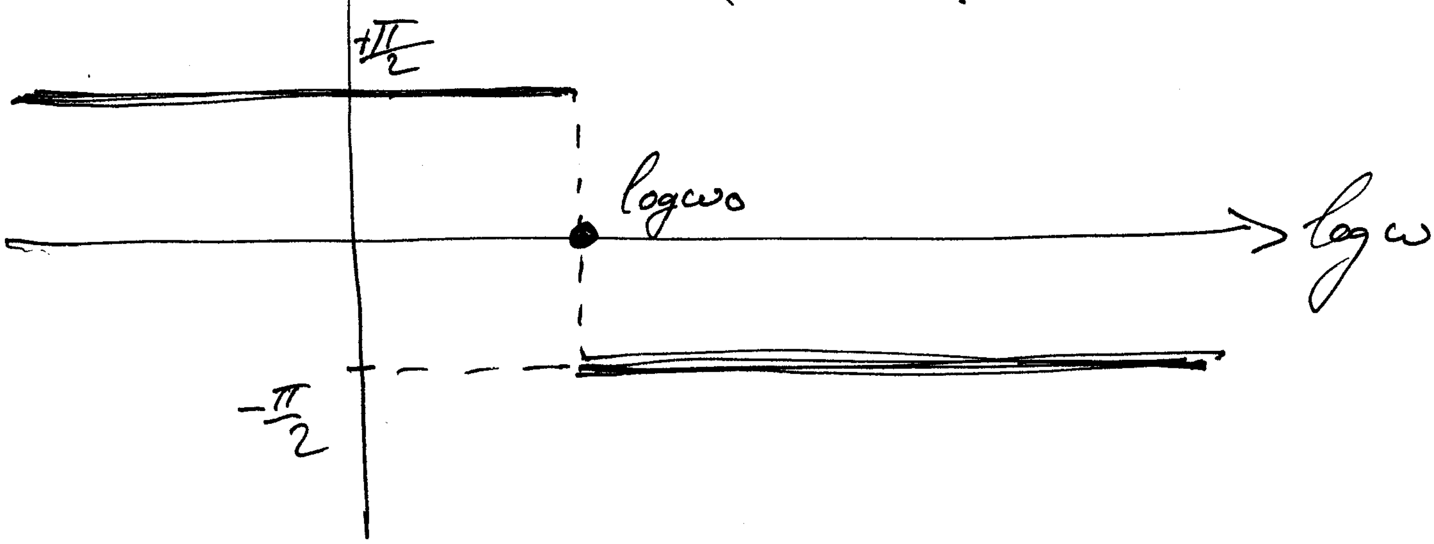
$$\boxed{\varphi = -\operatorname{Arctan}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}$$

$$\omega \to 0 \Rightarrow \varphi \to -\left(-\frac{\pi}{2}\right) = +\frac{\pi}{2}$$

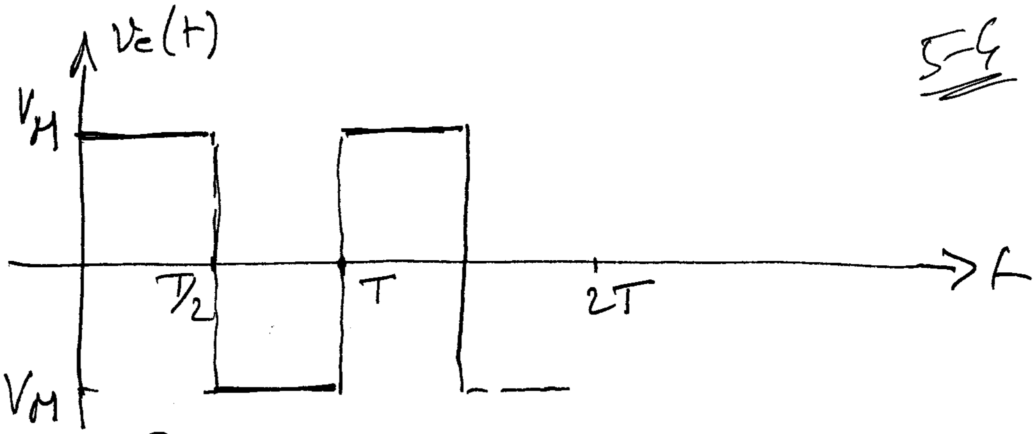
$$\omega \to \infty \Rightarrow \varphi \to -\left(\frac{\pi}{2}\right) = -\frac{\pi}{2}$$

$$\omega = \omega_0 \Rightarrow \varphi \Rightarrow 0$$

$$\varphi = -\operatorname{Arctan}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)$$



3)



Théorème de Fourier :

$$v_e(t) = \sum_{0}^{\infty} a_n \cos\left(n\frac{2\pi t}{T}\right) + b_n \sin\left(n\frac{2\pi t}{T}\right)$$

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} v_e(t)\,dt \; ; \quad a_n = \frac{2}{T}\int_{t_0}^{t_0+T} v_e(t)\cos(n\omega t)\,dt \; ; \quad b_n = \frac{2}{T}\int_{t_0}^{t_0+T} v_e(t)\sin(n\omega t)\,dt$$

fonction impaire $\Rightarrow \boxed{a_0 \text{ et } a_n = 0}$

$$b_1 = \frac{2}{T}\int_0^T v_e(t)\sin\frac{2\pi t}{T}\,dt = \frac{4V_M}{\pi}$$

$$b_2 = \frac{2}{T}\int_0^T v_e(t)\sin\frac{4\pi t}{T}\,dt = 0$$

$$b_3 = \frac{2}{T}\int_0^T v_e(t)\sin\frac{6\pi t}{T}\,dt = \frac{4V_M}{3\pi}$$

$$\Rightarrow \boxed{v_e(t) = \frac{4V_M}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \dots\right] \quad \omega = \frac{2\pi}{T}}$$

La question 3. est hors programme